සියලු ම හිමිකම් ඇවිරිණි /முழுப் பதிப்புரிமையுடையது /All Rights Reserved]

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இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka
ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
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Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2022(2023)
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2022(2023)
General Certificate of Education (Adv. Level) Examination, 2022(2023)

සංයුක්ත ගණිතය	I
இணைந்த கணிதம்	I
Combined Mathematics	**I**

10 E I

උත්තර පත්‍ර පරීක්ෂකවරයාගේ
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පැය තුනයි	
மூன்று மணித்தியாலம்	
Three hours	

අමතර කියවීම් කාලය	-	මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம்	-	10 நிமிடங்கள்
Additional Reading Time	-	**10 minutes**

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

Instructions:

* *This question paper consists of two parts*;
  Part A (Questions 1 - 10) and **Part B** (Questions 11 - 17).
* **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(10) Combined Mathematics I		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner	
Checked by: 1	
2	
Supervised by:	

[see page two

Part A

1. Using the **Principle of Mathematical Induction**, prove that $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$ for all $n \in \mathbb{Z}^+$.

2. Sketch the graphs of $y = 2 - |x-2|$ and $y = \big||x|-2\big|$ in the same diagram.
Hence or otherwise, find all real values of x satisfying the inequality $\big||x|-2\big| + |x-2| \leq 2$.

[see page three

3. Shade in an Argand diagram, the region consisting of points that represent the complex numbers z satisfying the inequality $|\overline{z} + 2i| \leq 1$.

Find the greatest value of Arg z for the complex numbers z represented by the points in this shaded region.

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4. Let $a \in \mathbb{R}$. Write down the expansion of $(2 + ax)^5$ in ascending powers of x up to and including x^2 term.

Hence, find the values of a for which the coefficient of x^2 in the expansion of $(4 - 5x)(2 + ax)^5$ is -80.

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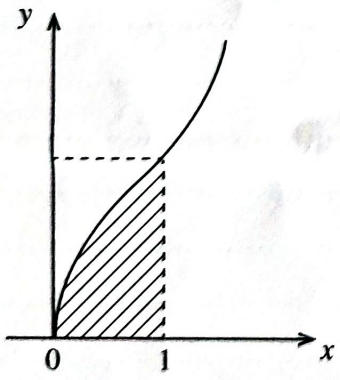
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[see page four

5. Show that $\lim\limits_{x\to 0} \dfrac{x\left((1+x)\operatorname{cosec} 2x - \cot 2x\right)}{\sqrt{1+2x} - \sqrt{1-2x}} = \dfrac{1}{4}$.

6. **Using** $\dfrac{\mathrm{d}}{\mathrm{d}x}\left\{x(x^2+1)\tan^{-1}x\right\} = (3x^2+1)\tan^{-1}x + x$, show that $\displaystyle\int_0^1 (3x^2+1)\tan^{-1}x\,\mathrm{d}x = \frac{1}{2}(\pi - 1)$.

The region enclosed by the curves $y = \sqrt{2(3x^2+1)\tan^{-1}x}$, $x = 1$ and $y = 0$ is rotated about the x-axis through 2π radians. Show that the volume of the solid thus generated is $\pi(\pi - 1)$.



[see page five

7. Let $a, b > 0$. A curve is parametrically given by $x = a\sec\theta$ and $y = b\tan\theta$ for $0 < \theta < \frac{\pi}{2}$. The tangent line to the curve at the point $P \equiv (a\sec\theta, b\tan\theta)$ passes through the point $(0, -b)$. Find the coordinates of P.

8. Let $ABCD$ be a square with $A \equiv (1, 3)$ and $C \equiv (7, 5)$. Find the x-coordinates of B and D.

[see page six

9. Find the equation of the circle that touches the circle $x^2 + y^2 - 6x - 4y - 12 = 0$ externally at the point (6, 6) and has its centre on the line $x = 12$.

10. Show that $\cos 5\theta = \cos 3\theta$ if and only if $\theta = \frac{n\pi}{4}$ for $n \in \mathbb{Z}$.

Show also that $\frac{\sin 5\theta - \sin 3\theta}{\cos 5\theta - \cos 3\theta} = -\cot 4\theta$ for $\theta \neq \frac{n\pi}{4}$ and $n \in \mathbb{Z}$.

[see page seven

සියලු ම හිමිකම් ඇවිරිණි/முழுப் பதிப்புரிமையுடையது/All Rights Reserved]

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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2022(2023)
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2022(2023)
General Certificate of Education (Adv. Level) Examination, 2022(2023)

සංයුක්ත ගණිතය	I
இணைந்த கணிதம்	I
Combined Mathematics	**I**

10 E I

Part B

* Answer **five** questions only.

11. (*a*) Let $0 < |p| < 1$. Show that the equation $p^2x^2 - 2x + 1 = 0$ has real distinct roots.

Let α and β $(> \alpha)$ be these roots. Show that α and β are both positive.

