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Department of Examinations - Sri Lanka

G.C.E. (A/L) Examination - 2022 (2023)

10 - Combined Mathematics II

Marking Scheme

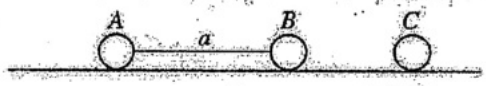
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This document has been prepared for the use of Marking Examiners. Some changes would be made according to the views presented at the Chief Examiners' meeting.

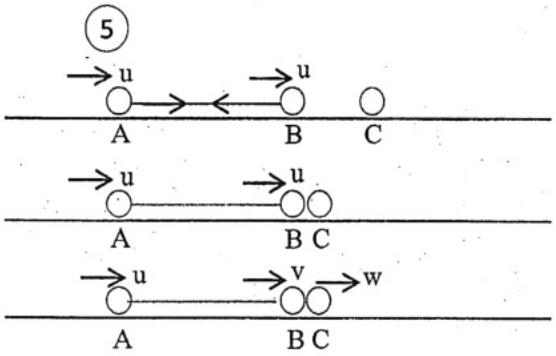
Amendments to be included

Part A

1. Three particles A, B and C, each of mass m, are placed in a straight line on a smooth horizontal table with A and B distance a apart, and connected by a light inextensible string of length a, as shown in the figure.



The particle B is given an impulse in the direction of $\overrightarrow{AB}$ such that its velocity just after the impulse is u. Show that the velocity of B just after it collides with C is $\frac{1}{2}(1-e)u$ in the direction of $\overrightarrow{AB}$, where e is the coefficient of restitution between B and C.
Also, find the time taken, after this collision, for A to collide with B.



Apply $\underline{I} = \Delta(m\underline{v})$ for B and C :

$\longrightarrow \quad 0 = mv + mw - mu$ (5)

$\therefore v + w = u$ ..... (1)

Apply Newton's Law of restitution:

$w - v = eu$ ..... (2) (5)

(1) − (2) : $2v = u - eu$

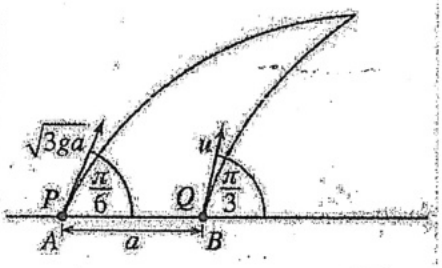
$\therefore v = \frac{1}{2}(1-e)u$ (5)

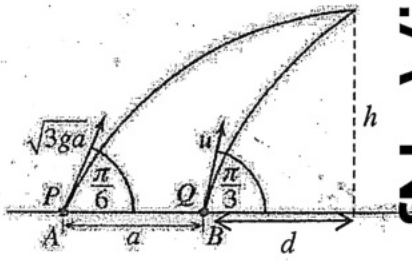
Required time taken $= \frac{a}{u-v}$

$= \frac{2a}{(1+e)u}$. (5)

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2. A and B are two points on a horizontal ground such that $AB = a$. Two particles P and Q are projected from the points A and B respectively, at the same instant and in the vertical plane that contains the line AB such that they collide with each other after a time T at a point in space. The initial velocities of P and Q are given in the figure.
Show that $u = \sqrt{ga}$ and find T in terms of a and g.





Apply $s = ut + \frac{1}{2}at^2$:

(P) $\uparrow$ $h = \sqrt{3ga} \cdot \frac{1}{2} T - \frac{1}{2}gT^2$ ----------(1) (5)

(Q) $\uparrow$ $h = u \cdot \frac{\sqrt{3}}{2} \cdot T - \frac{1}{2}gT^2$ ----------(2) (5)

(1) − (2) : $u \frac{\sqrt{3}}{2} T = \sqrt{3ga} \cdot \frac{1}{2} T$ (5)

$\Rightarrow u = \sqrt{ga}$

(P) $\rightarrow$ $a + d = \sqrt{3ga} \frac{\sqrt{3}}{2} T$

(Q) $\rightarrow$ $d = \sqrt{ag} \cdot \frac{1}{2} \cdot T$

(5) (for both)

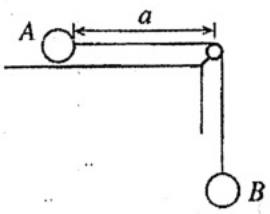
$\therefore a + \frac{\sqrt{ag}}{2} T = 3\frac{\sqrt{ag}}{2} T$

