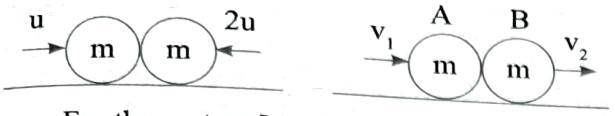
01.



For the system $\underline{I} = \Delta m\underline{v} \rightarrow$

$O = mv_1 + mv_2 - (2mu - mu)$

$v_1 + v_2 = u$ ——— (1) (05)

Newton's Low of Restitution

$v_2 - v_1 = e(2u + u)$

$v_2 - v_1 = 3eu$ ——— (2) (05)

(1) and (2) $\Rightarrow v_1 = \frac{1}{2}(1 - 3e)u$ (05)

$v_2 = \frac{1}{2}(1 + 3e)u$ (05)

$e = \frac{1}{3} \Rightarrow v_1 = 0$ 25

---

02. $S = ut + \frac{1}{2}at^2$: $\leftarrow 3a = u\cos\theta \cdot t$. (05)

$\leftarrow \frac{5a}{4} = u\sin\theta - \frac{1}{2}gt^2$ (05)

$\frac{5a}{4} = u\sin\theta \cdot \frac{3a}{u\cos\theta} - \frac{1}{2}g\frac{9a^2}{u^2\cos^2\theta}$ (05)

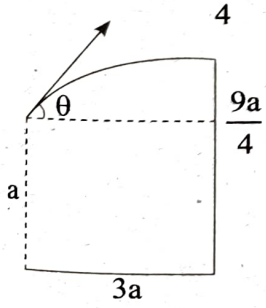
$\frac{5a}{4} = 3a\tan\theta - 9a(1 + \tan^2\theta)$

$4\tan^2\theta - 12\tan\theta + 9 = 0$

$(2\tan\theta - 3)^2 = 0$ (05)

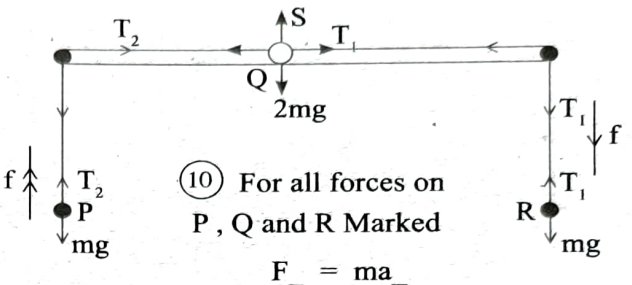
$\therefore \tan\theta = \frac{3}{2}$

$\theta = \tan^{-1}\left(\frac{3}{2}\right)$ (05)



25

03.



(10) For all forces on P, Q and R Marked

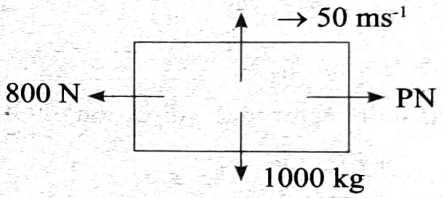
$F_{_} = ma_{_}$

(P) $\uparrow T_2 - mg = mf$ (05)

(R) $\uparrow 4mg - T_1 = 4mf$ (05)

(Q) $\rightarrow T_1 - T_2 = mf$ (05) 25

04.



$\rightarrow \underline{F} = m\underline{a}$

$P - 800 = 0$

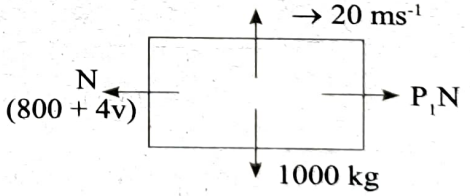
$P = 800$ (05)

Power $= P \times 50W$

$= 800 \times 50W$

$= 40000W$

$= 40\ KW$ (05)



$\uparrow \underline{F} = m\underline{a}$ $P_1 - (800 + 4v) = 1000a$ (05)

$P_1 = (800 + 80) + 1000a$

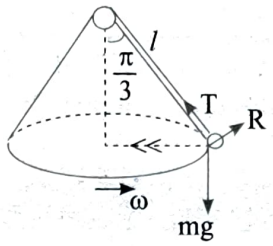
Also $P_1 \times 20 = 40000$ (05)

$880 + 1000a = 2000$

$a = 1.12$ (05)

25

05.



(05) For Forces on P

$\underline{F} = m\underline{a} \nwarrow T - mg\cos\frac{\pi}{3} = m(l\sin\frac{\pi}{3})\omega^2\cos\frac{\pi}{6}$

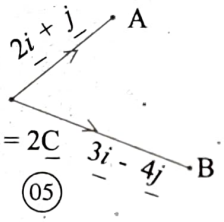
(05) (05) (05)

$T = \frac{mg}{2} + \frac{\sqrt{3}}{2}\, ml\,.\,\omega^2\,\frac{\sqrt{3}}{2}$ (05)

$= \frac{mg}{2} + \frac{3}{4}ml\omega^2 = \frac{mg}{2} + \frac{3}{4}m\,.\,2g = 2mg$ **25**

---

06. Let $\overrightarrow{OC} = \underline{C}$



$2\overrightarrow{OA} + \overrightarrow{BC} = 2\overrightarrow{OC}$

$2(2\underline{i} + \underline{j}) + \underline{C} - (3\underline{i} - 4\underline{j}) = 2\underline{C}$

(05) (05) (05)

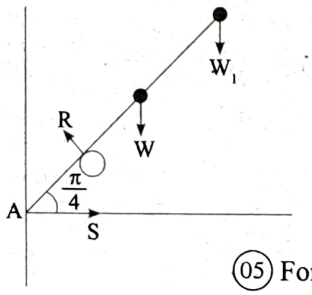
$\underline{C} = (4\underline{i} + 2\underline{j}) - (3\underline{i} - 4\underline{j})$

$= \underline{i} + 6\underline{j}$ (05)

The unit vector in the direction of $\overrightarrow{OC}$ is

$\frac{\underline{i} + 6\underline{j}}{\sqrt{37}}$ (05) **25**

07.



