Part A - Structured Essay

1. Figure (1) shows the motion of a simple pendulum of length ℓ.

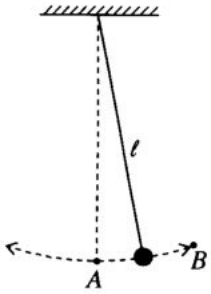


Figure (1)

(*a*) Write down an expression for the period of oscillation T of the simple pendulum in terms of ℓ and acceleration due to gravity, g.

(*b*) In the laboratory experiment to find the value of g, using the simple pendulum you are provided with a stop-watch which can measure the time with an accuracy of 0.5 s. If the estimated value of the period T is 2 s, determine the minimum number of oscillations you should take to reduce the percentage error of T down to 1%.

(*c*) A student has designed an electrical method to determine the period of oscillation T more accurately by using a 'detector system'.

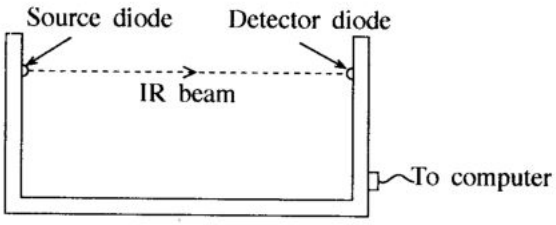
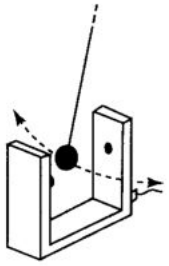


Figure (2)(*a*)


Figure (2)(*b*)

The detector system consists of a source diode and a detector diode. Source diode, emits a narrow beam of Infra-Red (IR) light with a constant intensity of I_0. This light beam is detected by the detector diode, and it also measures the intensity of the beam [see figure (2)(*a*)]. Detector system is placed in the path of the bob of the simple pendulum. While oscillating, the bob also crosses the IR beam [see figure (2)(*b*)]. Whenever the bob interrupts the IR beam, the detector diode signal becomes zero, otherwise it produces a signal of constant intensity I_0. When the bob is oscillating, the computer monitor displays a graph of the variation of the detector signal intensity (I) with time (t).

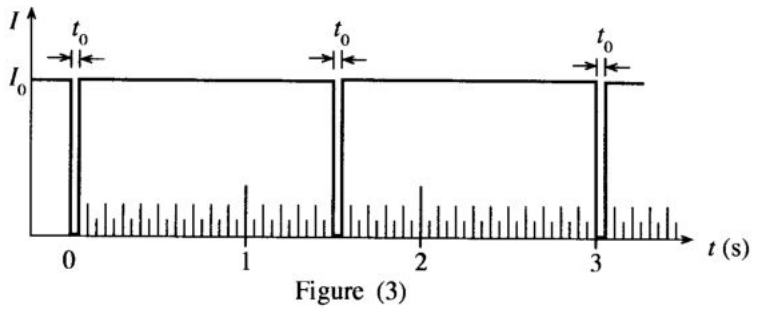


Figure (3)

Figure (3) shows such a graph displayed on the computer screen and is taken in a situation where the force due to **air drag is negligible**. The time interval corresponding to the zero detector signal is t_0 (see figure).

(i) Value of t_0 depends on the speed v with which the bob crosses the IR beam, and the diameter D of the bob. What will happen to the value of t_0 when (1) v is increased (2) D is increased?

(ii) Write down an expression to estimate v in terms of D and t_0.

(iii) What is the value of T according to the graph given in figure (3) above?

(*d*) Student placed the detector system at the most appropriate position of the path of the bob to determine the **maximum speed** v_m of the bob, and obtained a graph similar to the graph shown in figure (3).

(i) With respect to figure (1) above, give the position (*A* or *B*) at which the student should keep the detector system in order to determine v_m. Give a reason for your choice.

(ii) In order to carry out this experiment, the student says that the cylindrical bob shown in figure (4)(*a*) is better than the spherical bob shown in figure (4)(*b*). If both bobs have same diameter D, give a reason to justify his statement.

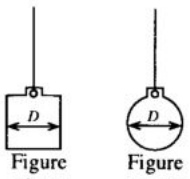


Figure (4)(*a*) Figure (4)(*b*)

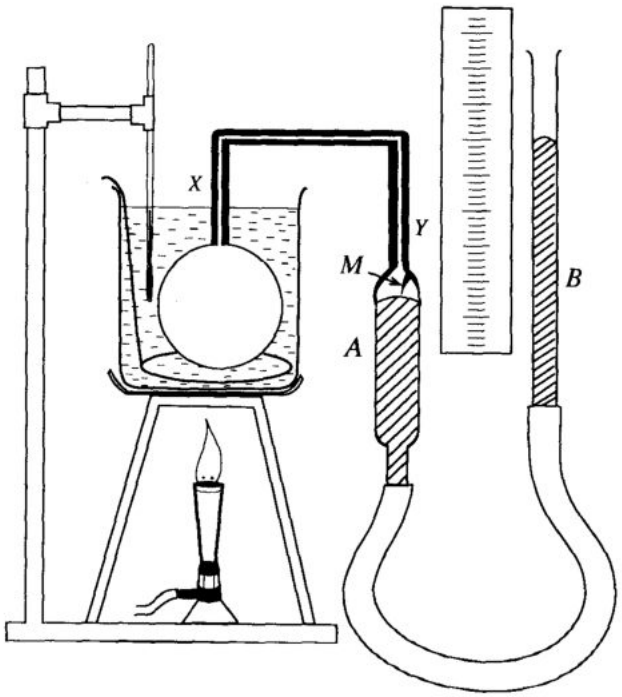
(iii) The student decided to calculate v_m using the graph mentioned above, and the expression in (*c*)(ii). Can he get the **exact** value for v_m by this method? Explain your answer.

