සියලු ම හිමිකම් ඇවිරිණි/முழுப் பதிப்புரிமையுடையது/All Rights Reserved]

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Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka
ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
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අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2022(2023)
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2022(2023)
General Certificate of Education (Adv. Level) Examination, 2022(2023)

සංයුක්ත ගණිතය II இணைந்த கணிதம் II **Combined Mathematics II**	**10 E II**

ප්‍රශ්න පත්‍ර පරිගණකවරුන්ගේ
ප්‍රයෝජනය සඳහා පමණි

පැය තුනයි மூன்று மணித்தியாலம் ***Three hours***	අමතර කියවීම් කාලය - මිනිත්තු 10 යි மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள் **Additional Reading Time - 10 minutes**

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number								

Instructions:

* *This question paper consists of two parts;*
  Part A (Questions 1 – 10) *and* **Part B** (Questions 11 – 17)
* **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

* In this question paper, g denotes the acceleration due to gravity.

For Examiners' Use only

(10) Combined Mathematics II		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
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In Numbers	
In Words	

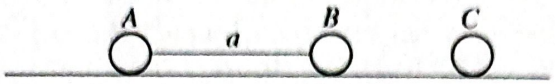
Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Three particles A, B and C, each of mass m, are placed in a straight line on a smooth horizontal table with A and B distance a apart, and connected by a light inextensible string of length a, as shown in the figure.



The particle B is given an impulse in the direction of $\overrightarrow{AB}$ such that its velocity just after the impulse is u. Show that the velocity of B just after it collides with C is $\frac{1}{2}(1-e)u$ in the direction of $\overrightarrow{AB}$, where e is the coefficient of restitution between B and C.
Also, find the time taken, after this collision, for A to collide with B.

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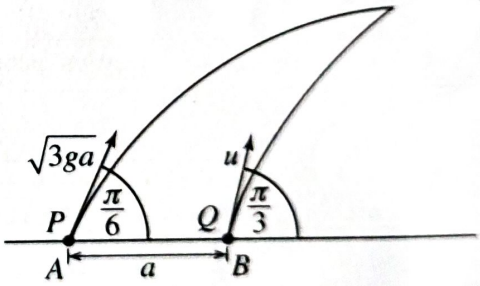
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2. A and B are two points on a horizontal ground such that $AB = a$. Two particles P and Q are projected from the points A and B respectively, at the same instant and in the vertical plane that contains the line AB such that they collide with each other after a time T at a point in space. The initial velocities of P and Q are given in the figure.



Show that $u = \sqrt{ga}$ and find T in terms of a and g.

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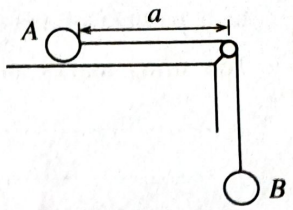
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[see page three

3. Two particles A and B of masses m and $3m$, respectively, are attached to the ends of a light inextensible string. The particle A is held at rest on a horizontal table with the string passing over a small smooth pulley fixed at the edge of the table. The particle B hangs vertically below the pulley. The system is released from rest with the particle A at a distance a from the pulley. In the subsequent motion, a constant frictional force of magnitude $\frac{1}{2}mg$ acts on A. Find the acceleration of A.
Also, find the speed of A at the instant when A reaches the pulley.



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4. A car of mass 1500 kg working with a constant power 80 kW moves on a horizontal road against a constant resistance. The acceleration of the car is 2 m s^{-2}, when it moves with the speed 20 m s^{-1}. Obtain equations sufficient to determine the acceleration of the car when it moves upward on a road of inclination $\sin^{-1}\left(\frac{2}{3}\right)$ to the horizontal with speed 8 m s^{-1} working with the same constant power against the same constant resistance.

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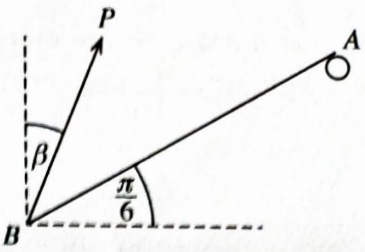
[see page four

5. One end of a light inextensible string of length a is attached to a fixed point and the other end to a particle of mass m. The particle moves in a horizontal circle with constant angular speed ω. The string makes an angle $\theta\left(0<\theta<\frac{\pi}{2}\right)$ with the downward vertical. Show that $\omega>\sqrt{\frac{g}{a}}$.