(05) For the Forces

$A \curvearrowright R\,\frac{3a}{2} = W.2a\cos\frac{\pi}{4} + W_1 \times 4a\cos\frac{\pi}{4}$ (10)

$R = \frac{2}{3}\left(W \times \frac{2}{\sqrt{2}} + W_1 \times \frac{4a}{\sqrt{2}}\right)$

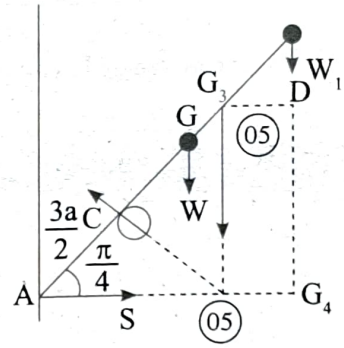
$\uparrow R\cos\frac{\pi}{4} + W + W_1$ (05)

$\frac{2}{3}\left(\frac{2W}{\sqrt{2}} + \frac{4W_1}{\sqrt{2}}\right).\frac{2}{\sqrt{2}} = W + W_1$

$2W + 4W_1 = 3W + 3W_1$

$W_1 = W$ (05) **25**

Aliternative Method



$\Delta ACG_1 \quad = AG_1 = \frac{3a}{2} = \sec\frac{\pi}{4}$

$= \frac{3a}{2}\sqrt{2} = \frac{3a}{\sqrt{2}}$ (05)

$AG_2 \quad = 2a\cos\frac{\pi}{4} = \frac{2a}{\sqrt{2}}$

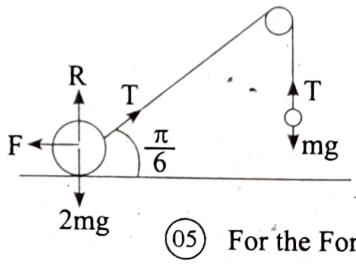
$\therefore\ G_2G_1 \quad = AG_1 - AG_2 = \frac{a}{\sqrt{2}}$

$G_1G_4 = G_2 \times G_4 - G_2 \times G_1 = \frac{2a}{\sqrt{2}} - \frac{a}{\sqrt{2}} = \frac{a}{\sqrt{2}}$ (05)

$G_1 \curvearrowright W_1 \times G_1G_4 - W \times G_2G_1 = 0$

$\therefore \quad W_1 = W$ (05) **25**

08.



(05) For the Forces

(P) $\uparrow R = 2mg - \frac{T}{2}$ (05)

(Q) $T = mg$

$R = 2mg - \frac{mg}{2} = \frac{3mg}{2}$ (05)

$\rightarrow T\cos\frac{\pi}{6} = F$

$F = \frac{\sqrt{3}mg}{2}$ (05)

$\mu \geq \frac{F}{R} = \frac{\frac{\sqrt{3}mg}{2}}{\frac{3mg}{2}} = \frac{1}{\sqrt{3}}$ (05) **25**

---

09. $B = (A \cup A') \cap B$

$= (A \cap B) \cup (A' \cap B)$ and $(A \cap B) \cap (A' \cap B) = \phi$

$\therefore P(B) = P(A \cup B) + P(A' \cap B)$ (05)

$= \frac{1}{3} + \frac{1}{12} = \frac{5}{12}$ (05)

A and B are independent

$\Rightarrow P(A \cap B) = P(A) \cap P(B)$ (05)

$\therefore \frac{1}{3} = P(A) \frac{5}{12}$

$\therefore P(A) \frac{4}{5}$

Now,

$(A \cup B) = P(A) + P(B) - P(A \cap B)$ (05)

$= \frac{4}{5} + \frac{5}{12} - \frac{1}{3}$

$P(A \cup B) = \frac{4}{5} + \frac{1}{12} = \frac{53}{60}$ (05)

25

10. $\frac{\sum_{i=1}^{10} x_i}{10} = 30$ (05)

$\sum_{i=1}^{10} x_i = 300$

$\sum_{i=1}^{10} (x_i - 20)^2 = 1040$

$\sum_{i=1}^{10} x_i^2 - 40 \sum_{i=1}^{10} x_i^1 + 400 = 1040$ (05)

$\therefore \sum_{i=1}^{10} x_i^2 = 1040 - 400 + 40 \times 300$ (05)

$= 12\,640$

$\therefore$ The Variance $= \frac{\sum_{i=1}^{10} x_i^2}{10} - 30^2$ (05)