(*e*) The student observed that in a situation where the force due to air drag is significant, the maximum speed v_m that he obtained, decreases considerably from oscillation to oscillation and the bob finally comes to rest.

(i) For such a situation, complete the graph of (I) with (t) that you would expect for a period of time T on the figure given below.

(ii) If the maximum speeds of the bob at $t = 0$ and $t = T$ are 0.44 m s^{-1} and 0.42 m s^{-1} respectively, estimate the energy loss of the pendulum due to air drag during the time $t = 0$ to $t = T$. Mass of the bob is 100 g.

2.



The experimental setup shown in the above figure is used to verify the pressure law for a gas.

(*a*) The pressure law can be applied to a gas only if two variable quantities pertaining to the gas are kept constant. What are those quantities?

(*b*) What is the reason for using the capillary tube *XY* in this setup?

(*c*) Explain why it is necessary to increase the temperature of the water bath slowly in this experiment.

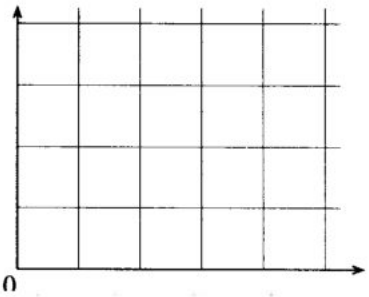
(*d*) Even if the temperature of water is maintained at a certain value it does not mean that the temperature of the gas inside the bulb has reached the same value. In this experiment, how would you make sure that the temperature of the gas inside the bulb has reached the temperature of

(*e*) Write down the **two** main steps used in the experimental procedure to maintain the temperature of water at a suitable value before measuring that temperature in this experiment.

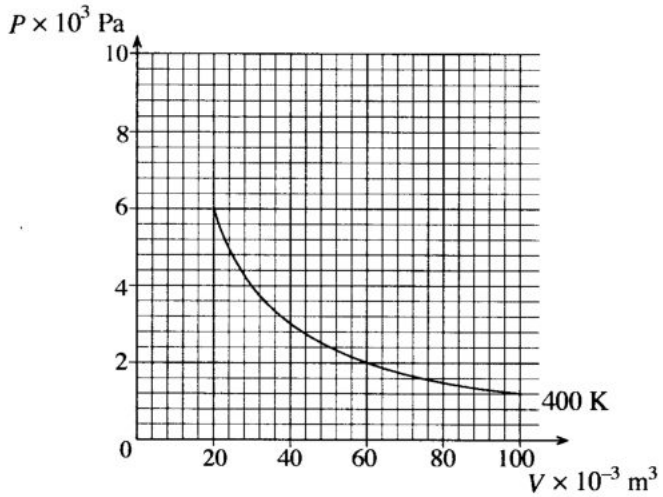
(*f*) Write down the main step in the experimental procedure that you would follow before taking the relevant readings to obtain the pressure of the gas.

By **moving the tube** B up and down until the mercury level in tube touches the tip of M / fixed mark / pointer ..... **(01 mark)**

(*g*) If the atmospheric pressure is H centimetres of mercury and the height difference of the two mercury levels of the tubes A and B is h centimetres, draw a rough sketch of the graph that you would plot in the given diagram in order to verify the pressure law. Label the axes correctly.



(*h*) The graph below shows the variation of pressure P with volume V for an ideal gas at temperature 400 K.



(i) Calculate the values P_1 and P_2 of pressures corresponding to the volumes $20 \times 10^{-3}\ m^3$ and $60 \times 10^{-3}\ m^3$ of the gas at temperature 600 K.

(ii) Mark the points corresponding to the values that you have obtained in (*h*)(i) above, on the graph given under (*h*) above and draw a rough sketch of a curve to show the variation of the pressure with volume at 600 K of the gas on the same graph.

3. You are asked to determine the focal length of a convex lens experimentally using the no-parallax method. Assume that you are provided with all the items necessary to carry out this experiment.

(*a*) Draw a diagram to show how you would setup all the necessary items on the table to carry out this experiment and label the items. (Stands on which the items are mounted should be clearly drawn.)

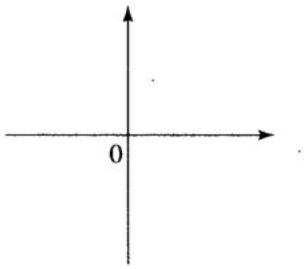
(*b*) Before setting up the items for the experiment, it is convenient to know a certain data pertaining to a certain item given. What is this data? Describe a simple method to obtain an approximate value for this data.