Find $(\alpha - 1)(\beta - 1)$ in terms of p, and **deduce** that $\alpha < 1$ and $\beta > 1$.

Show that $\sqrt{\beta} - \sqrt{\alpha} = \frac{1}{|p|}\sqrt{2(1-|p|)}$

It is **given that** $\sqrt{\beta} + \sqrt{\alpha} = \frac{1}{|p|}\sqrt{2(1+|p|)}$. Show that the quadratic equation whose roots are $\left|\sqrt{\alpha} - 1\right|$ and $\left|\sqrt{\beta} - 1\right|$ is $|p|x^2 - \sqrt{2(1-|p|)}\,x + \sqrt{2(1+|p|)} - |p| - 1 = 0$.

(*b*) Let $p(x) = 2x^3 + ax^2 + bx - 4$, where $a, b \in \mathbb{R}$. It is given that $(x+2)$ is a factor of both $p(x)$ and $p'(x)$, where $p'(x)$ is the derivative of $p(x)$ with respect to x. Find the values of a and b. For these values of a and b, completely factorise $p(x) - 3p'(x)$.

12. (*a*) Six mangoes and four oranges are to be distributed among eight students so that each student receives at least one fruit.

Find the number of different ways in which

(i) six students get one fruit each and out of the remaining two students one gets **two mangoes** and the other gets **two oranges**,

(ii) seven students get one fruit each, and the other student gets **three mangoes**,

(iii) seven students get one fruit each, and the other student gets **three fruits**.

(*b*) Let $U_r = \frac{4(2r+7)}{(2r+1)(2r+3)(2r+5)}$ for $r \in \mathbb{Z}^+$. Also, let $f(r) = \frac{A}{(2r+1)} + \frac{B}{(2r+3)}$ for $r \in \mathbb{Z}^+$, where A and B are real constants. Determine the values of A and B such that $U_r = f(r) - f(r+1)$ for $r \in \mathbb{Z}^+$.

Hence or otherwise, show that $\sum_{r=1}^{n} U_r = \frac{4}{5} - \frac{3}{2n+3} + \frac{1}{2n+5}$ for $n \in \mathbb{Z}^+$.

Deduce that the infinite series $\sum_{r=1}^{\infty} U_r$ is convergent and find its sum.

Hence, find the value of the real constant k such that $\sum_{r=1}^{\infty} (U_r + kU_{r+1}) = 1$.

[see page eight

13.(*a*) Let $\mathbf{A} = \begin{pmatrix} a & -2 \\ 1 & a+2 \end{pmatrix}$. Show that $\mathbf{A}^{-1}$ exists for all $a \in \mathbb{R}$.

The matrices $\mathbf{P} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} 2 & 3 & 2 \\ -1 & 7 & 4 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ are such that $\mathbf{A} = \mathbf{PQ}^{\mathrm{T}} + \mathbf{R}$. Show that $a = 1$.

For this value of a, write down $\mathbf{A}^{-1}$ and **hence**, find the values of x and y such that $\mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \end{pmatrix}$.

(*b*) Let $z, w \in \mathbb{C}$. Show that $z\bar{z} = |z|^2$ and **hence**, show that $|z+w|^2 = |z|^2 + 2\,\mathrm{Re}(z\bar{w}) + |w|^2$.

Deduce that $|z+w|^2 + |z-w|^2 = 2\left(|z|^2 + |w|^2\right)$ and give a geometric interpretation for it when the points representing z, w and 0 in the Argand diagram are non-collinear.

(*c*) Let $z = -1 + \sqrt{3}i$. Express z in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $\frac{\pi}{2} < \theta < \pi$.

Let $z^n = a_n + ib_n$, where $a_n, b_n \in \mathbb{R}$ for $n \in \mathbb{Z}^+$. Write down $\mathrm{Re}\left(z^m \cdot z^n\right)$ in terms of a_m, a_n, b_m and b_n for $m, n \in \mathbb{Z}^+$.

Considering z^{m+n} and using De Moivre's theorem, show that $a_m a_n - b_m b_n = 2^{m+n}\cos(m+n)\frac{2\pi}{3}$, for $m, n \in \mathbb{Z}^+$.

14.(*a*) Let $f(x) = \frac{2x+3}{(x+2)^2}$ for $x \neq -2$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x) = \frac{-2(x+1)}{(x+2)^3}$ for $x \neq -2$.

Hence, find the interval on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

Also, find the coordinates of the turning point of $f(x)$.

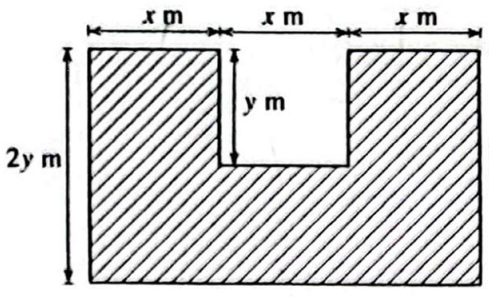
It is given that $f''(x) = \frac{2(2x+1)}{(x+2)^4}$ for $x \neq -2$. Find the coordinates of the point of inflection of the graph of $y = f(x)$.