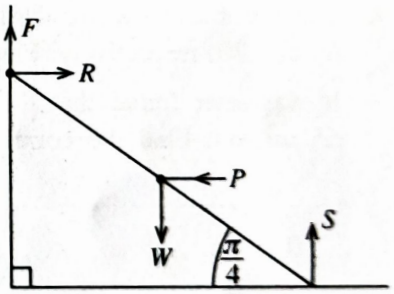
6. In the usual notation, the position vectors of two points A and B with respect to a fixed origin O are $3\mathbf{i}+2\mathbf{j}$ and $2\mathbf{i}+4\mathbf{j}$, respectively. Show that O, A and B are non-collinear.
Let C be the point such that $\overrightarrow{BC}=\lambda\overrightarrow{OA}$, where $\lambda\in\mathbb{R}$. Find $\overrightarrow{OC}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and λ.
Show that if $B\hat{O}C=\frac{\pi}{2}$, then $\lambda=-\frac{10}{7}$.

[see page five

7. A uniform rod AB is kept in equilibrium with its upper end A resting on a smooth peg by applying a force P to its lower end B at an angle β to the vertical, as shown in the figure. The rod makes an angle $\frac{\pi}{6}$ with the horizontal. Show that $\tan\beta = \frac{\sqrt{3}}{5}$.



8. A uniform ladder of weight W and length $2a$ is kept in equilibrium against a rough vertical wall with its lower end on a smooth horizontal ground, by a horizontal force of magnitude P applied at the mid-point of the ladder as shown in the figure. The ladder makes an angle $\frac{\pi}{4}$ with the ground. The coefficient of friction between the ladder and the wall is $\frac{1}{6}$. Show that, $\frac{3W}{4} \leq P \leq \frac{3W}{2}$.



[see page six

9. Let A and B be two events of a sample space Ω. It is given that $P(A)=\frac{2}{7}$, $P(A\cup B)=\frac{11}{14}$ and $P(A'\cup B')=\frac{4}{5}$. Find $P(B)$ and show that A and B are independent events.

10. The mean and the standard deviation of marks obtained by 100 students for an examination are 60 and 20, respectively. Find the z-score of a student who obtained 56 marks for this examination.

It was later found that this mark of 56 has been entered erroneously and it should have been 65 instead. Find the correct value of the mean of the marks obtained for this examination.

* *

[see page seven

සියලු ම හිමිකම් ඇවිරිණි/முழுப் பதிப்புரிமையுடையது/All Rights Reserved]

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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2022(2023)
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2022(2023)
General Certificate of Education (Adv. Level) Examination, 2022(2023)

සංයුක්ත ගණිතය II இணைந்த கணிதம் II **Combined Mathematics II**	**10 E II**

Part B

* Answer **five** questions only.

(In this question paper, g denotes the acceleration due to gravity.)

11. (*a*) A car P that begins its journey from rest from a point O on a straight horizontal road travels with a constant acceleration $2f$ m s^{-2} up to a point A on that road, where $OA = a$ m. It maintains the velocity attained at A throughout its remaining journey. At the instant when car P reaches the point A, another car Q begins its journey, along the same road in the same direction, from rest at the point O and moves with a constant acceleration f m s^{-2}. Sketch the velocity-time graphs for the motion of P and Q in the same diagram.

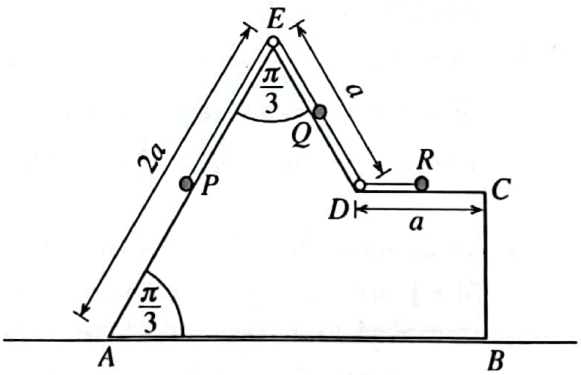
Hence, show that the time taken by Q to the instant when the velocities of P and Q are equal is $2\sqrt{\frac{a}{f}}$ s.