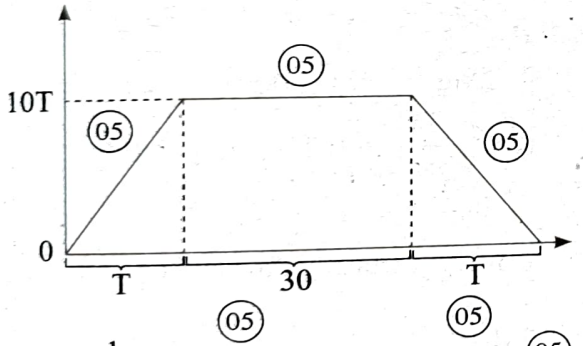
$= \frac{12\,640}{10} - 30^2$

$= 364$ (05)

25

Part B

11. (a)



$\frac{1}{2} \times (60 + 2T) \times 10T = 1750$ (05)

$(30 + T)\ 10T = 1750$

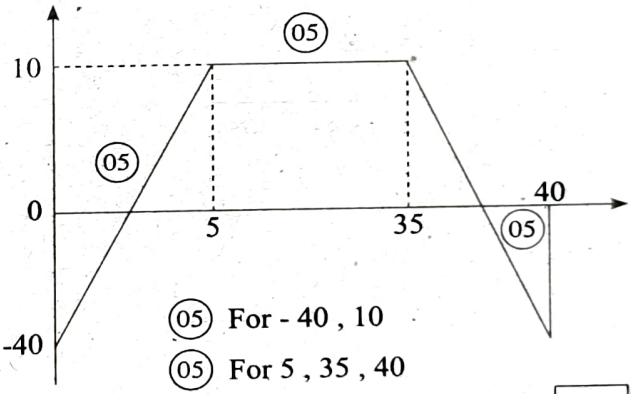
$T^2 + 30T - 175 = 0$ (05)

$(T - 5)(T + 35) = 0$ (05)

$T = 5\ (\because T > 0)$ (05)

The total time taken $= (5 + 30 + 5)S = 40S$ (05)

50



25

(b) $\underline{v}(W, E) = \rightarrow u$ (05)

$\underline{v}(P, W) = \sqrt{2}u$ (05)

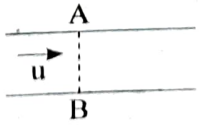
$\underline{v}(P, E) = \downarrow$ (05)

$\underline{v}(P, W) = \underline{v}(P, E) + \underline{v}(E, W)$

$= \underline{v}(E, W) + \underline{v}(P, E)$

$= \overrightarrow{xy} + \overrightarrow{yz}$

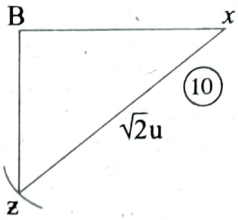
$= \overrightarrow{xz}$



$yz^2 = 2u^2 - u^2 = u^2$

$\underline{v}(A, B) = \underline{v}(A, E) + \underline{v}(E, B)$

$= \underline{v}(A, E) - \underline{v}(B, E)$

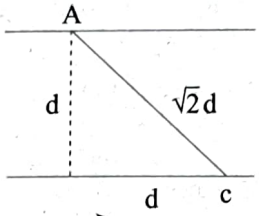


$\therefore$ yz = u (05)

The required time

$= \frac{d}{yz} S$

$= \frac{d}{u} S$ (05) 35



$\underline{v}(Q, P) = 45^0$ (05)

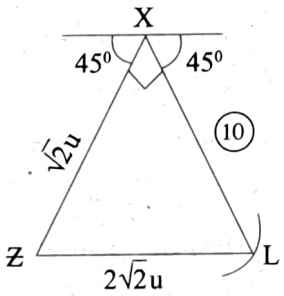
$\underline{v}(Q, W) = 2\sqrt{2}u$ (05)

$\underline{v}(P, W) = 45^0$, $\sqrt{2}u$ (05)

$\underline{v}(Q, W) = \underline{v}(Q, P) + \underline{v}(P, W)$

$= \vec{LX} + \vec{XZ}$

$= \vec{LZ}$



$LX^2 = (2\sqrt{2}u)^2 - (\sqrt{2}u)^2$

$LX^2 = 6u^2$

$LX = \sqrt{6}u$ (05)

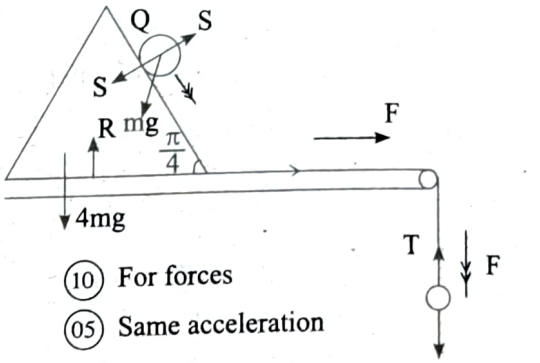
The time taken by Q to meet P $= \frac{AC}{XL} = \frac{\sqrt{2}d}{\sqrt{6}u}$ (05)

$= \frac{1d}{\sqrt{3}u} < \frac{d}{u}$ (05)

$\therefore$ Q meets P before P reaches B 40

---

12. (a)



(10) For forces

(05) Same acceleration

Apply F_ = ma_

(P) ↓ mg - T = MF ——— (1) (10)

(Q) ↘ $mg\sin\frac{\pi}{4} + m(f + F\cos\frac{\pi}{4})$ ——— (2) (10)

Q and the wedge →

$T = 4mF + m(F + f\cos\frac{\pi}{4})$ ——— (3) (15)