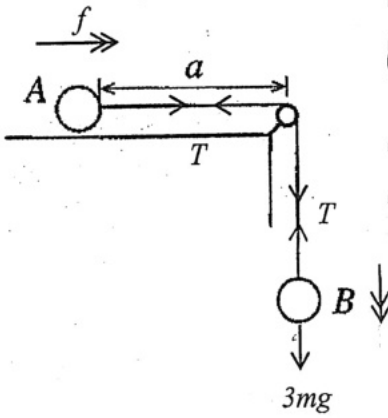
$\Rightarrow a = 2\frac{\sqrt{ag}}{2} T$

$\Rightarrow T = \sqrt{\frac{a}{g}}$ (5)

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3. Two particles A and B of masses m and $3m$, respectively, are attached to the ends of a light inextensible string. The particle A is held at rest on a horizontal table with the string passing over a small smooth pulley fixed at the edge of the table. The particle B hangs vertically below the pulley. The system is released from rest with the particle A at a distance a from the pulley. In the subsequent motion, a constant frictional force of magnitude $\frac{1}{2}mg$ acts on A. Find the acceleration of A.
Also, find the speed of A at the instant when A reaches the pulley.





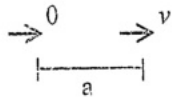
Apply $\vec{F} = m\vec{a}$

For B : $\downarrow 3mg - T = 3mf$ ----------(1) (5)

For A : $\rightarrow T - \frac{1}{2}mg = mf$ ----------(2) (5)

(1) − (2) : $\frac{5}{2}mg = 4mf$

$$f = \frac{5}{8}g \quad (5)$$



this wrong

Apply $v^2 = u^2 + 2as$:

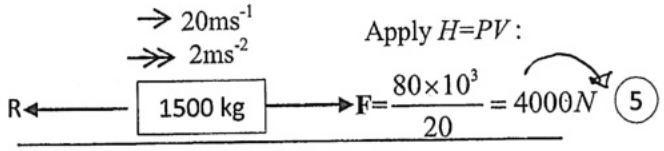
$v^2 = 2fa$. (5)

$\therefore v^2 = 2 \times \frac{\sqrt{5ag}}{8}$.

$\therefore v = \frac{\sqrt{5ag}}{2}$. (5)

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4. A car of mass 1500 kg working with a constant power 80 kW moves on a horizontal road against a constant resistance. The acceleration of the car is 2 m s^{-2}, when it moves with the speed 20 m s^{-1}. Obtain equations sufficient to determine the acceleration of the car when it moves upward on a road of inclination $\sin^{-1}\left(\frac{2}{3}\right)$ to the horizontal with speed 8 m s^{-1} working with the same constant power against the same constant resistance.



Apply $H=PV$:

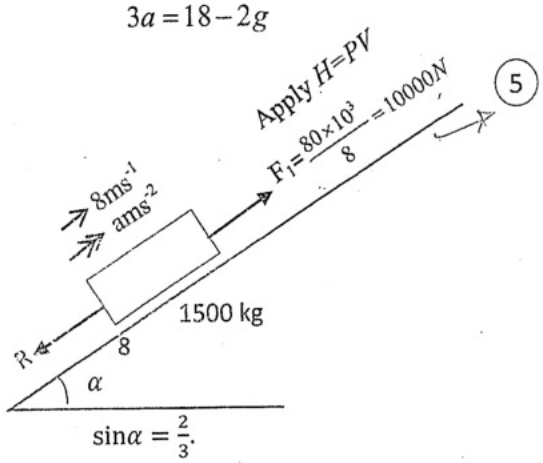
Apply $\vec{F} = m\vec{a}$: $\rightarrow$ $4000 - R = 1500 \times 2$ (5)

$\therefore R = 1000\text{N}$

$$10000 - 1000 - 1500 \times \frac{2}{3} g = 1500a \quad (10)$$

$$9000 - 1000g = 1500a$$

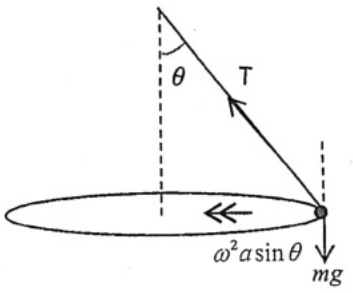
$$3a = 18 - 2g$$



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5. One end of a light inextensible string of length a is attached to a fixed point and the other end to a particle of mass m. The particle moves in a horizontal circle with constant angular speed ω. The string makes an angle $\theta\left(0<\theta<\frac{\pi}{2}\right)$ with the downward vertical. Show that $\omega>\sqrt{\frac{g}{a}}$.