Now, let $a = 50$ and $f = 2$, and let B be the point on the road at which the car Q passes the car P. Show that $AB = 50\left(5+2\sqrt{6}\right)$ m.

(*b*) A ship P is sailing due South with a uniform speed 60 m s^{-1} relative to earth and a ship Q is sailing due East with a uniform speed $30\sqrt{3}$ m s^{-1} relative to earth. A third ship R appears to move in the direction 30° North of East when it is observed from P and ship R appears to move due South when it is observed from Q. Show that the ship R moves in the direction 30° South of East with a speed 60 m s^{-1} relative to earth.

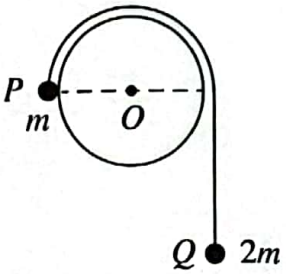
Suppose that initially the ship R is located 24 km away from P in a direction 60° South of West and 6 km away from Q in due West. Show that the distance between Q and R is 12 km, when P and R are the shortest distance apart.

12. (*a*) The vertical cross-section $ABCDE$ through the centre of gravity of a smooth uniform block of mass $4m$ is shown in the figure. The face containing AB is placed on a smooth horizontal floor. Also, AE and ED are the lines of greatest slope of the faces containing them. $AE = 2a$, $ED = a$, $DC = a$ and $E\hat{A}B = A\hat{E}D = \frac{\pi}{3}$. Three particles P, Q and R of masses $3m$, $2m$ and m, respectively, are placed at the mid-points of AE, ED and DC. The particles P and Q are attached to the ends of a light inextensible string passing over a smooth light small pulley fixed to the block at E, and the particles Q and R are attached to the ends of another light inextensible string passing through a smooth light small ring fixed to the block at D. Strings are taut in the position shown in the diagram and the system is released from rest from this position. Obtain equations sufficient to determine the time taken for the particle Q to reach E.



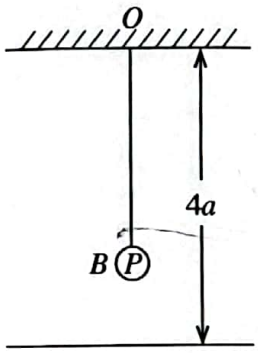
[see page eight

(*b*) A cylinder of radius a is fixed with its axis horizontal and the adjoining figure shows a vertical cross-section of the cylinder perpendicular to its axis. Two particles P and Q of masses m and $2m$, respectively connected by a light inextensible string are held with the string taut and OP horizontal in the position as shown in the figure and released from rest. Assuming that the particle Q moves vertically downwards, show that the speed v of the particle P when $\overrightarrow{OP}$ has turned through an angle θ $(0 \le \theta \le \frac{\pi}{6})$ is given by $v^2 = \frac{2ga}{3}(2\theta - \sin\theta)$.



The string is cut when $\theta = \frac{\pi}{6}$ and it is given that the particle P moving on the cylinder comes to instantaneous rest before it reaches the highest point of the cylinder. In the subsequent motion, find the speed of P when it is at a distance a vertically below its initial position.

13. One end of a light elastic string of natural length $2a$ and modulus of elasticity $2mg$ is attached to a fixed point O which is at a distance of $4a$ above a smooth horizontal floor, and the other end to a particle P of mass m. The particle P hangs in equilibrium at B. Show that the extension of the string is a.



Now, the particle P is given an impulse of mv vertically downwards.

Show that the equation of motion of P is $\ddot{x} + \omega^2 x = 0$ where $\omega = \sqrt{\frac{g}{a}}$ and $BP = x$.

Using the formula $\dot{x}^2 = \omega^2(c^2 - x^2)$, where c is the amplitude, show that if $v > \sqrt{ag}$, P hits the floor.

Now, suppose that $v = 3\sqrt{ag}$.

Find the velocity with which P hits the floor.

The coefficient of restitution between P and the floor is e. If $e < \frac{1}{\sqrt{2}}$, show that the particle P will not reach O.

If it is given that $e = \frac{1}{2}$, find the velocity of P when the string becomes slack for the first time.

Find the total time taken by P to come to instantaneous rest for the first time, from the instant it was given the impulse at B.