(1) + (3) $Mg = (M + 5m)F + \frac{mf}{\sqrt{2}}$

(2) : $\frac{g}{\sqrt{2}} = f + \frac{F}{\sqrt{2}}$

$Mg = (M + 5m)F + \frac{m}{\sqrt{2}}\left(\frac{g}{\sqrt{2}} - \frac{F}{\sqrt{2}}\right)$

$2Mg = (2M + 10m)F + mg - mF$

$F = \left(\frac{2M - m}{2M + 9m}\right) > 0$ if $m < 2M$

(05) (05)

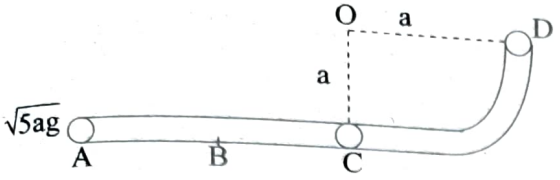
P moves vertically downwards 60

2M = m , F = 0 (05)

P and the wedge remain at rest. (system was released from rest)

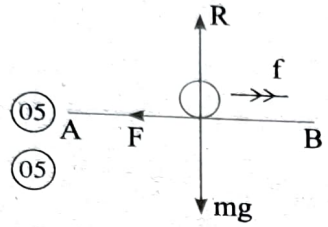
Q moves down along the line of greatest slop of the face contacting it (with acceleration $\frac{g}{\sqrt{2}}$ (05)

10



From A to B :

$\underline{F} = m\underline{a}$



$\uparrow$ $R - mg = 0$

$R = mg$ (05)

$F = \frac{1}{2}R$ (05)

$F = \frac{mg}{2}$

$\rightarrow \underline{F} = m\underline{a}$

$-F = mf$ (05)

$-\frac{mg}{2} = mf$

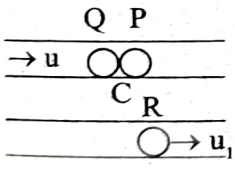
$f = -\frac{g}{2}$ (05)

$u^2 = 5ga - 2\frac{g}{2}a$

$u^2 = 5ga - ga = 4ga$

$u = 2\sqrt{ga}$ (05)

The velocity of the particle Q just before it coolides with P at C = u. (05)



$\rightarrow \underline{I} = \Delta m\underline{v}$

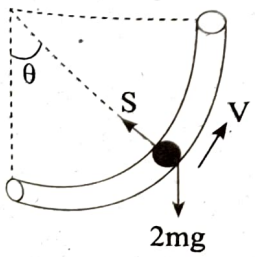
$\Rightarrow 2mu_1 = mu$ (05)

$u_1 = \frac{u}{2}$

$u_1 = \frac{g}{2} = \sqrt{ga}$ 35

Conservation of energy :

$\frac{1}{2}mu_1^2 - 2mga = \frac{1}{2}(2m)V^2 - 2mga\cos\theta$



KE (05)

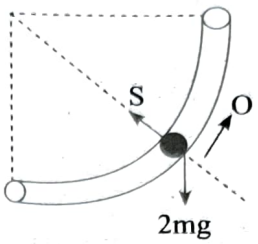
PE (05)

Equation (05)

$\frac{1}{2}ga - ga = \frac{1}{2}V^2 - ga\cos\theta$

$V^2 = 2ga\cos\theta - ga$ (05)

$V^2 = ga(2\cos\theta - 1)$ 20



$V = 0 \Leftrightarrow \cos\theta = \frac{1}{2}$ (05)

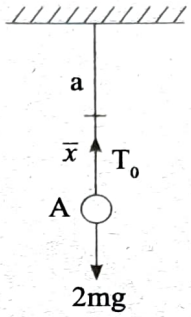
$\Leftrightarrow \theta = \frac{\pi}{3}$ (05)

$\nwarrow \underline{F} = m\underline{a}$

$S - 2mg\cos\frac{\pi}{3} = 0$ (10)

$S = mg$ (05) 25

13.



$T_0 = 2mg\frac{\bar{x}}{a}$ (05)

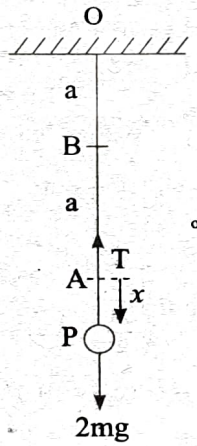
$\uparrow T_0 = 2mg$ (05)

$\therefore 2mg\frac{\bar{x}}{a} = 2mg$

$\bar{x} = a$ (05)

The extension = a

15



$\downarrow \underline{F} = m\underline{a}$ apply (05)

$2mg - \frac{2mg(a+x)}{a} = 2m\ddot{x}$ (10)

$\ddot{x} + \frac{g}{a}x = 0, \; -\frac{a}{2} \le x \le \frac{a}{2}$ (05)

i.e. $\ddot{x} + \omega^2 x = 0, \; -\frac{a}{2} \le x \le \frac{a}{2}$

where $\omega = \sqrt{\frac{g}{a}}$

20

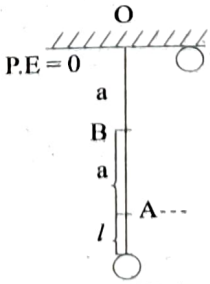
The required maximum value of l = a (10)

10

Consevation of energy

$\frac{1}{2}(2mg)\frac{(a+l)^2}{a} - 2mg(2a+l) = 0 + \frac{1}{2}(2m)ga$ (05)

(05) (05) (05)



$\therefore \frac{(a+l)^2}{a} - 2(2a+l) = a$

$\therefore a^2 + 2al + l^2 - 4a^2 - 2la = a^2$

$\therefore l^2 = 4a^2$ (05)

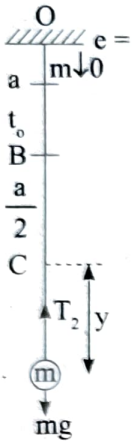
$l = 2a$ (05)

30

From O to B.

