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ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව இலங்கைப் பரீட்சைத் திணைக்களம் Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2024
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2024
General Certificate of Education (Adv. Level) Examination, 2024

සංයුක්ත ගණිතය I இணைந்த கணிதம் I **Combined Mathematics I**	**10 E I**

පැය තුනයි மூன்று மணித்தியாலம் ***Three hours***	අමතර කියවීම් කාලය - මිනිත්තු 10 යි மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள் **Additional Reading Time - 10 minutes**

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number									

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Instructions:

* *This question paper consists of two parts;* **Part A** (Questions **1 - 10**) and **Part B** (Questions **11 - 17**).
* **Part A:** *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:** *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(10) Combined Mathematics I		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Using the **Principle of Mathematical Induction**, prove that $7^n - 1$ is divisible by 6 for all $n \in \mathbb{Z}^+$.

2. Sketch the graphs of $y = |x-3|+1$ and $y = 5-|x|$ in the same diagram.
Hence, find the area of the rectangular region enclosed by these graphs.

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3. Shade in an Argand diagram, the region consisting of points that represent the complex numbers z satisfying the inequalities $|z-2i| \leq 2$ and $0 \leq \text{Arg}\left(z+2\sqrt{3}\right) \leq \frac{\pi}{6}$.
Find the greatest value of $|z|$ for the complex numbers z represented by the points in this shaded region.

4. Show that the constant term in the expansion of $\left(1+x^3\right)\left(x-\frac{1}{\sqrt{x}}\right)^9$ is 93.

[see page four

5. Show that $\lim_{x\to 4} \frac{\left(\sqrt{x-3}-1\right)}{(x-4)^2} \sin\left(\sqrt{x}-2\right) = \frac{1}{8}$.

6. The region enclosed by the curves $y = \frac{2}{x\sqrt[4]{4-x^2}}$, $y = 0$, $x = 1$ and $x = \sqrt{2}$ is rotated about the x-axis through 2π radians. Show that the volume of the solid thus generated is $\pi\left(\sqrt{3}-1\right)$.

[see page five

7. Let C be the curve given parametrically by $x = \ln t$ and $y = e^t + t\ln t$ for $t > 0$.

Show that $\frac{dy}{dx} = t(e^t + \ln t + 1)$.

If the tangent drawn to the curve C at the point corresponding to $t = 1$, passes through the point $(1, a)$, show that $a = 1 + 2e$.

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8. Find the equations of the two straight lines passing through the point $A \equiv (-1, 2)$ having a perpendicular distance of 1 from the origin.

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9. Let $A \equiv (-1, 1)$ and $B \equiv (3, 3)$. Write down the equation of the circle S with AB as a diameter. Show that the circle $x^2 + y^2 - 4x - 5y + 9 = 0$ touches the circle S internally at B.

10. Show that $\dfrac{\cot\theta}{1+\sin\theta} + \dfrac{\cot\theta}{1-\sin\theta} \equiv 4\operatorname{cosec} 2\theta$.

Hence, solve $\dfrac{\cot\theta}{1+\sin\theta} + \dfrac{\cot\theta}{1-\sin\theta} = 8\cos 2\theta$.

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[see page seven

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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2024
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2024
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සංයුක්ත ගණිතය I இணைந்த கணிதம் I **Combined Mathematics I**	**10 E I**

Part B

* Answer **five** questions only.

11. (*a*) Let $f(x) = x^2 + 2x + c$, where $c \in \mathbb{R}$.

It is given that the equation $f(x) = 0$ has two real distinct roots. Show that $c < 1$.

Let α and β be the roots of $f(x) = 0$.

Show that $\alpha^2 + \beta^2 = 4 - 2c$.

Let $c \neq 0$ and $\lambda \in \mathbb{R}$. The quadratic equation with $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$ as its roots is $2x^2 + 12x + \lambda = 0$. Find the values of c and λ.