Apply $\vec{F}=m\vec{a}$



$\uparrow\ T\cos\theta=mg$ ----------(1) (5)

$\leftarrow T\sin\theta=m\omega^2 a\sin\theta$ ----------(2) (5)

$\therefore T=m\omega^2 a$

(1) and (2) : $\Rightarrow\ \cos\theta=\frac{g}{\omega^2 a}$ (5)

Since $0<\theta<\frac{\pi}{2}$, we have $\cos\theta<1$. (5)

$\therefore \frac{g}{\omega^2 a}<1.$

$\therefore \omega>\sqrt{\frac{g}{a}}$ (5)

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6. In the usual notation, the position vectors of two points A and B with respect to a fixed origin O are $3\mathbf{i}+2\mathbf{j}$ and $2\mathbf{i}+4\mathbf{j}$, respectively. Show that O, A and B are non-collinear.
Let C be the point such that $\overrightarrow{BC}=\lambda\overrightarrow{OA}$, where $\lambda\in\mathbb{R}$. Find $\overrightarrow{OC}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and λ.
Show that if $B\hat{O}C=\frac{\pi}{2}$, then $\lambda=-\frac{10}{7}$.

Since $3:2\neq 2:4$, O, A and B are non-conlinear. (5)

$$\overrightarrow{OC}=\overrightarrow{OB}+\overrightarrow{BC} \quad (5)$$
$$=2\underline{i}+4\underline{j}+\lambda\left(3\underline{i}+2\underline{j}\right).$$

$$\therefore \overrightarrow{OC}=(2+3\lambda)\underline{i}+(4+2\lambda)\underline{j}. \quad (5)$$

since $B\hat{O}C=\frac{\pi}{2}$, we have $\overrightarrow{OB}\cdot\overrightarrow{OC}=0$.

$$\therefore (2i+4j)\cdot\left((2+3\lambda)\underline{i}+(4+2\lambda)\underline{j}\right)=0. \quad (5)$$

$$\therefore\ 4+6\lambda+16+8\lambda=0.$$

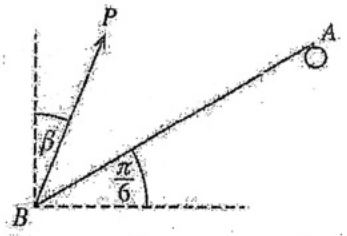
$$\therefore\ \lambda=\frac{-10}{7}. \quad (5)$$

$\overrightarrow{OA}=\lambda\overrightarrow{OB}$

$(3\underline{i}+2\underline{j})\neq\lambda(2\underline{i}+4\underline{j})$

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7. A uniform rod AB is kept in equilibrium with its upper end A resting on a smooth peg by applying a force P to its lower end B at an angle β to the vertical, as shown in the figure. The rod makes an angle $\frac{\pi}{6}$ with the horizontal. Show that $\tan\beta = \frac{\sqrt{3}}{5}$.



ΔBMN; $BM = a\cos\frac{\pi}{6} = a\frac{\sqrt{3}}{2}$ (5)

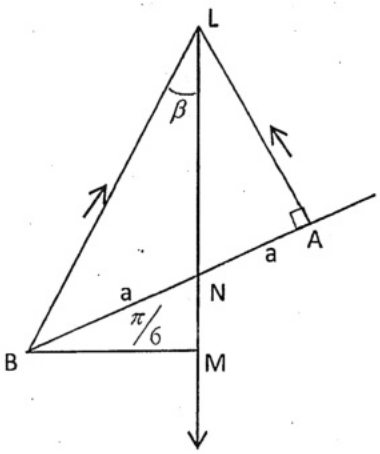
$MN = a\sin\frac{\pi}{6} = \frac{a}{2}$ (5)

ΔALN; $LN = \frac{a}{\cos\frac{\pi}{3}} = 2a$ (5)

$\therefore LM = 2a + \frac{a}{2} = \frac{5a}{2}$. (5)

ΔBLM; $\tan\beta = \frac{BM}{LM} = \frac{a\frac{\sqrt{3}}{2}}{5\frac{a}{2}} = \frac{\sqrt{3}}{5}$. (5)

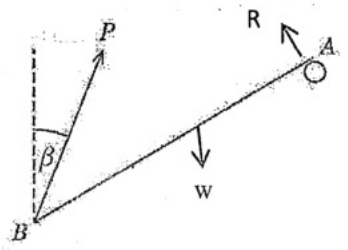
$\tan\beta = \frac{\sqrt{3}}{5}$.