(*c*) Suppose that when looking at the image after setting up all the items as indicated in (*a*), you have observed that the image and the observation pin are not in the same vertical line. Give **two** reasons, one related to the pins and the other related to the lens, as to why this has happened.

(*d*) In this experiment, suppose you have observed that, as the eye is moved sideways across the optical axis, the image moves opposite to the direction of the eye movement. In this situation, state whether the observation pin should be moved towards the eye or away from the eye in order to locate the exact position of the image.

(*e*) If the object distance, image distance and the focal length of the convex lens are u, v and f respectively, rearrange the lens formula in order to determine the focal length of the lens by plotting a linear graph. State the sign convention that you have used for the lens formula.

(*f*) Mark the independent variable of the equation obtained in (*e*) above on the horizontal axis and the dependent variable on the vertical axis of the given diagram.



(*g*) Draw a rough sketch of the expected graph on the same diagram. Use the signs for the object distance and image distance according to the sign convention used in (*e*).

4. (*a*) An incomplete diagram of a potentiometer circuit that is being used in the laboratory to determine the internal resistance r_0 of a standard cell of e.m.f. E_0 ($< E$) is shown in figure (1).

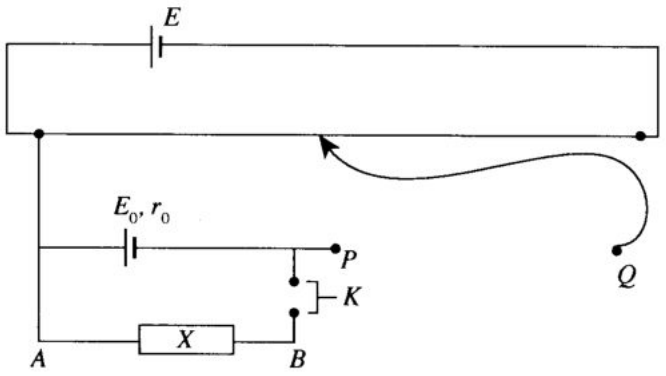


Figure (1)

(i) Complete the section of the circuit between P and Q using **standard circuit symbols**.

(ii) What is the item used in the laboratory for X to obtain a resistance R?

(iii) If ℓ is the balance length of the potentiometer wire, and k is the potential drop per unit length of the potentiometer wire, derive an expression for the product $k\ell$ in terms of E_0, r_0 and R.

(*b*) A student decided to modify the above setup to determine the resistance per unit length (m_0) of a nichrome wire by replacing the item X of the circuit with the nichrome wire of length ℓ_1.

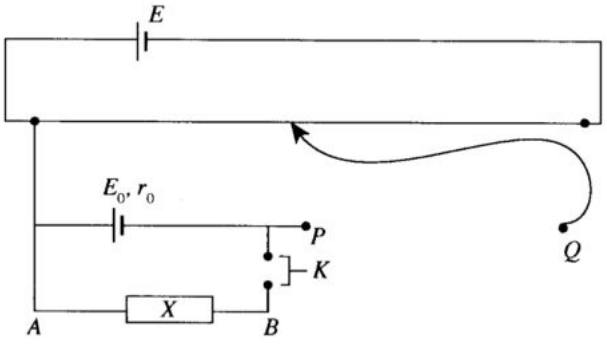
(i) If the balance length of the potentiometer wire in this case is ℓ_2, modify the expression that you have given under (*a*)(iii), and write down an expression for product $k\ell_2$ in terms of E_0, m_0, ℓ_1 and r_0.

(ii) Rearrange the expression that you have given under (*b*)(i) in a suitable manner to plot a graph between $\frac{1}{\ell_2}$ and $\frac{1}{\ell_1}$, taking $\frac{1}{\ell_1}$ as the independent variable.

(iii) How would you determine m_0 using the data obtained from the graph mentioned in (*b*)(ii) above and the value of r_0?

(iv) If the nichrome wire provided to the student has a diameter of 1.6×10^{-4} m, calculate the length of the wire necessary to obtain a resistance of 50 Ω. Resistivity of nichrome is 10^{-6} Ω m. (Take π as 3)

(v) The nichrome wire of resistance 50 Ω is fixed onto a metre ruler. You are asked to obtain a set of measurements from the potentiometer to determine m_0 using the graph mentioned in (*b*)(ii) above. By completing the circuit in figure (2) given below, show as to how you would connect the nichrome wire to the potentiometer in order to obtain the relevant measurement for a wire length approximately corresponding to 25 Ω.



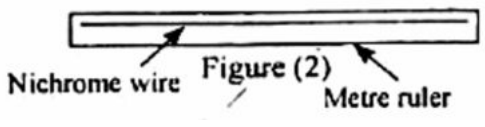
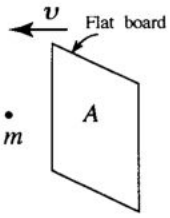


Figure (2)

Part B - Essay

5. (*a*) A vertical flat board of cross-sectional area A moves in still air at a constant speed v as shown in the figure. Consider the relative motion of the board and the air molecules. Under this condition, assume that the air molecules collide with the surface of the board perpendicularly and after colliding, bounce back in the opposite direction with the same speed v with respect to the board.