(*b*) Let $f(x) = x^3 + px^2 + qx + p$, where $p, q \in \mathbb{R}$. The remainder when $f(x)$ is divided by $(x - 2)$ is 36 more than the remainder when $f(x)$ is divided by $(x - 1)$. Show that $3p + q = 29$.

It is also given that $(x + 1)$ is a factor of $f(x)$. Show that $p = 6$ and $q = 11$, and factorize $f(x)$ completely.

Hence, solve $f(x) = 3(x + 2)$.

12. (*a*) The parents of a family decide to invite 6 out of 15 of their close relatives for a dinner. While the father has 5 close female relatives and 3 close male relatives, the mother has 3 close female relatives and 4 close male relatives.

Find the number of different ways in which

(i) the father can invite 3 of his close female relatives and the mother can invite 3 of her close male relatives,

(ii) the father can invite 3 of his close relatives and the mother can invite 3 of her close relatives so that 3 males and 3 females are invited.

[see page eight

(*b*) Let $U_r = \dfrac{1}{r(r+2)(r+4)}$ and $f(r) = \dfrac{1}{r(r+2)}$ for $r \in \mathbb{Z}^+$.

Determine the value of the real constant A such that $f(r) - f(r+2) = AU_r$ for $r \in \mathbb{Z}^+$.

Hence, show that $\sum_{r=1}^{n} U_r = \dfrac{11}{96} - \dfrac{1}{4(n+1)(n+3)} - \dfrac{1}{4(n+2)(n+4)}$ for $n \in \mathbb{Z}^+$.

Show further that the infinite series $\sum_{r=1}^{\infty} U_r$ is convergent and find its sum.

Find the value of the real constant m such that $\lim_{n\to\infty} \sum_{r=1}^{n} (mU_r + U_{n+1-r}) = \dfrac{11}{32}$.

13. (*a*) Let $a, b \in \mathbb{R}$, $\mathbf{A} = \begin{pmatrix} 1 & 2 & -1 \\ 3 & a & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 & a & b \\ 3 & b & a \end{pmatrix}$. It is given that $2\mathbf{A} + \mathbf{B} = \begin{pmatrix} 2 & 4 & 3 \\ 9 & 5 & 4 \end{pmatrix}$.

Show that $a = 0$ and $b = 5$.

With these values for a and b, let $\mathbf{C} = \mathbf{AB}^{\mathrm{T}}$.

Find $\mathbf{C}$ and write down $\mathbf{C}^{-1}$.

Find the matrix $\mathbf{D}$ such that $\mathbf{DC} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$.

(*b*) Let $z_1, z_2 \in \mathbb{C}$. Show that

(i) $\overline{z_1 + z_2} = \overline{z}_1 + \overline{z}_2$

(ii) $\overline{z_1 z_2} = \overline{z}_1 \overline{z}_2$

(iii) $z_1 \overline{z}_1 = |z_1|^2$

Using the result that $\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\overline{z}_1}{\overline{z}_2}$ for $z_2 \neq 0$, show that if $|z_1| = 1$ and $z_1 \neq \pm 1$, and also if $\dfrac{z_1 + z_2}{1 + z_1 z_2}$ is real, then $|z_2| = 1$.

(*c*) Express $\sqrt{3} + i$ is the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $0 < \theta < \dfrac{\pi}{2}$.

Using De Moivre's theorem, show that $\dfrac{(\sqrt{3}+i)^{24}}{2^{23}(1+i)} = 1 - i$.

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14. (*a*) Let $f(x)=\dfrac{px+q}{(x-1)(x-2)}$ for $x \in \mathbb{R} - \{1, 2\}$, where $p, q \in \mathbb{R}$. It is given that the graph of $y = f(x)$ has a stationary point at (0, 1). Show that $p = -3$ and $q = 2$.