Aliter

(5)

$B\circlearrowleft$ $W\,a\cos\frac{\pi}{6} = R.(2a) \Rightarrow R = \frac{\sqrt{3}W}{4}$.



$\uparrow$ $P\cos\beta + R\cos\frac{\pi}{6} = W$ (5)

$P\cos\beta = W - \frac{\sqrt{3}}{4} \cdot \frac{\sqrt{3}W}{2} = \frac{5W}{8}$ (5)

$\rightarrow$ $P\sin\beta = R\sin\frac{\pi}{6}$ (5)

$= \frac{\sqrt{3}W}{4}\left(\frac{1}{2}\right)$

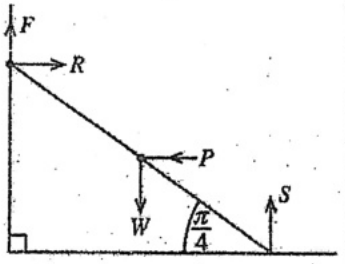
$= \frac{\sqrt{3}W}{8}$

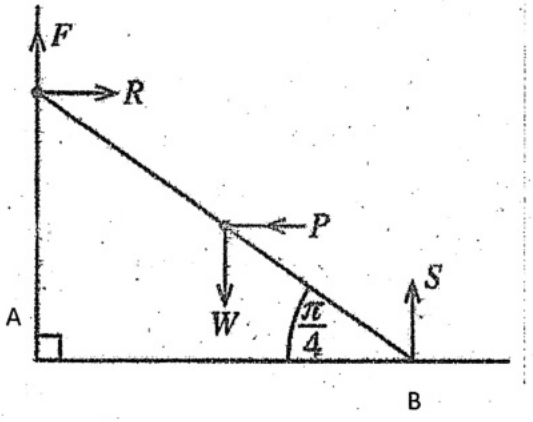
$\therefore \tan\beta = \frac{\sqrt{3}W}{8} \div \frac{5W}{8} = \frac{\sqrt{3}}{5}$ (5)

Aliter: (m+n) ale-rule

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8. A uniform ladder of weight W and length $2a$ is kept in equilibrium against a rough vertical wall with its lower end on a smooth horizontal ground, by a horizontal force of magnitude P applied at the mid-point of the ladder as shown in the figure. The ladder makes an angle $\frac{\pi}{4}$ with the ground. The coefficient of friction between the ladder and the wall is $\frac{1}{6}$. Show that, $\frac{3W}{4} \le P \le \frac{3W}{2}$.





For the equilibrium of the ladder :

$\uparrow \quad F + S = W$ (5)

$\leftarrow \quad P = R$ (5)

A) $W a \cos\frac{\pi}{4} + P \cdot a \cdot \sin\frac{\pi}{4} - S \cdot 2a \cos\frac{\pi}{4} = 0$ (5)

$\therefore \quad S = \frac{W+P}{2}$,

and $F = \frac{W-P}{2}$.

Now, $\frac{1}{6} \ge \frac{|F|}{R}$

$\Rightarrow -\frac{1}{6} \le \frac{W-P}{2P} \le \frac{1}{6}$

$\Rightarrow -P \le 3(W-P) \le P$

$\Rightarrow \frac{3W}{4} \le P \le \frac{3W}{2}$. (10)

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9. Let A and B be two events of a sample space Ω. It is given that $P(A)=\frac{2}{7}$, $P(A\cup B)=\frac{11}{14}$ and $P(A'\cup B')=\frac{4}{5}$. Find $P(B)$ and show that A and B are independent events.

$$P(A\cup B)=P(A)+P(B)-P(A\cap B) \quad (5)$$

$$\Rightarrow \frac{11}{14}=\frac{2}{7}+P(B)-\frac{1}{5}.$$

$$\therefore P(B)=\frac{7}{10}. \quad (5)$$

$$P(A\cap B)=1-P(A'\cup B') \quad (5)$$

$$=\frac{1}{5} \quad (5)$$

$$P(A)P(B)=\frac{2}{7}\times\frac{7}{10}=\frac{1}{5}=P(A\cap B). \quad (5)$$

$\therefore A$ and B are independent.