14. (*a*) Let the position vectors of four points A, B, C and D be $\mathbf{a}$, $\mathbf{b}$, $3\mathbf{a}$ and $4\mathbf{b}$, respectively with respect to a fixed origin O, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel vectors. E is the point of intersection of AD and BC. Using the triangle law of addition for the triangle OAE, show that $\overrightarrow{OE} = \mathbf{a} + \lambda(4\mathbf{b} - \mathbf{a})$ for $\lambda \in \mathbb{R}$.

Similarly, show also that $\overrightarrow{OE} = \mathbf{b} + \mu(3\mathbf{a} - \mathbf{b})$ for $\mu \in \mathbb{R}$.

Hence, show that $\overrightarrow{OE} = \frac{1}{11}(9\mathbf{a} + 8\mathbf{b})$.

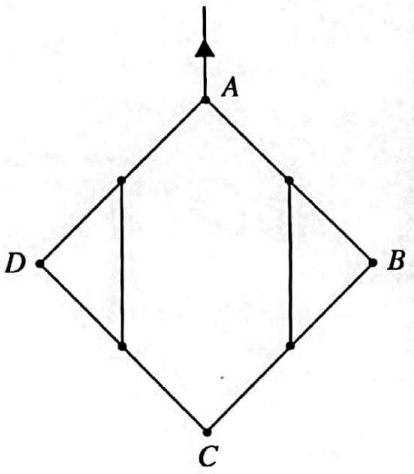
(*b*) Three forces $\alpha\mathbf{i} + 2\mathbf{j}$, $-3\mathbf{i} + \beta\mathbf{j}$ and $\mathbf{i} + 5\mathbf{j}$ act through the points with position vectors $\mathbf{i} + \mathbf{j}$, $3\mathbf{i} + \mathbf{j}$ and $2\mathbf{i} + 2\mathbf{j}$, respectively, where $\alpha, \beta \in \mathbb{R}$. It is given that this system of forces is equivalent to a couple. Find the values of α and β, and the moment of this couple.

Now, a new force $3\gamma\mathbf{i} + 4\gamma\mathbf{j}$ acting through the origin O is added to the above system of forces, where $\gamma > 0$. Show that the new system consisting of 4 forces is equivalent to a resultant force and, find its magnitude, direction and the equation of its line of action.

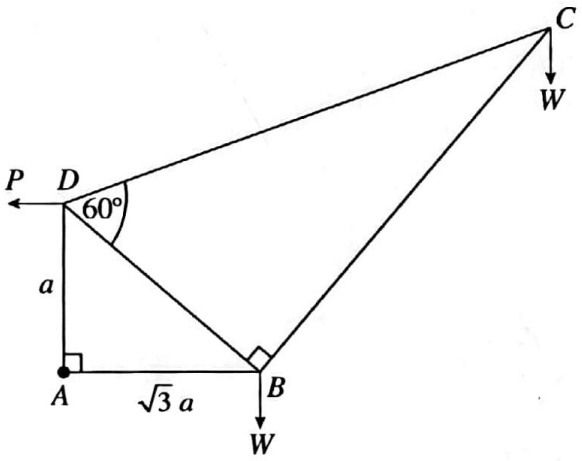
Next, it is given that when a force $p\mathbf{i} + q\mathbf{j}$ acting through the point with position vector $2\mathbf{i} + 3\mathbf{j}$ is added, this system consisting of 5 forces is in equilibrium. Find the values of γ, p and q.

[*see page nine*

15.(*a*) Four uniform rods AB, BC, CD and DA each of length $2a$ and weight W are smoothly jointed at their ends A, B, C and D. The midpoints of AB and BC are joined by a light inextensible string of length a. Similarly, midpoints of AD and DC are also joined by a light inextensible string of length a. The system is suspended in a vertical plane from the point A and stays in equilibrium as shown in the figure. Find the tensions in the strings and the reaction exerted on AB by BC at the joint B.



(*b*) The framework shown in the figure consists of five light rods AB, BC, CD, DA and DB smoothly jointed at their ends. It is given that $AD = a$, $AB = \sqrt{3}\,a$, $B\hat{A}D = 90°$, $C\hat{B}D = 90°$ and $B\hat{D}C = 60°$. At each of the joints B and C, a load W is suspended and the framework is smoothly hinged at A to a fixed point and kept in equilibrium in a vertical plane with AB horizontal by a horizontal force P applied to it at the joint D.