For these values of p and q, show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x)=\dfrac{x(3x-4)}{(x-1)^2(x-2)^2}$ for $x \neq 1, 2$, and find the intervals on which $f(x)$ is decreasing and the intervals on which $f(x)$ is increasing.

Sketch the graph of $y = f(x)$ indicating the asymptotes and the turning points.

Hence, find the number of real solutions to the equation $x^2(x-1)(x-2) = 2 - 3x$.

(*b*) A cylinder with a top and a bottom is made to have a volume of 1024π cm^3. Let r cm be the radius of the cylinder. Show that the total surface area S cm^2 of the cylinder is given by $S=2\pi\left(\dfrac{1024}{r}+r^2\right)$ for $r > 0$.

Show that S is minimum when $r = 8$.

15. (*a*) Find the values of the real constants A and B such that $3t^2 + 4 = A(t^2 - 2t + 4) + Bt(t + 1)$ for all $t \in \mathbb{R}$.

Hence or otherwise, find $\displaystyle\int \frac{3t^2+4}{(t+1)(t^2-2t+4)}\,dt$.

(*b*) Using the substitution $u = x + \sqrt{x^2+3}$, show that $\displaystyle\int_0^1 \frac{1}{\sqrt{x^2+3}}dx = \frac{1}{2}\ln 3$.

Let $J = \displaystyle\int_0^1 \sqrt{x^2+3}\,dx$. Using integration by parts, show that $2J = 2 + \displaystyle\int_0^1 \frac{3}{\sqrt{x^2+3}}dx$.

Deduce that $J = 1 + \dfrac{3}{4}\ln 3$.

(*c*) Using the formula $\displaystyle\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, where a is a constant, show that

$$\int_0^{\frac{\pi}{4}} \ln\left(\frac{\cos x}{\cos x+\sin x}\right)dx = \frac{\pi}{8}\ln\left(\frac{1}{2}\right).$$

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16. Let $A \equiv (1, 2)$ and $B \equiv (a, b)$, where $a, b \in \mathbb{R}$. It is given that the perpendicular bisector l of the line segment AB has the equation $x + y - 4 = 0$. Find the values of a and b.

Let $C \equiv (3, 1)$. Show that the point C lies on the line l and find $A\hat{C}B$.

Let S be the circle through the points A, B and C. Show that the centre of S is given by $\left(\frac{13}{6}, \frac{11}{6}\right)$ and find the equation of S.

Hence, find the equation of the circle passing through the points A, B and the point $D \equiv (0, 3)$.

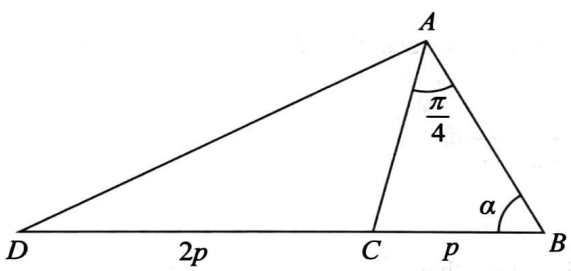
17. (*a*) Express $6\cos 2x - 8\sin 2x$ in the form $R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Hence, solve $6\cos 2x - 8\sin 2x = 5$.

Express $24\cos^2 x - 32\sin x\cos x$ in the form $a\cos 2x + b\sin 2x + c$, where a, b, c $(\in \mathbb{R})$ are constants to be determined.

Deduce the minimum value of $24\cos^2 x - 32\sin x\cos x$.

(*b*)



In the triangle ABC shown in the figure, $BC = p$, $B\hat{A}C = \frac{\pi}{4}$ and $A\hat{B}C = \alpha$. The point D lies on the extended line BC such that $CD = 2p$.

Show that $AB = p(\cos\alpha + \sin\alpha)$.

Find AD^2 in terms of p and α.

Deduce that if $AD = 3p$, then $\alpha = \tan^{-1}(5)$.

(*c*) Solve the equation $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$.

* * *