(i) If m is the mass of an air molecule, write down an expression for the change in momentum of the molecule.

(ii) Considering the number of air molecules colliding with the board per unit time or otherwise, show that the magnitude of the force F exerted on the board by the air can be given by $F = 2Adv^2$, where d is the density of air. **This force is known as the drag force**.

(*b*) The drag force (F_D) on an object moving in a fluid depends on the shape of the object. A more accurate expression for F_D can be given as $F_D = KAdv^2$, where K is a constant which depends on the shape of the object. Drag force plays an important role in the design of the external shape of vehicles.

Consider a motor vehicle moving in still air on a flat road with a constant speed v. Take $K = 0.20$, $A = 2.0\ m^2$ for the motor vehicle and $d = 1.3\ kg\ m^{-3}$.

(i) Write down an expression for the power (P) needed to overcome the drag force F_D.

(ii) Calculate the power P when the motor vehicle is moving with a speed of 90 km h^{-1} (= 25 m s^{-1}).

(iii) If the power needed to overcome other external frictional forces acting on the motor vehicle is **constant** and is 6 kW, what should be the total power supplied by the drive wheels of the motor vehicle in order to maintain a constant speed of 90 km h^{-1}?

(iv) If the speed of the motor vehicle has been increased from 90 km h^{-1} to 126 km h^{-1} (= 35 m s^{-1}), calculate the **additional power** required to maintain the speed of the motor vehicle at that value.

(v) If the motor vehicle climbs at a constant speed of 90 km h^{-1} on a road of slope of 3°, calculate the **additional power** that should be supplied by the drive wheels. Consider that the mass of the motor vehicle is 1 200 kg. (Take sin 3° = 0.05)

(*c*) Consider a motor vehicle moving on a flat road as described in (*b*)(iii) above. Consider that the energy released by burning one litre of petrol is 4×10^7 J and only 15% of this energy is used to drive the wheels. Under following conditions, calculate the fuel efficiency of this motor vehicle in kilometres per litre.

(i) When it moves in still air.

(ii) When it moves in opposite direction to a wind blowing at constant speed of 36 km h^{-1} (= 10 m s^{-1}).

6. Read the following passage and answer the questions.

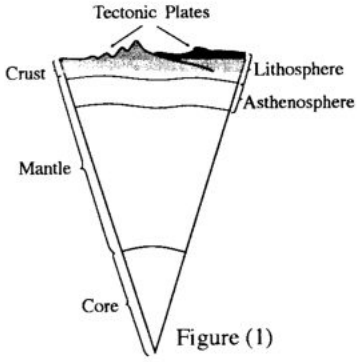


Figure (1)

Earthquakes are one of the powerful natural phenomena on Earth. The internal structure of the Earth is one of the important parameters needed to understand the major seismic activities around the globe. The Earth may be considered to have three major concentric parts, namely the crust, the mantle and the core [see figure (1)]. The lithosphere and asthenosphere are the two outer layers of the Earth. The lithosphere consists of 10 major rigid lithospheric plates called tectonic plates which are considered to be floating on the asthenosphere.

Heat is transferred towards asthenosphere due to the high temperature in the core. The convection currents thus produced in the asthenosphere cause the movements of tectonic plates. When two tectonic plates move with respect to each other, friction sometimes causes two plates to get stuck. When this happens elastic strain energy builds up, until eventually the plates give way creating an earthquake. This stored energy is released creating energetic waves called seismic waves. These seismic waves travel in all directions from the point where the energy is released, and this point is known as the focus of the earthquake. The corresponding point on the Earth's surface, directly above the focus, is called epicentre of the earthquake.

The Earth's crust supports propagation of travelling waves. The waves which travel through the crust are called body waves and those travel on the surface are called surface waves. The body waves consist of P (primary) waves and S (secondary) waves. P waves are longitudinal whereas the S waves are transverse. Since any material, solid or fluid, can be subjected to compression, the P waves can travel through any kind of material. However, S waves which depend upon shear force, do not exist in a fluid. The absence of S waves at large distances from an earthquake was the first indication that the Earth has a liquid region also. The P waves from an earthquake arrive at a given location before the S waves and surface waves.

There is a large number of seismic data recording stations throughout the world. In order to find the distance d from such a station to the epicentre, one needs to measure the difference in arrival times Δt of P and S waves to the station [see figure (2)]. The distance d is given by $d = \left[\dfrac{v_P v_S}{v_P - v_S}\right]\Delta t$, where v_P and v_S are speeds of P and S waves respectively. The location of the epicentre can be found using the d values from at least three recording stations. By drawing three circles with radii corresponding to the distances (d values) measured, and using the common point of intersection of the circles (triangulation), one can find the location of the epicentre.