Now, $2mg\ \frac{\bar{y}}{a} = mg$ (05)

$\bar{y} = \frac{a}{2}$ (05)



Apply $\downarrow \underset{\sim}{F} = m\underset{\sim}{a}$

$mg - \frac{2mg(\bar{y}+y)}{a} = m\ddot{y}$ (10)

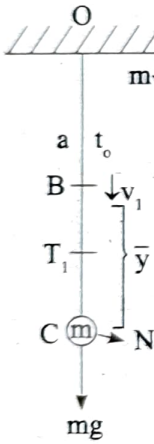
$mg - \frac{2mg(\frac{a}{2}+y)}{a} = m\ddot{y}$

$\ddot{y} + \frac{2g}{a}y = 0$

$\ddot{y} + \frac{2g}{a}y = 0$ (05)

$\ddot{y} + \omega_1^2 y = 0,\ \omega_1 = \sqrt{\frac{2g}{a}}$ 25

From O to B



Apply $\downarrow$ $V^2 = u^2 + 2as$

$V_1^2 = 0 + 2ga$

$V_1 = \sqrt{2ga}$ (05)

Apply $\downarrow$ $s = ut + \frac{1}{2}at^2$

$a = \frac{1}{2}gt_0^2$ (05)

$t_0 = \sqrt{\frac{2a}{g}}$ (05)

$\dot{y}^2 = \omega_1^2(c^2 - y^2)$ (05)

$\dot{y} = \sqrt{2ga}$ when $y = -\frac{a}{2}$

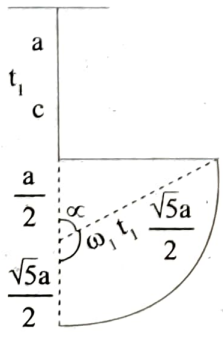
$\therefore 2ga = \frac{2g}{a}\left(c^2 - \frac{a^2}{4}\right)$

(05)

$a^2 = c^2 - \frac{a^2}{4}$

$\therefore c^2 = \frac{5a^2}{4}$

$c = \frac{\sqrt{5}a}{2}$ (05)



$\cos\alpha = \frac{\frac{a}{2}}{\frac{\sqrt{5}a}{2}} = \frac{1}{\sqrt{5}}$ (05)

(05)

$\omega_1 t_1 = \pi - \alpha$ (05)

$\therefore t_1 = \sqrt{\frac{2g}{a}}\left\{\pi - \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right\}$

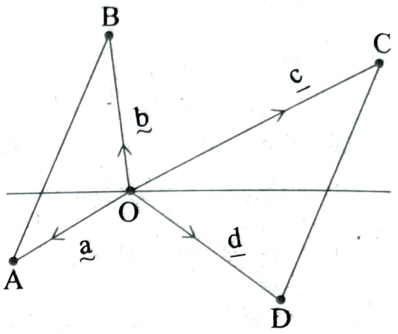
The required time $= t_0 + t_1 = \sqrt{\frac{2g}{a}}\left[1 + \pi - \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right]$

(05)

50

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14. (a)



$\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a} = 2\underset{\sim}{i} + 5\underset{\sim}{j}$ (05)

$\overrightarrow{DC} = \underset{\sim}{c} - \underset{\sim}{d} = (8\underset{\sim}{i} + \alpha\underset{\sim}{j}) - (4\underset{\sim}{i} + 2\underset{\sim}{j})$

$= 4\underset{\sim}{i} + (\alpha + 2)\underset{\sim}{j}$ (05)

$\overrightarrow{AB} \parallel \overrightarrow{DC} \Leftrightarrow \overrightarrow{AB} = k\overrightarrow{DC}$ for $k \in \mathbb{R}$

$\Leftrightarrow 4\underset{\sim}{i} + (\alpha + 2)\underset{\sim}{j} = k(2\underset{\sim}{i} + 5\underset{\sim}{j})$ (05)

$4 = 2k$ and $\alpha + 2 = 5k$ (05)

$\therefore k = 2$ and $\alpha + 2 = 10$

$\alpha = 8$ (05) 25

$\overrightarrow{AE} \parallel \overrightarrow{AC}$

$\overrightarrow{AE} = \lambda(\overrightarrow{AC})$ for $\lambda \in \mathbb{R}$

$\underset{\sim}{e} - \underset{\sim}{a} = \lambda(\underset{\sim}{c} - \underset{\sim}{a})$ (05)

$\underset{\sim}{e} = (1 - \lambda)\underset{\sim}{a} + \lambda\underset{\sim}{c}$ (05)

$\overrightarrow{BE} \parallel \overrightarrow{BD}$

$\overrightarrow{BE} = \mu\overrightarrow{BD}$ for $\mu \in \mathbb{R}$ (05)

$\underset{\sim}{e} - \underset{\sim}{b} = \mu(\underset{\sim}{d} - \underset{\sim}{b})$

$\underset{\sim}{e} = (1 - \mu)\underset{\sim}{b} + \mu\underset{\sim}{d}$ (05) 20