25

10. The mean and the standard deviation of marks obtained by 100 students for an examination are 60 and 20, respectively. Find the z-score of a student who obtained 56 marks for this examination. It was later found that this mark of 56 has been entered erroneously and it should have been 65 instead. Find the correct value of the mean of the marks obtained for this examination.

$$z=\frac{56-60}{20}=\frac{-4}{20}=\frac{-1}{5}=-0.2 \quad (5)$$

$$60=\mu_{old}=\frac{\sum_{i=1}^{100}x_i}{100} \Rightarrow \left(\sum_{i=1}^{100}x_i\right)_{old}=6000 \quad (5)$$

$$\therefore \mu_{correct}=\frac{\left(\sum_{i=1}^{100}x_i\right)_{correct}}{100}=\frac{6000-56+65}{100}=\frac{6009}{100}=60.09 \quad (5)$$

(5)

25

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Part B

* Answer **five** questions only.

(In this question paper, g denotes the acceleration due to gravity.)

11. (*a*) A car P that begins its journey from rest from a point O on a straight horizontal road travels with a constant acceleration $2f$ m s^{-2} up to a point A on that road, where $OA = a$ m. It maintains the velocity attained at A throughout its remaining journey. At the instant when car P reaches the point A, another car Q begins its journey, along the same road in the same direction, from rest at the point O and moves with a constant acceleration f m s^{-2}. Sketch the velocity-time graphs for the motion of P and Q in the same diagram.

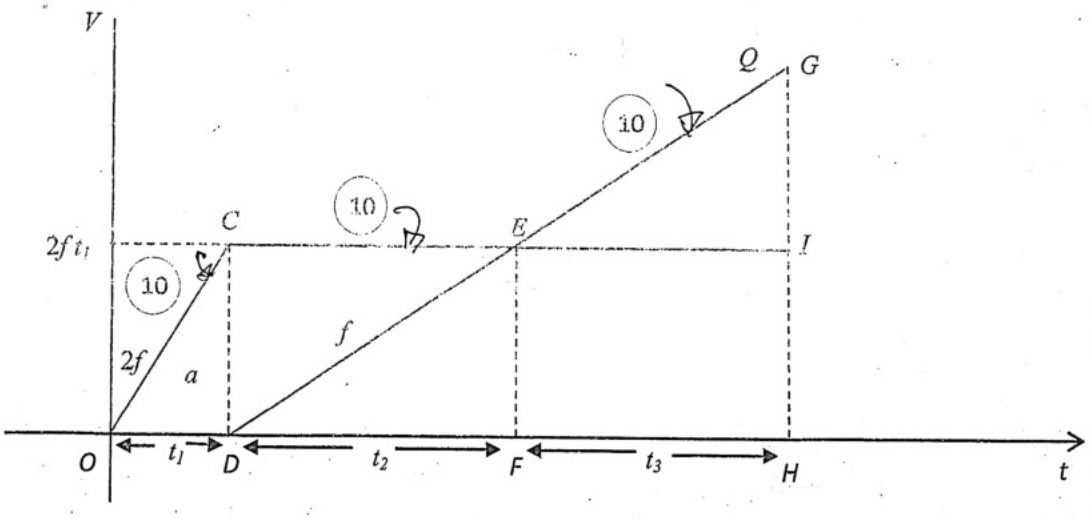
Hence, show that the time taken by Q to the instant when the velocities of P and Q are equal is $2\sqrt{\frac{a}{f}}$ s.

Now, let $a = 50$ and $f = 2$, and let B be the point on the road at which the car Q passes the car P. Show that $AB = 50\left(5+2\sqrt{6}\right)$ m.

(*b*) A ship P is sailing due South with a uniform speed 60 m s^{-1} relative to earth and a ship Q is sailing due East with a uniform speed $30\sqrt{3}$ m s^{-1} relative to earth. A third ship R appears to move in the direction 30° North of East when it is observed from P and ship R appears to move due South when it is observed from Q. Show that the ship R moves in the direction 30° South of East with a speed 60 m s^{-1} relative to earth.

Suppose that initially the ship R is located 24 km away from P in a direction 60° South of West and 6 km away from Q in due West. Show that the distance between Q and R is 12 km, when P and R are the shortest distance apart.

(*a*)



30

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From ΔOCD :

$\frac{1}{2}(t_1)(2f\ t_1) = a$ (5)

$\Rightarrow \quad t_1^2 = \frac{a}{f}$

$\therefore \quad t_1 = \sqrt{\frac{a}{f}}$ *as* $t_1 > 0$. (5)

From ΔDEF :

$f = \frac{2f\ t_1}{t_2}$. (5)

$\therefore t_2 = 2t_1$.