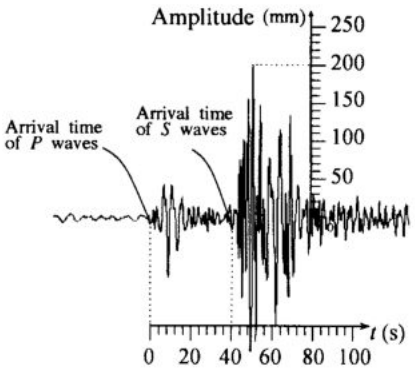


Figure (2)

Richter scale is the most accepted method used to estimate the strength of an earthquake. Distance d of the epicentre from the station and the **maximum** amplitude A_m of the seismic waves recorded at the station can be used to estimate the Richter scale magnitude M of an earthquake using the nomogram shown in figure (3). The magnitude M of an earthquake is related to the released energy E (in joules) by the equation, $\log_{10} E = 4.4 + 1.5M$.

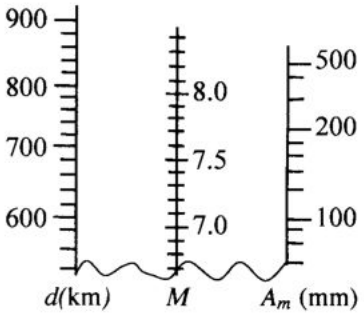


Figure (3)

(*a*) What are the **three** major parts of the interior of the Earth?

(*b*) Explain, why the tectonic plates are in continuous motion?

(*c*) What is the relationship between focus and epicentre of an earthquake?

(*d*) Even though P waves can travel through any part of the Earth, S waves can only travel in the solid parts of the Earth. Explain why?

(*e*) Draw **two** separate diagrams for the propagation of P and S waves indicating the direction of propagation and the direction of vibration of particles in the medium by arrows. Label them clearly.

(*f*) What was the first experimental observation which indicates the existence of a liquid region in the internal structure of the Earth?

(*g*) Using an appropriate diagram, illustrate the triangulation method used in seismology. Clearly mark the location of the epicentre as point O and S_1, S_2 and S_3 as the locations of the corresponding stations in your diagram.

(*h*) If the graph in figure (2) is a seismogram obtained by a certain station with regard to the recent earthquake in Nepal, find the value of Δt in seconds, and calculate the value of d in kilometres for this station. Take $v_P = 5 \text{ km s}^{-1}$ and $v_S = 4 \text{ km s}^{-1}$.

(*i*) Using the nomogram in figure (3) above, estimate the Richter scale magnitude M of the earthquake mentioned in (*h*) above.

Hint: Mark the values of d and A_m on the correct axes. Draw the line connecting the two points (d and A_m) and read the value of the point of intersection of the line with the M axis. You **do not need** to copy the nomogram to your answer script.

(*j*) Calculate the total energy E_N released from the earthquake in Nepal in joules.

(*k*) If E_S is the total energy released and $M = 9.1$ for 2004 Sumatra earthquake, calculate the ratio, $\frac{E_S}{E_N}$ Take $10^{1.8} = 63$.

7. (*a*) In the human body, if the length of a bone is larger than its width then it is classified as a 'long bone'. The tensile stress $\left(\frac{F}{A}\right)$ – strain $\left(\frac{\Delta \ell}{\ell}\right)$ curve for a certain 'long bone' is shown in figure (1). Here, all the symbols have their usual meaning.

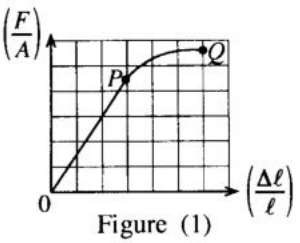


Figure (1)

(i) Identify the points P and Q marked on the curve shown in figure (1).

(ii) Assume that the 'long bone' is a uniform rod of area of cross-section 3×10^{-4} m^2 . If a tensile force of magnitude 4.5×10^3 N is applied, calculate the tensile stress on the bone.

(iii) If the Young's modulus of the 'long bone' is 1.5×10^{10} N m^{-2}, calculate the tensile strain of the bone.

(iv) If the initial length of the 'long bone' is 25 cm, what is the length when the tensile force is applied?

(*b*) Table given below shows the elastic characteristics of one of the long bones of the human body; the femur (the long bone in the thigh), obtained under tension and compression.

Elastic characteristics	Tensile value	Compressive value
Young's Modulus	1.60×10^{10} N m^{-2}	1.00×10^{10} N m^{-2}
Stress corresponding to the fracture point	1.20×10^{8} N m^{-2}	1.65×10^{8} N m^{-2}
Strain corresponding to the fracture point	1.50×10^{-2}	1.75×10^{-2}

(i) Using the values given in the table above for a femur, show that for the same value of stress, the compressive strain is 1.6 times the tensile strain.

(ii) Under which condition (tension or compression) the femur is more susceptible to fracture? Use the values given in the table above to justify your answer.

(*c*) When a person stands on a single leg the total weight of the person creates a compressive effect on the leg. Consider a situation that one of the femurs supports the entire body mass of 75 kg of a person who is walking. Consider that the femur is a thick-walled cylinder of uniform cross-section with an inner cavity. Its outer and inner radii are 1.5 cm and 0.5 cm respectively. For the following calculations, use the values given in the table above.

(i) When the person stands on a single leg, find the compressive stress applied to his femur. (Take π as 3)

(ii) Find the strain corresponding to the situation in (*c*)(i) above.