$\therefore (1 - \lambda)\underset{\sim}{a} + \lambda\underset{\sim}{c} = (1 - \mu)\underset{\sim}{b} + \mu\underset{\sim}{d}$ (05)

$(1-\lambda)(-\underset{\sim}{i} - \underset{\sim}{j}) + \lambda(8\underset{\sim}{i} + 8\underset{\sim}{j}) = (1-\mu)(\underset{\sim}{i} + 4\underset{\sim}{j}) + \mu(4\underset{\sim}{i} - 2\underset{\sim}{j})$

$(9\lambda - 1)\underset{\sim}{i} + (9\lambda - 1)\underset{\sim}{j} = (3\mu + 1)\underset{\sim}{i} + (-6\mu + 4)\underset{\sim}{j}$ (10)

$(9\lambda - 1) = (3\mu + 1)$ and $(9\lambda - 1) = -6\mu + 4$

$3\mu + 1 = -6\mu + 4$ (05)

$9\mu = 3$

$\mu = \frac{1}{3}$ $\therefore \lambda = \frac{1}{3}$ (05)

$\therefore \underset{\sim}{e} = 2\underset{\sim}{i} + 2\underset{\sim}{j}$ (05) 30

Let $A\hat{E}D = \theta$

$\vec{EA} \cdot \vec{ED} = |\vec{EA}|\,|\vec{ED}|\cos\theta$ (05)

$\underline{a} - \underline{e} = 3\underline{i} - 3\underline{j}$

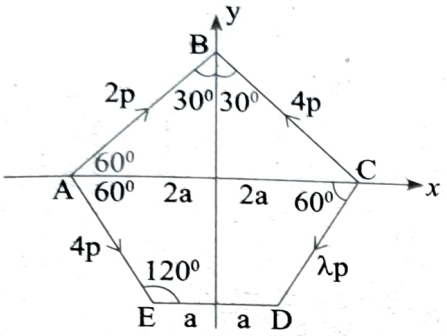
$\underline{d} - \underline{e} = 2\underline{i} - 4\underline{j}$

$(\underline{a} - \underline{e}) \cdot (\underline{d} - \underline{e}) = |\vec{EA}|\,|\vec{ED}|\cos\theta$ (10)

$-6 + 12 = \sqrt{18} \cdot \sqrt{20}\cos\theta$

$\therefore \theta = \cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$ (05) **20**

(b)



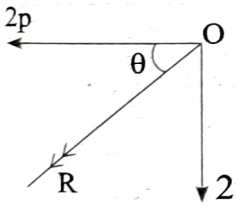
O↘ $(2p\sin60^0 \times 2a) - (4p\sin60^0 \times 2a) - (4p\sin60^0 \times 2a) + (\lambda p\sin60^0 \times 2a) = 0$ (10)

$-6p + \lambda p = 0$

$\lambda = 6\ (p \neq 0)$ (05)

$\uparrow Y = 2p\sin60^0 - 4p\sin60^0 + 4p\sin60^0 - 6p\sin60^0 = 2\sqrt{3}p$ (05)

$\rightarrow x = 2p \times \frac{1}{2} - 6p \times \frac{1}{2} = -2p$ (05)

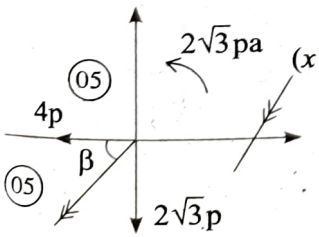


$|\underline{R}| = \sqrt{4 + 12p} = 4p$ (05)

$\tan\theta = \frac{2\sqrt{3}p}{2p} = \sqrt{3}$

$\theta = 60^0$ (05) **35**

$|\underline{R}_1| = \sqrt{16 + 12p} = 2p$ (05)



$\tan\beta = \frac{\sqrt{3}}{2}$

$\beta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

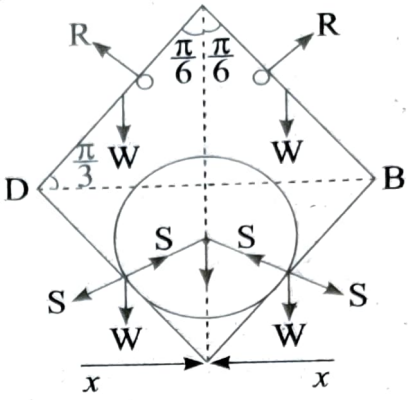
Clockwise moment about (x, y)

$4py - 2\sqrt{3}px - 2\sqrt{3}pa = 0$ (05)

i.e. $2y - \sqrt{3}x - \sqrt{3}a = 0$ **20**

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15. (a)



(10) For Forces

Let the distance between the pegs be 2b

For the disc ↑ $2\cos\frac{\pi}{3} = W$

$S = W$ (05)

For the system: ↑ $2R\cos\frac{\pi}{3} = 5W$

$R \cdot \frac{1}{2} = \frac{5}{2}W$

$R = 5W$ (05)

For BC : B↘ $W.a\cos\frac{\pi}{3} + S \times a - x \times 2a\sin\frac{\pi}{3} = 0$ (10)

$W \times \frac{1}{2} + W = x \times \sqrt{3}$

$\frac{3W}{2} = \sqrt{3}\,x$

$x = \frac{\sqrt{3}W}{2}$ (05)

For ABC :

A↘ $R.b\,\mathrm{cosec}\frac{\pi}{6} - 2W \times a\sin\frac{\pi}{6} - x \times 4a\cos\frac{\pi}{6} = 0$ (10)

