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සියලු ම හිමිකම් ඇවිරිණි /முழுப் பதிப்புரிமையுடையது/All Rights Reserved]

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இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2025
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2025
General Certificate of Education (Adv. Level) Examination, 2025

සංයුක්ත ගණිතය II
இணைந்த கணிதம் II
Combined Mathematics II

10 E II

Part B

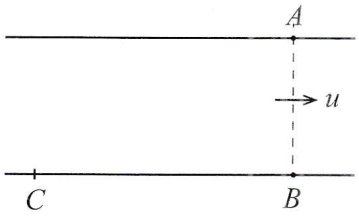
* Answer **five** questions only.

(In this question paper, g denotes the acceleration due to gravity.)

11. (*a*) A particle P, moving with a constant deceleration of f m s^{-2} along a straight line, passes a point O with a velocity of 60 m s^{-1} at time $t = 0$ s, where $f > 0$. Just after the particle P comes to rest, it moves towards O with a constant acceleration of f m s^{-2}. At time $t = \frac{30}{f}$ s, another particle Q which was at rest at O, starts to move towards P along the same straight line with a constant acceleration of f m s^{-2} and after reaching a velocity of 30 m s^{-1} continues to move with this constant velocity. The particle Q meets the particle P, 10 seconds after the particle Q reaches the constant velocity. Sketch the velocity-time graphs for the motions of P and Q from $t = 0$ s until they meet, in the same diagram.

Show that $f = 3$, and that the distance from O to the point at which the particles meet is 450 m.

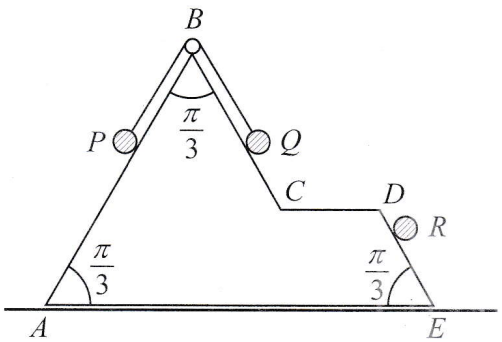
(*b*) A river of breadth a with parallel straight banks flows with a uniform velocity u. The points A, B and C lie on the banks such that AB is perpendicular to the banks and $BC = 2a$, as shown in the figure. Two boats P and Q begin their journeys at A and B respectively, at the same instant. The boat P travels relative to the water with a velocity of $2\sqrt{5}\,u$ in the direction of $\overrightarrow{AC}$. The boat Q travels with a speed of $\sqrt{2}\,u$ relative to the water in the direction of $\overrightarrow{BA}$ relative to earth. Sketch the velocity triangles for the motions of P and Q in the same diagram.



Find the velocity of P relative to earth, and the speed of Q relative to earth.

Also, find the direction of the velocity of P relative to Q, and **hence** find the shortest distance between P and Q.

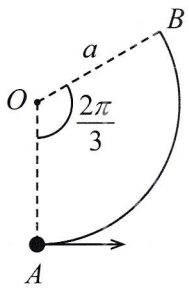
12. (*a*) The vertical cross-section $ABCDE$ through the centre of gravity of a smooth uniform block of mass $10m$ is shown in the figure. The face containing AE is placed on a smooth horizontal floor. Also, AB, BC and DE are lines of greatest slope of the faces containing them, CD is parallel to AE, and $E\hat{A}B = A\hat{B}C = D\hat{E}A = \frac{\pi}{3}$.



Three particles P, Q and R of masses $5m$, m and m respectively, are held on AB, BC and DE respectively. The particles P and Q are attached to the ends of a light inextensible string passing over a smooth light small pulley fixed to the block at B. The system is released from rest with the string taut from the position shown in the figure. Obtain equations sufficient to determine the acceleration of the block and the magnitude of the reaction on P by the block.

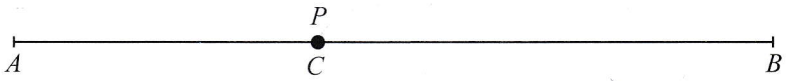
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(*b*) A smooth thin rigid wire AB in the shape of a circular arc of radius a, centre O and $A\hat{O}B = \frac{2\pi}{3}$ is fixed in a vertical plane with OA vertical, as shown in the figure. A smooth small bead of mass m is kept at A and projected along the wire with a speed of $\sqrt{\frac{7ga}{2}}$. Show that the speed v of the bead when it has turned through an angle $\theta \left(0 < \theta \leq \frac{2\pi}{3}\right)$ about O is given by $v^2 = \frac{ga}{2}(3 + 4\cos\theta)$.