$= 2\sqrt{\frac{a}{f}}$ (5)

20

Let $a = 50$ and $f = 2$.

Then $t_1 = \sqrt{\frac{50}{2}} = 5$ and $t_2 = 10$. (5)

Since Q meets P at B, area of $OCED$ = area of EGI.

$\therefore \frac{1}{2}(5+10)(2\cdot 2\cdot 5) = \frac{1}{2}+3\cdot 2t_3$ (5)

$t_3^2 = 150$

$t_3 = \sqrt{150} = 5\sqrt{6}$. (5)

15

$AB = \frac{1}{2}(t_2 + t_3)(2f\ t_1 + f\ t_3)$ (5)

$= \frac{1}{2}\left(10+5\sqrt{6}\right)\left(5\times 2+5\sqrt{6}\right)\cdot(2) = 50\left(5+5\sqrt{6}\right)$ (5)

Full Marks

10

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Any two correct give (05) marks

(*b*)

$\underline{V}(P,E) = \downarrow 60$

$\underline{V}(Q,E) = \rightarrow 30\sqrt{3}$

$\underline{V}(R,P) = \nearrow 30^0$

$\underline{V}(R,Q) = \downarrow$ (10)

$\underline{V}(R,E) = \underline{V}(R,P) + \underline{V}(P,E)$

$= \underline{V}(P,E) + \underline{V}(R,P)$

$= \underline{V}(R,P) + \underline{V}(P,E)$

$= \overrightarrow{AB} + \overrightarrow{AC}$ $= \overrightarrow{AB} + \overrightarrow{BC}$

$\triangle ABC$ (15)

$= \overrightarrow{AC}.$

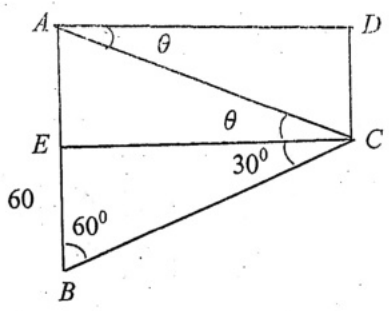
Also, $\underline{V}(R,E) = \underline{V}(R,Q) + \underline{V}(Q,E)$

$= \underline{V}(Q,E) + \underline{V}(R,Q)$

$= \overrightarrow{AD} + \overrightarrow{DC}$

$\triangle ADC$ (15)

$= \overrightarrow{AC}$



$BE = 30\sqrt{3} \cdot \frac{1}{\sqrt{3}}$

$= 30.$

$\therefore AE = 30.$

Also, $CE = 30\sqrt{3}.$

$\tan\theta = \frac{AE}{CE} = \frac{1}{\sqrt{3}}$ (5)

$\therefore \theta = 30^0$ (5)

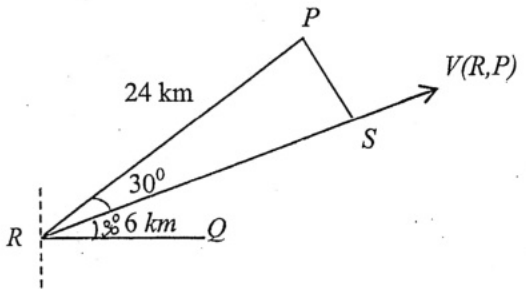
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Now $V^2 = \left(30\sqrt{3}\right)^2 + 30^2$ (5)

$V^2 = 30^2(4)$

$\therefore V = 60\text{ms}^{-1}$ (5)

60



$RS = 24000\sqrt{\frac{3}{2}}$

$= 12000\sqrt{3}$

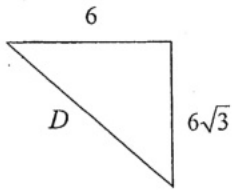
$t = \frac{12000\sqrt{3}}{60}$

$= 200\sqrt{3}$ S (5)

Let $d = 30 \times 200\sqrt{3} = 6000\sqrt{3}$

$= 6\sqrt{3}km$ (5)