Sketch the graph of $y = f(x)$ indicating the asymptotes, the turning point and the point of inflection.

State the smallest value of k for which $f(x)$ is one-one on $[k, \infty)$.

(*b*) The shaded region shown in the figure is of area 45 m^2. It is obtained by removing a rectangle of length x m and width y m from a rectangle of length $3x$ m and width $2y$ m. Show that the perimeter L m of the shaded region is given by $L = 6x + \frac{54}{x}$ for $x > 0$.

Find the value of x such that L is minimum.



[see page nine

15. (*a*) Find the values of the constants A, B and C such that

$x^2+x+2=A(x^2+x+1)+(Bx+C)(x+1)$ for all $x\in\mathbb{R}$.

Hence, write down $\dfrac{x^2+x+2}{(x^2+x+1)(x+1)}$ in partial fractions and find $\displaystyle\int\frac{x^2+x+2}{(x^2+x+1)(x+1)}\,dx$.

(*b*) Show that $1+\sin 2x=2\cos^2\left(\dfrac{\pi}{4}-x\right)$ and **hence**, show that $\displaystyle\int_0^{\frac{\pi}{2}}\frac{1}{1+\sin 2x}\,dx=1$.

(*c*) Let $I=\displaystyle\int_0^{\frac{\pi}{2}}\frac{x^2\cos 2x}{(1+\sin 2x)^2}\,dx$. Using integration by parts, show that $I=-\dfrac{\pi^2}{8}+J$, where $J=\displaystyle\int_0^{\frac{\pi}{2}}\frac{x}{1+\sin 2x}\,dx$.

Using the relation $\displaystyle\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$ and the result in (*b*), evaluate J and show that $I=\dfrac{\pi}{8}(2-\pi)$.

16. Let $P\equiv(x_0,y_0)$ and l be the straight line given by $ax+by+c=0$. Show that the perpendicular distance from P to l is $\dfrac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$.

Let l_1 and l_2 be two straight lines given by $4x-3y+8=0$ and $3x-4y+13=0$, respectively. Show that l_1 and l_2 intersect at $A\equiv(1,\,4)$.

Also, show that the parametic equations of the bisector of the acute angle between l_1 and l_2 can be written as $x=t$ and $y=t+3$, where $t\in\mathbb{R}$.

Hence, show that the equation of any circle touching both straight lines l_1 and l_2, and lying in the region between l_1 and l_2 that contains the acute angle, is given by $(x-t)^2+(y-t-3)^2=\dfrac{1}{25}(t-1)^2$, where $t\in\mathbb{R}$ and $t\neq 1$.

From among the above circles, find the equations of the circles that intersect the circle centred at A of radius 1, orthogonally.

[see page ten

17. (*a*) Write down $\cos(A+B)$ in terms of $\cos A$, $\cos B$, $\sin A$ and $\sin B$, and obtain a similar expression for $\sin(A-B)$.

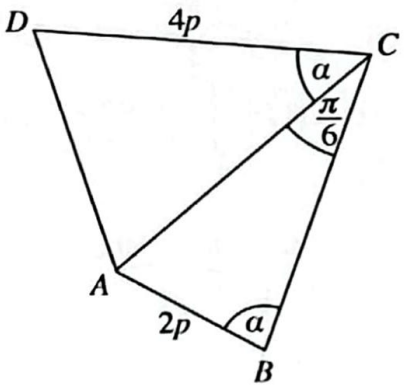
Let $k \in \mathbb{R}$ and $k \neq 1$. By separately considering the cases $k > 1$ and $k < 1$, express

$2k\cos\left(\theta+\frac{\pi}{3}\right)+2\sin\left(\theta-\frac{\pi}{6}\right)$ in the form $R\cos(\theta+\alpha)$, where $R(>0)$ in terms of k, and $\alpha(0<\alpha<2\pi)$ are real constants to be determined.

Hence, solve $2k\cos\left(\theta+\frac{\pi}{3}\right)+2\sin\left(\theta-\frac{\pi}{6}\right)=|k-1|$.

(*b*) In the quadrilateral $ABCD$ shown in the figure $AB = 2p$, $CD = 4p$, $A\hat{C}B = \frac{\pi}{6}$ and $A\hat{B}C = A\hat{C}D = \alpha$. Show that $AD^2 = 16p^2(\sin^2\alpha - \sin 2\alpha + 1)$.

Hence, show that if $AD = 4p$, then $\alpha = \tan^{-1}(2)$.



(*c*) Solve, $\tan^{-1}(\ln x^{\frac{2}{3}})+\tan^{-1}(\ln x)+\tan^{-1}(\ln x^2)=\frac{\pi}{2}$ for $x > 1$.

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