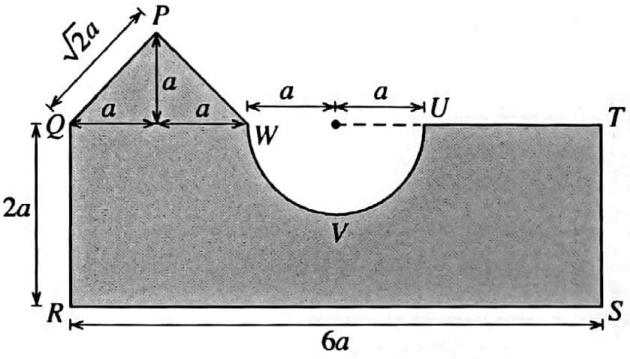
(i) Find the value of P.

(ii) Draw the stress diagram using Bow's notation for the joints C, B and D.

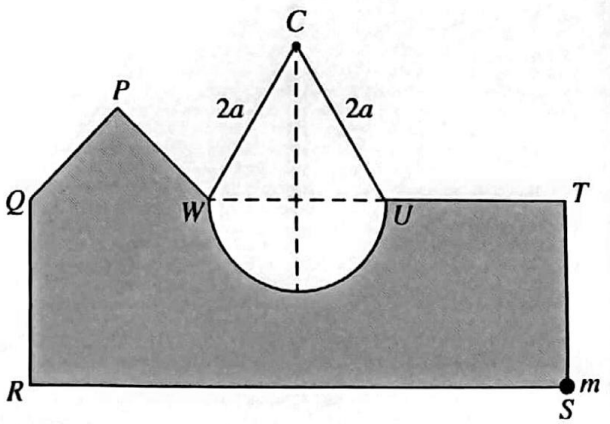
Hence, find the stresses in the rods, stating whether they are tensions or thrusts.

16. Show that the centre of mass of a uniform semi-circular lamina of radius r and centre O is at a distance $\frac{4r}{3\pi}$ from O.

A plane lamina is made from a uniform thin sheet metal of surface density σ by removing a semi-circle of radius a from the rectangle $QRST$ and by adding an isosceles triangle PQW with equal side-lengths $\sqrt{2}a$ to it, as shown in the adjoining figure. $QR = 2a$, $RS = 6a$ and $QW = 2a$. The centre of mass of this lamina lies at a distance $\bar{x}$ from QR and $\bar{y}$ from RS. Show that $\bar{x} = \frac{(74-3\pi)a}{(26-\pi)}$ and $\bar{y} = \frac{2(15-\pi)a}{(26-\pi)}$.



The lamina with a particle of mass m fixed to it at S, hangs in equilibrium by a light inextensible string of length $4a$ whose ends are attached to U and W and passing over a small smooth fixed peg C with side RS horizontal as shown in the figure. Find the value of m and the tension of the string in terms of a and σ.



[see page ten

17. (*a*) Four identical boxes B_1, B_2, B_3 and B_4, each contains 4 pens which are identical in all aspects except for their colour. Each box B_k contains k red pens and $4-k$ black pens for $k = 1, 2, 3, 4$. One of the four boxes is chosen at random and 2 pens are drawn from that box. Find the probability that

(i) the two pens drawn are red,

(ii) the pens are drawn from box B_4, given that the two pens drawn are red.

(*b*) The data sets $\{x_1, x_2, \ldots, x_n\}$ and $\{y_1, y_2, \ldots, y_m\}$ have the same mean, and their standard deviations are σ_x and σ_y, respectively. Show that the variance of the combined data set $\{x_1, \ldots, x_n, y_1, \ldots, y_m\}$ is given by $\dfrac{n\sigma_x^2 + m\sigma_y^2}{n+m}$.

Diameters of bolts produced at a factory is summarised in the following table:

Diameter (mm)	Number of bolts (in thousands)
2 – 6	2
6 – 10	5
10 – 14	8
14 – 18	4
18 – 22	1

Estimate the mean, the median and the variance of the distribution given above.

The diameters of another 40 000 bolts produced by a neighboring factory has the same mean, while the variance is 22.53 mm^2. Estimate the combined variance of the diameters of the bolts produced by both factories.

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