$\therefore$ The required distance D km given by



$D^2 = 6^2 + 6^2(3)$

$= 6^2(4)$

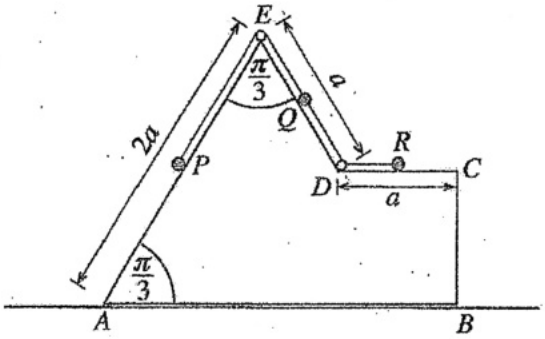
$\therefore D = 12$ km. (5)

15

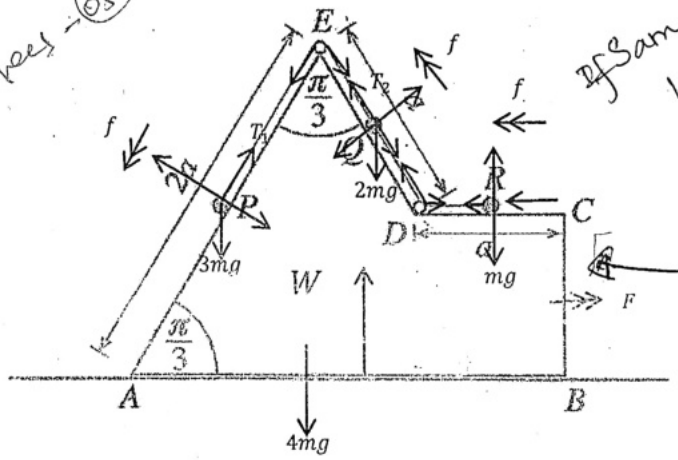
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Q11 ⇒ | 30 | 20 | 15 | 10 | 60 | 15 |

12. (*a*) The vertical cross-section *ABCDE* through the centre of gravity of a smooth uniform block of mass $4m$ is shown in the figure. The face containing *AB* is placed on a smooth horizontal floor. Also, *AE* and *ED* are the lines of greatest slope of the faces containing them. $AE = 2a$, $ED = a$, $DC = a$ and $E\hat{A}B = A\hat{E}D = \frac{\pi}{3}$. Three particles *P*, *Q* and *R* of masses $3m$, $2m$ and m, respectively, are placed at the mid-points of *AE*, *ED* and *DC*. The particles *P* and *Q* are attached to the ends of a light inextensible string passing over a smooth light small pulley fixed to the block at *E*, and the particles *Q* and *R* are attached to the ends of another light inextensible string passing through a smooth light small ring fixed to the block at *D*. Strings are taut in the position shown in the diagram and the system is released from rest from this position. Obtain equations sufficient to determine the time taken for the particle *Q* to reach *E*.



(*a*)



If same tension less -05

If denoted as f_1, f_2 less -05 marks

10 for the forces. (T, mg)

$\vec{V}(W,E) = \rightarrow F$

$\vec{V}(P,W) = \swarrow f.$

Then $\vec{V}(Q,W) = \nwarrow f$ (5)

$\vec{V}(R,W) = \leftarrow f$ (5)

Apply $F = ma$:

For P: $\swarrow 3mg\cos\frac{\pi}{6} - T_1 = 3m(f - F\cos\frac{\pi}{3})$ (15)

For Q: $\nwarrow T_1 - T_2 - 2mg\cos\frac{\pi}{6} = 2m(f - F\cos\frac{\pi}{3})$ (15)

For R: $\leftarrow T_2 = m(f - F)$ (10) /0

One mistake less -05

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One mistake loss 5
Two " " less 1
Three " " NOM

For the system :

$\rightarrow$

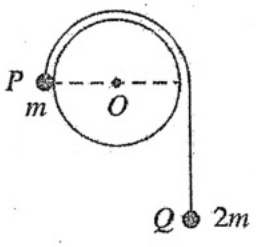
$$0 = 4mF + m(F - f) + 2m(F - f\cos\frac{\pi}{3}) + 3m(F - f\cos\frac{\pi}{3}) \quad (20)$$

For Q $\nwarrow$ $s = ut + \frac{1}{2}ft^2$

$$\Rightarrow \frac{a}{2} = \frac{1}{2}ft^2. \quad (10)$$

90

(*b*) A cylinder of radius a is fixed with its axis horizontal and the adjoining figure shows a vertical cross-section of the cylinder perpendicular to its axis. Two particles P and Q of masses m and $2m$, respectively connected by a light inextensible string are held with the string taut and OP horizontal in the position as shown in the figure and released from rest. Assuming that the particle Q moves vertically downwards, show that the speed v of the particle P when $\overrightarrow{OP}$ has turned through an angle θ $(0 \le \theta \le \frac{\pi}{6})$ is given by $v^2 = \frac{2ga}{3}(2\theta - \sin\theta)$.