(iii) For a human to stand on a single leg without feeling uncomfortable under ordinary standing conditions, the strain on the femur must be less than 1% of the value of the strain indicated in the table above. Hence show that when the above mentioned person stands on one leg he does **not** feel uncomfortable.

(iv) Consider a person having all body dimensions doubled including all bones compared to an ordinary person. Let the mass of such a person be 600 kg. If the scaled-up person now stands on one leg, does he feel uncomfortable? Justify your answer. Assume that the elastic characteristics given in the table above remain unchanged for the situation.

8. (*a*) A long thin conducting straight cylindrical wire *A* of radius *a* has a charge per unit length $+\lambda$. Practically this can be done by connecting the wire to a positive potential with respect to ground.

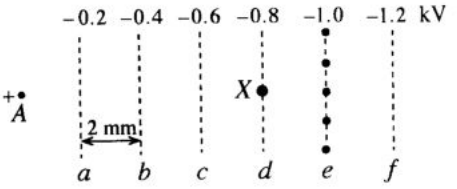
(i) Where does the given charge of the wire physically reside?

(ii) Considering an appropriate Gaussian surface around the wire, show that the magnitude of the intensity of the electric field *E* at a distance $r (\geq a)$ from the axis of the wire is given by $E = \frac{\lambda}{2\pi\varepsilon_0 r}$, where ε_0 is the permittivity of free space.

(iii) Draw a cross-section of the wire, and draw the equipotential lines around it.

(iv) If $a = 10\ \mu m$ and $\lambda = 8.1 \times 10^{-8}\ C\ m^{-1}$, calculate the magnitude of the intensity of the electric field on the surface of the wire. (Take ε_0 to be $9 \times 10^{-12}\ F\ m^{-1}$, and π as 3.)

(v) This wire *A* is now brought close to a region of a uniform electric field in which the equipotential surfaces are planar and normal to the plane of the paper. The axis of the wire is also normal to the plane of the paper. The dashed lines *a*, *b*, *c*, *d*, *e* and *f* shown in figure represent the cross-sections of the above mentioned equipotential surfaces as seen on the plane of the paper. These dashed lines represent the equipotential lines corresponding to the electric field, and the respective voltages (in kV) of these equipotential lines are also shown in the figure. Distance between any two equipotential lines is 2 mm.



In this arrangement the wire *A* is connected to a positive potential with respect to the ground, and can be considered as an anode.

(1) Copy the anode and the equipotential lines to the answer script and draw the electric field lines from the positions marked with dots on the equipotential line *e* up to the anode wire *A*.

(2) Calculate the intensity of the electric field E_0 between two equipotential lines.

(*b*) Part of an arrangement used for the detection of high energy particles and photons is similar to the one described in part (*a*)(v) above. Suppose that such an arrangement with an anode *A* having a charge per unit length $+\lambda = 8.1 \times 10^{-8}\ C\ m^{-1}$ is housed in a chamber filled with an inert gas (Argon) at atmospheric pressure.

Consider a situation in which a photon enters the chamber and collides with an argon atom at *X* creating a photoelectron and an argon ion. Such an electron is called a primary electron. The energy needed to create one such electron-ion pair in argon gas is 30 eV. ($1\ eV = 1.6 \times 10^{-19}\ J$, charge of an electron $e = 1.6 \times 10^{-19}\ C$).

(i) Write down an expression for the magnitude of the initial acceleration of the primary photoelectron due to the electric field mentioned in (*a*)(v)(1) above in terms of *m*, *e* and E_0, where *m* and *e* are the mass and charge of an electron respectively.

(ii) Explain, why the electron moves towards anode *A* with a drift velocity v_d without accelerating continuously.

(iii) Suppose that the primary electron which started from rest is moving along the electric field mentioned in (*a*)(v)(1) above. If the average distance moved by the primary electron between two successive collisions with argon atoms is 0.5 μm, calculate the increase in kinetic energy of the primary electron between two collisions in eV due to the electric field between two collisions, and show that the primary electron having this energy is unable to remove another electron upon colliding with another argon atom. (Consider that the energy required for an electron to remove an electron from an argon atom is 30 eV.)

(iv) When this primary electron is close to the anode it experiences a high electric field given by the expression stated in (*a*)(ii) above. Under this condition, the primary electron gains sufficient energy between collisions to create electron-ion pairs and the secondary electrons produced in this manner in turn create more electron-ion pairs before getting collected at the anode. Total number of secondary electrons produced by a single primary electron in this manner is called the **amplification factor** for the gas. Ability to collect charges by the anode wire indicates that it has the property of capacitance. This capacitance is called the detector capacitance. When the charges are collected by the anode, a small voltage is generated across this capacitor.

If the detector capacitance is 5 pF and the voltage generated across this capacitor due to secondary electrons produced by the primary electron is 0.96 mV find the charge collected by the anode.

(v) Hence find the amplification factor for the gas.

9. Answer either part (A) or part (B) only.

(A) (*a*) In the circuit shown in figure (1), X is an accumulator of e.m.f. E and internal resistance r. L is an electric lamp connected across AB, and the current through the lamp is I.