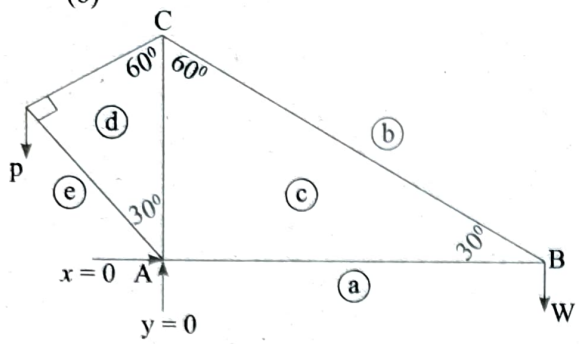
$5W \times 2b - Wxa - \frac{\sqrt{3}W}{2} \times 4a\,\frac{\sqrt{3}}{2} = 0$ (05)

$\therefore b = \frac{2a}{5}$

$\therefore$ The distance between the pegs $= \frac{4a}{5}$ (05)

55

(b)

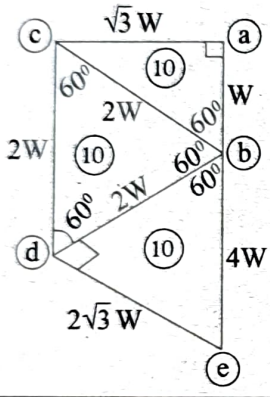


$AB = 2a\cot 30^0 = 2\sqrt{3}\,a$

$AD = 2a\cos 30^0 = \sqrt{3}\,a$

$A \curvearrowleft \; W \times 2\sqrt{3}\,a - P \times \sqrt{3}\,a \sin 30 = 0$ (10)

$P = 4W$ (05) **15**



(30)

Rod		
AB	$\sqrt{3}$ W (05)	Thrust (05)
BC	2W (05)	Tension (05)
AC	2W (05)	Thrust (05)
CD	2W (05)	Tension (05)
AD	$2\sqrt{3}$ W (05)	Thrust (05)

(50)

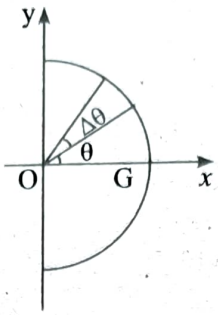
80

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16. (i) By symmetry the center of mass lies on the x - axis (05)

$\Delta m = a\,\Delta\theta\rho$, where ρ is the mass per unit length.

Let $OG = \bar{x}$



$$\bar{x} = \frac{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} a\rho a\cos\theta\, d\theta}{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} a\rho\, d\theta} \quad (05) \quad (05)$$

$$= \frac{\left[a\sin\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}{\left[\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}} \quad (05)$$

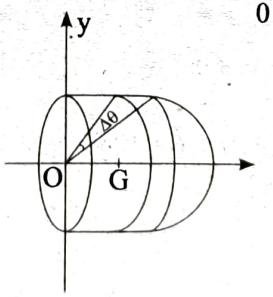
$$= \frac{2a}{\pi} \quad (05) \qquad \boxed{25}$$

By symmetry the center of mass G lies on the x - axis (05)

$\Delta m = 2\pi (a\sin\theta)\; a\rho\theta$, ρ where is the mass per unit area.

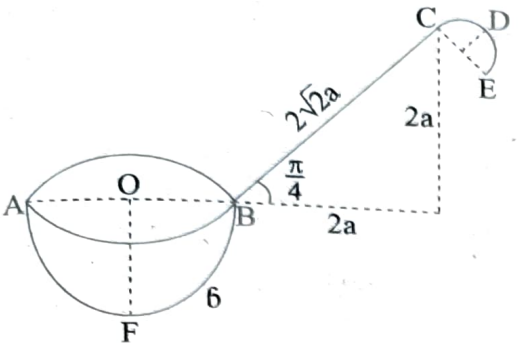
Let $OG = \bar{x}$

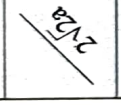
$$\bar{x} = \frac{\int_0^{\frac{\pi}{2}} 2\pi (a\sin\theta)\; a\rho a\cos\theta\, d\theta}{\int_0^{\frac{\pi}{2}} 2\pi (a\sin\theta)\; a\rho\, d\theta} \quad (05) \quad (05)$$



$$= \frac{\left[\frac{a\sin\theta}{2}\right]_0^{\frac{\pi}{2}}}{\left[-\cos\theta\right]_0^{\frac{\pi}{2}}} \quad (05) \quad (05)$$

$$= \frac{a}{2} \quad (05) \qquad \boxed{30}$$



Objects	Mass	Distance to the CM from OB ↑	from OF →
	$2\pi(2a^2)6$ $8\pi a^2 6$ (05)	$\frac{-2a}{3\pi} = -a$ (05)	0 (05)
	$(2\sqrt{2}a)\sqrt{2}a6$ $= 4a^2 6$ (05)	$\sqrt{2}a\sin\frac{\pi}{4} = a$ (05)	3a (05)
	$(\pi\sqrt{2}a)(\sqrt{2}a6)$ $= 2\pi a^2 6$ (05)	$a + \frac{2a}{\pi}$ (05)	$5a + \frac{2a}{\pi}$ (05)
Spoon	$(10\pi + 4)\,a^2 6$ $(5\pi + 2)\,2a^2 6$ (05) K	$\bar{y}$	$\bar{x}$

(50)