Also, show that after leaving the wire, the bead comes back to A, and find the speed of the bead when it reaches A.

13. Two fixed points A and B lie on a smooth horizontal table such that $AB = 16a$. One end of a light elastic string S_1 of natural length $2a$ and modulus of elasticity $3mg$ is attached to a particle P of mass m and the other end of S_1 is attached to A. Also, one end of a second light elastic string S_2 of natural length $4a$ and modulus of elasticity $4mg$, is also attached to P and the other end of S_2 is attached to B. The particle P is in equilibrium at a point C on the table, as shown in the figure.



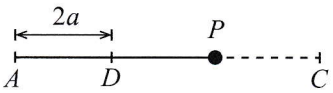
Find the lengths of AC and BC.

Now, the particle P is pulled by a distance $2a$ towards B and released from rest. Show that the equation of motion of P is given by $\ddot{x} + \omega^2(x - 6a) = 0$, where $AP = x$ and $\omega\ (> 0)$ is a constant to be determined.

By taking $X = x - 6a$, show that $\ddot{X} + \omega^2 X = 0$ and state the period of this simple harmonic motion.

Using the formula $\dot{X}^2 = \omega^2(c^2 - X^2)$, find the amplitude c and the maximum speed of P during this motion.

At the first instant when P reaches C during this motion, the string S_2 is cut.



Let D be the point on AC such that $AD = 2a$ as shown in the figure. Show that the equation of motion of P from C to D is given by $\ddot{y} + \omega_1^2 y = 0$, where $DP = y$ and $\omega_1\ (> 0)$ is a constant to be determined.

Show that the amplitude of this simple harmonic motion is $\sqrt{\frac{68}{3}}a$.

Find the total time elapsed from the instant when P was set in motion until it reaches the point A.

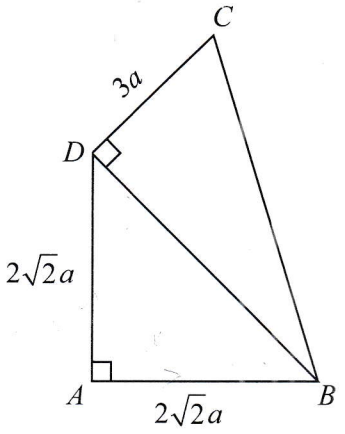
14.(*a*) In the usual notation, let $\mathbf{a} = -\mathbf{i} + \mathbf{j}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j}$ be the position vectors of the points A and B respectively, with respect to the origin O. Let C be the point such that $\overrightarrow{OC} = 2\overrightarrow{OB}$. Also, let D be the point such that DC is parallel to AB, and AD is perpendicular to AB. Show that $\overrightarrow{OD} = -\frac{8}{5}\mathbf{i} + \frac{11}{5}\mathbf{j}$.

Let E be the point of intersection of AB and OD.

Show that $\overrightarrow{AE} = \frac{1}{10}\overrightarrow{AB}$.

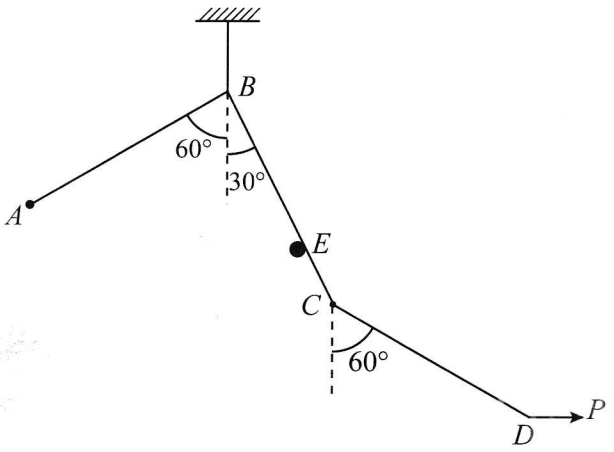
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(*b*) $ABCD$ is a plane quadrilateral with $B\hat{A}D = B\hat{D}C = \frac{\pi}{2}$, $AB = AD = 2\sqrt{2}\,a$ and $CD = 3a$ as shown in the figure. Forces of magnitudes $3P$, $3P$, $2\sqrt{2}P$, $5\sqrt{2}P$ and $3\sqrt{2}P$ act along AB, AD, BD, BC and DC respectively, in the directions indicated by the order of the letters. This system of forces is equivalent to two forces of magnitudes αP and βP acting along AB and AD respectively, in the directions indicated by the order of the letters, together with a couple whose moment is M acting in the counter clockwise sense. Find the values of α, β and M.



Now, a couple acting in the plane of the quadrilateral is added to the above system of forces such that the resultant of the new system of forces goes through D. Find the moment of couple added.