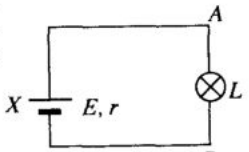


Figure (1)

(i) Show that the power P consumed by the electric lamp can be given as

$$P = EI - I^2r.$$

(ii) Using the definitions for E and I, explain why the product EI is equal to the power generated by the accumulator.

(iii) Electric lamp in the figure (1) is now replaced by another accumulator of e.m.f. E_1 and internal resistance r_1 as shown in figure (2). $E > E_1$ and the current in the circuit is now I_1.

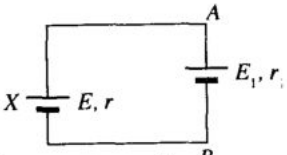


Figure (2)

(1) Show that $EI_1 - I_1^2r = E_1I_1 + I_1^2r_1$.

(2) Physically what quantities do the products EI_1 and E_1I_1 in the above expression represent? Explain your answer.

(*b*) A circuit similar to the one given in figure (2) above can be used to re-charge a run-down rechargeable battery. In this context X is a source capable of delivering a constant power output, and is known as the battery charger. Y represents the run-down battery. Consider such a circuit shown in figure (3).

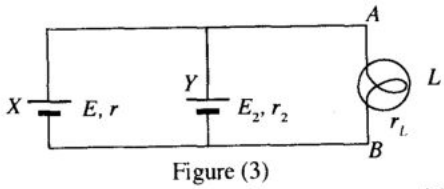


Figure (3)

X is a 12 V battery charger. For the purpose of calculations consider it as a constant power source with e.m.f 12 V, and an internal resistance $r = 2\ \Omega$. L is an indicator lamp of resistance $r_L = 2\ \Omega$ connected across the battery charger. E_2 and r_2 represent the e.m.f. of the battery Y and its internal resistance at a **particular instant** in the charging process. If $r_2 = 1\ \Omega$, and the current through Y is 1 A at that instant,

(i) calculate the e.m.f. E_2 of the battery Y at that instant.

(ii) calculate the power generated by the battery charger, and the power dissipated in r, r_2 and r_L at that instant.

(iii) apply the principle of conservation of energy to the charging process at that instant, and explain what has happened to the excess power generated by the battery charger.

(B) (*a*) Draw current (I) – voltage (V) characteristic for a silicon diode, indicating its forward bias voltage of 0.7 V on the voltage axis.

(*b*) Instead of the characteristic you have drawn under (*a*), hypothetical diode characteristic given in figure (1) is also used frequently in the analysis and design of circuits with silicon diodes. According to the figure (1), current through the diode is zero until its voltage becomes 0.7 V at which the current increases sharply parallel to the I-axis.

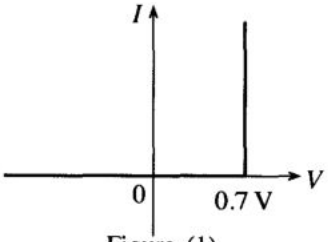


Figure (1)

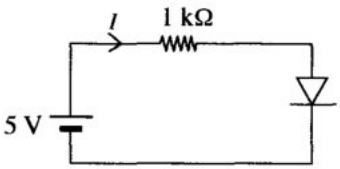


Figure (2)

Use the $I-V$ characteristic given in figure (1) and calculate the current I in the circuit shown in figure (2).

Also use the characteristic given in figure (1) above to answer all the following questions.

(*c*) In the figure (3) shown, D_1 and D_2 are silicon diodes, and the input voltages A and B can have either 5 V or 0 V.

(i) Find the voltage (V_F) at the output F for various combinations of the input voltages and fill in the following table (Copy the table onto your answer script for this purpose)

A(V)	B(V)	V_F(V)
0	0	
5	0	
0	5	
5	5	

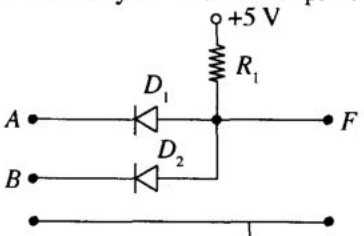


Figure (3)

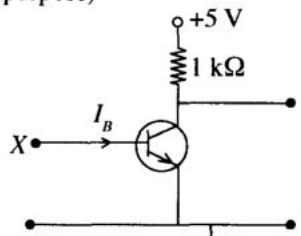


Figure (4)

(ii) As far as F output is considered, if 0.7 V represents binary 0 and 5 V represents binary 1, identify the gate corresponding to the circuit given in figure (3) and write down its truth table.

(iii) Calculate a suitable value for R_1 which will limit the sum of the currents through **both** diodes to 0.5 mA.

(*d*) Suppose the terminal X of the circuit shown in figure (4) is now connected to the output F of the circuit shown in figure (3).

(i) When the inputs A and B represent binary 1, what is the base current I_B?

(ii) Show that the transistor operates as a closed switch under the input conditions given in (*d*)(i) above. Assume that the current gain, β, of the transistor is 50.

(iii) Show that the transistor, however, does **not** operate as an open switch when F in figure (3) represents binary 0.

(iv) With the aid of a circuit diagram, show how you would convert the composite circuit consisting of circuits given in figures (3) and (4) to perform as a NAND gate by inserting another silicon diode at an appropriate place in the circuit given in figure (4) above.