$k = 5\pi + 2$

$$2ka^2 6\bar{x} = 4a^2 6 \times 3a + 2\pi a^2 6\left(5a + \frac{2a}{\pi}\right) \quad (10)$$

$$k\bar{x} = 6a + 5\pi a + 2a$$

$$\bar{x} = \left(\frac{8 + 5\pi}{2 + 5\pi}\right) a \quad (05)$$

$$2ka^2 6\bar{y} = 8\pi a^2 6\,(-a) + 4a^2 6 \times a + 2\pi a^2 6\left(a + \frac{2a}{\pi}\right) \quad (10)$$

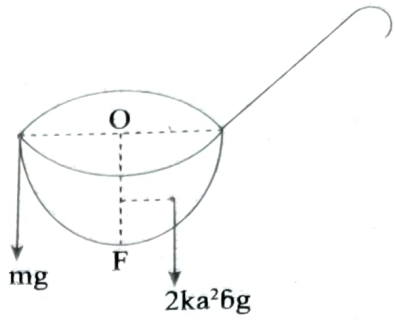
$$k\bar{y} = -4\pi a + 2a + \pi a + 2a$$

$$k\bar{y} = 4a - 3\pi a$$

$$\bar{y} = \frac{(4 - 3\pi)\,a}{2 + 5\pi}$$

$$= -\frac{(3\pi - 4)\,a}{(2 + 5\pi)} \quad (05)$$

$\therefore$ Below OB a distance $\frac{3\pi - 4}{2 + 5\pi}$ 80



O ↻ $mg \times 2a = 2ka^2\,6g \times \bar{x}$ (10)

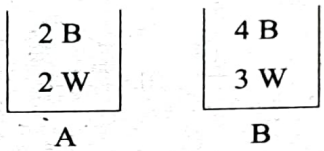
$\therefore m = ka6\left(\frac{8 + 5\pi}{k}\right) a$

$\therefore m = (8 + 5\pi)\,a^2 6$ (05)

15

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17. (a)



1 \ 2	1	2	3	4	5	6
1	(2)	(3)	4	(5)	6	(7)
2	(3)	4	(5)	6	(7)	8
3	4	(5)	6	(7)	8	9
4	(5)	6	(7)	8	9	10
5	6	(7)	8	9	10	(11)
6	(7)	8	9	10	(11)	12

P(sum is prime) $= \frac{1 + 2 + 4 + 6 + 2}{36} = \frac{15}{36} = \frac{5}{12}$

A (15)

P(sum is not a prime) $= \frac{7}{12}$ (05)

Let C = ball drawn in black

(i) Total probability law

$$P(C) = P(A \cap C) + P(B \cap C)$$

$$= P(C|A) \cdot P(A) + P(C|B) \cdot P(B) \quad (05)$$

$$= \frac{3}{5} \times \frac{5}{12} + \frac{4}{7} \times \frac{7}{12}$$

(10) (10)

$$= \frac{1}{12}(3 + 4)$$

$$= \frac{7}{12} \quad (05)$$

50

(ii) Bays Theorem ;

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{P(C/A) \cdot P(A)}{P(C)}$$ (05)

(05)

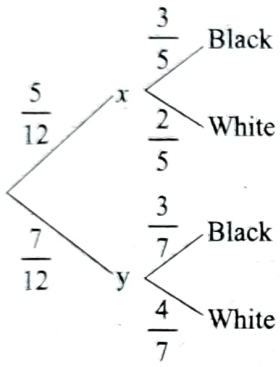
$$= \frac{\frac{3}{5} \times \frac{5}{12}}{\frac{7}{12}} \quad (05) \qquad = \frac{3}{7}$$ (05)

20

Aliter :

x = sum is prime

y = sum is not prime



(10) (10)

$$P(C) = \frac{5}{12} \times \frac{3}{5} + \frac{7}{12} \times \frac{4}{7}$$ (05)

$$= \frac{1}{4} + \frac{1}{3}$$

$$= \frac{7}{12}$$ (05)

(10)

$$P(A/C) = \frac{\frac{5}{12} \times \frac{3}{5}}{\frac{7}{12}} \quad (05) \qquad = \frac{3}{7}$$ (05)

(b)

(05) (05) (05) (05)

Time (S)	fi	xi	$fi\,xi$	$fi\,xi^2$
0 - 10	10	5	50	250
10 - 20	20	15	300	4500
20 - 30	35	25	875	21875
30 - 40	20	35	700	24500
40 - 50	15	45	675	30375
	100		2600	81500

(05) (05)

$$\text{Median} = 20 + \frac{10}{35} \times 20 = 20 + \frac{40}{7} = 25.714$$ (05)

$$\bar{x} = \frac{\sum_{i=1}^{5} f_i x_i}{\sum_{i=1}^{5} f_i}$$ (05)

$$= \frac{2600}{100}$$ (05)

$$= 26$$ (05)

$$\text{Variance} = \frac{\sum_{i=1}^{5} f_i x_i}{\sum_{i=1}^{5} f_i} - \bar{x}^2$$ (05)

$$= \frac{81500}{100} - 26^2$$ (05)

$$= 815 - 676$$

$$= 139$$ (05)

65

Let y be the new mean

$$125\,\bar{y} = 100\bar{x} + 5\,(5 + 15 + 25 + 35 + 45)$$

(10)

$$= 2600 + 5 \times 125$$

$$= 3225$$

$$\therefore \bar{y} = \frac{3225}{125} = 25.8$$ (05)

15