15. (*a*) Three uniform rods AB, BC and CD of equal length $4a$ and weights W, $2W$ and W respectively, are smoothly jointed at the end points B and C. The end A is smoothly hinged to a fixed point. One end of a light inextensible string is attached to the joint B and other end of the string is attached to a point lying vertically above B on a horizontal ceiling. The three rods are kept in equilibrium in a vertical plane by placing the rod BC on a fixed smooth peg at E, where $BE = 3a$, and by applying a horizontal force of magnitude P at D such that the string is taut and each of the rods AB and CD is making an angle of 60° with the vertical and the rod BC is making an angle of 30° with the vertical as shown in the figure.

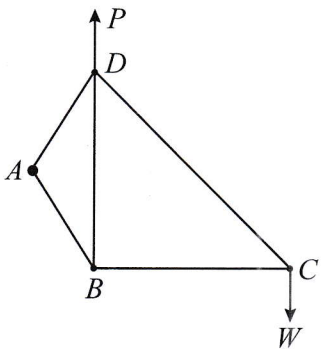


(i) Find the value of P.

(ii) Show that the magnitude of the reaction on the rod BC from the peg is $\frac{W}{3}$.

(iii) Find the tension of the string.

(*b*) The framework shown in the figure consists of five light rods AB, BC, CD, DA and DB that are smoothly jointed at their ends. It is given that $AB = AD = 2a$, $BC = BD = 2\sqrt{3}a$, and $C\hat{B}D = \frac{\pi}{2}$.



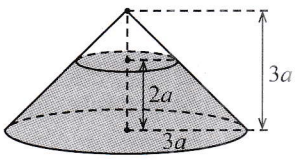
A load W is suspended at joint C and the framework is smoothly hinged to a fixed point at A and kept in equilibrium in a vertical plane with BC horizontal, by a force of magnitude P applied vertically upwards to it at the joint D. Draw a stress diagram using Bow's notation for the joints C, B and D.

Hence, find the stresses in the rods, stating whether they are tensions or thrusts.

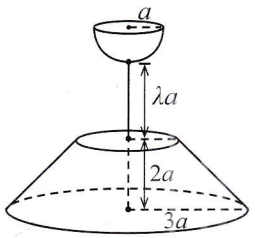
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16. Show that the centre of mass of a uniform hemispherical shell of radius a is at a distance $\frac{a}{2}$ from its centre and the centre of mass of a solid uniform right-circular cone of height h is at a distance $\frac{h}{4}$ from the centre of its base.

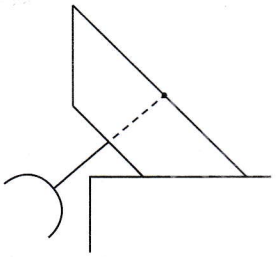
A frustum of height $2a$ is made from a solid uniform right-circular cone of base radius $3a$ and height $3a$, by cutting it through a plane parallel to its base and removing the smaller right-circular cone. (See the adjoining figure.) The mass of the right-circular cone removed to form the frustum is m. Show that the mass of the frustum is $26m$.



A uniform hemispherical shell of radius a and mass m, and the above frustum are rigidly fixed to the ends of a uniform rod of length λa and mass m to form a composite object such that the rod, the centre of the hemispherical shell and the axis of the frustum are all lying on the same straight line, as shown in the adjoining figure. Show that the centre of mass of the composite object is at a distance $\frac{3}{56}(15+\lambda)a$ from the centre of the large circular base of the frustum.



The composite object is placed on a horizontal table with the curved surface of the frustum touching the table. The adjoining figure shows the vertical cross-section through the axis of symmetry of the composite object. If the composite object is in equilibrium, show that $\lambda \leq \frac{11}{3}$.



17. (*a*) A box C contains 2 black balls and 2 white balls, and a box D contains 2 black balls and 1 white ball. These balls are identical in all aspects except for their colours. First, one ball is transferred at random from box C into box D. After that, a ball is transferred at random from box D into box C. Now, a ball is drawn at random from box D.

Find the probability that

(i) the ball drawn from box D is white,

(ii) the ball transferred first from box C into box D is black, given that the ball drawn from box D is white.

(*b*) The following table gives the data distribution of the annual savings of 100 customers of a bank.

Savings in thousands of rupees	Frequency
10 – 30	35
30 – 50	40
50 – 70	15
70 – 90	10

Find the mean, median, mode and variance of the above data distribution.

Now, an additional x number of customers are added to the data distribution and it is found that they all belong to a single class interval. The mean of the annual savings of $(100 + x)$ customers is given to be Rs. 40 000. Show that the class interval the new x customers belong to is 30 – 50.

* * *