10. Answer either part (A) or part (B) only.

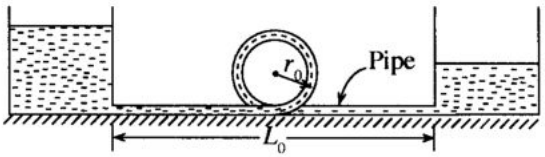
(A) (*a*) A pipe made of copper having a length L_0 at room temperature θ_0 is heated to a temperature θ. Write down an expression for the increase in length of the pipe. Linear expansivity of copper is α.

When answering the following questions always consider steady conditions.

(*b*) An insulated straight copper pipe of length L_0 and internal area of cross-section A_0 at room temperature θ_0 is laid between two oil tanks separated by a large distance to transport heated oil from one tank to the other tank.

If the distance between the tanks is kept fixed at L_0, a compressive stress builds-up in the pipe when heated oil is sent through it. Write down an expression for the maximum temperature θ_M of the oil that can be sent through the pipe without exceeding the compressive elastic limit for copper. Assume that the compressed length corresponding to the elastic limit of copper is ΔL_0.

(*c*) In order to avoid the compression of the pipe as stated in (*b*) and use it to transport oil at a higher temperature θ_H ($>\theta_M$) it was decided to modify the pipe by introducing an additional small circular section made of copper with mean radius r_0 at room temperature θ_0 so that it forms a part of the pipe as shown in the figure.



(i) Explain how such a modification would avoid the compression of the tube with temperature as stated in (*b*) above.

(ii) What is the total length of the pipe at room temperature θ_0?

(iii) Derive an expression for the total length (L_H) of the pipe when oil at temperature θ_H is sent through it.

(iv) Derive an expression for the new mean radius (R_H) of the circular section when oil at temperature θ_H is sent through the pipe. Assume that the shape of the circular section remains as circular.

(v) Derive an expression for the increase in the volume of oil in the pipe at θ_H when compared to the volume at room temperature θ_0.

(vi) If the variations of the area of cross-section of the inlet of the pipe and the density of oil with temperature are negligible, derive an expression for the ratio, $\frac{\text{Flow speed of oil at } \theta_H}{\text{Flow speed of oil at } \theta_0}$ in the pipe when its temperature is increased from room temperature θ_0 to θ_H. Assume that the pressure difference of oil between the inlet and outlet of the pipe is constant.

(vii) Even if the pipe is insulated, suppose there is a small **linear** drop in temperature θ_H across the entire length of the pipe. If this drop is $\Delta\theta$, derive an expression for the mean radius of the circular section. Assume that the circular section is located at the middle of the pipe and neglect temperature variation of that section.

(**B**) (*a*) Using the Einstein's mass-energy relation, determine the energy equivalence of the atomic mass unit (1 u) in MeV. (1 MeV = 1.6×10^{-13} J, 1 u = 1.66×10^{-27} kg, and speed of light = 3×10^8 m s^{-1})

(*b*) $^{235}_{92}U$ nucleus undergoes fission when a neutron is absorbed. One of the modes of fission is given in the following reaction.

$$n + {}^{235}_{92}U \longrightarrow {}^{96}_{37}Rb + {}^{138}_{55}Cs + 2n$$

The masses of $^{235}_{92}U$, $^{96}_{37}Rb$, $^{138}_{55}Cs$ and a neutron are approximately, 235.0440 u, 95.9343 u, 137.9110 u and 1.0087 u respectively.

(i) Find the mass loss of the above fission reaction in terms of atomic mass units.

(ii) Hence determine the energy released in the above fission reaction in MeV.

(*c*) In a large nuclear reactor the thermal power generated due to the fission of $^{235}_{92}U$ fuel is 3 200 MW. The corresponding electrical power generated is 1 000 MW. Different modes of fission reactions release different amounts of energy as heat. In these fission reactions the average heat energy generated per fission is 200 MeV.

(i) Determine the efficiency of the nuclear reactor.

(ii) Determine the number of fissions per second (fission rate) at the steady state of the nuclear reactor.

(iii) Hence find the $^{235}_{92}U$ consumption rate in kg per year of the nuclear reactor. (Take Avagadro number as 6.0×10^{23} mol^{-1}.)

(*d*) Natural uranium contains 0.7% of $^{235}_{92}U$ and 99.3% of $^{238}_{92}U$ by weight. Only $^{235}_{92}U$ is required as fuel for the above nuclear reactor to generate electricity. The above reactor requires uranium fuel of 2% enriched uranium (uranium fuel consisting of 2% $^{235}_{92}U$ by weight).

Determine the 2% enriched uranium fuel required to run the 1 000 MW reactor mentioned under (*c*) above for one year.

(*e*) In coal power plants, burning of carbon produces the heat energy required to produce electricity.

$$C + O_2 \longrightarrow CO_2 + 4 \text{ eV}$$

The efficiency of a coal power plant is mostly the same as the efficiency of a nuclear power plant. Determine the amount of carbon in kg required to run a 1 000 MW coal power plant for one year. Assume that the efficiency of the coal power plant is same as the efficiency determined in (*c*)(i) above.
(Molar mass of C = 12 g mol^{-1}).