The string is cut when $\theta = \frac{\pi}{6}$ and it is given that the particle P moving on the cylinder comes to instantaneous rest before it reaches the highest point of the cylinder. In the subsequent motion, find the speed of P when it is at a distance a vertically below its initial position.

(*b*) By the conservation of energy, we have

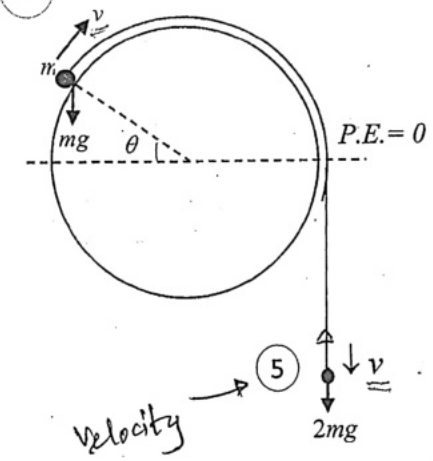
Eqn — (05)

$$\frac{1}{2}mv^2 + \frac{1}{2}2mv^2 + mga\sin\theta - 2ma\theta g = 0. \quad (25)$$

(05) (05) (05) (05)

$$\Rightarrow 3v^2 = 2ag(2\theta - \sin\theta).$$

$$\Rightarrow v^2 = \frac{2ag}{3}(2\theta - \sin\theta). \quad (5)$$



(5)

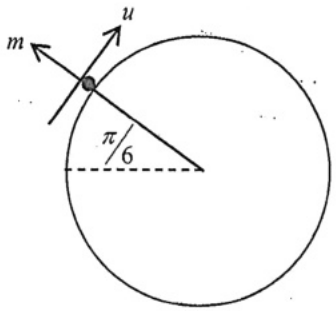
velocity

35

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$v = u$ when $\theta = \frac{\pi}{6}$ is given by $u^2 = \frac{2ag}{3}\left(\frac{\pi}{3} - \frac{1}{2}\right)$. (10)

$= \frac{ag}{9}(2\pi - 3)$.



Any By

By the conservation of energy

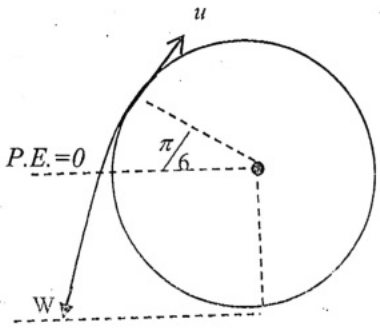
$\frac{1}{2}mw^2 - mga = mg\frac{a}{2} + \frac{1}{2}mu^2$ (10)

$\frac{1}{2}mw^2 = \frac{3mga}{2} + \frac{1}{2}m\frac{ag}{9}(2\pi - 3)$

$\frac{1}{2}mw^2 = \frac{1}{2}mag\left[3 - \frac{1}{3} + \frac{2\pi}{9}\right]$

$w^2 = ag\left[\frac{8}{3} + \frac{2\pi}{9}\right] = \frac{ag}{9}[24 + 2\pi]$

$w = \frac{\sqrt{2ga(\pi + 12)}}{3}$. (5)

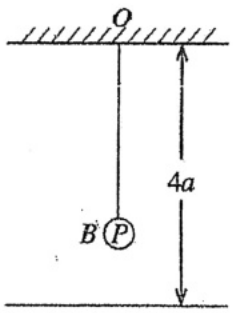


25

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Q12 = D | 90 | 35 | 25 |

13. One end of a light elastic string of natural length $2a$ and modulus of elasticity $2mg$ is attached to a fixed point O which is at a distance of $4a$ above a smooth horizontal floor, and the other end to a particle P of mass m. The particle P hangs in equilibrium at B. Show that the extension of the string is a.



Now, the particle P is given an impulse of mv vertically downwards.

Show that the equation of motion of P is $\ddot{x}+\omega^2 x=0$ where $\omega=\sqrt{\frac{g}{a}}$ and $BP=x$.

Using the formula $\dot{x}^2=\omega^2(c^2-x^2)$, where c is the amplitude, show that if $v>\sqrt{ag}$, P hits the floor.

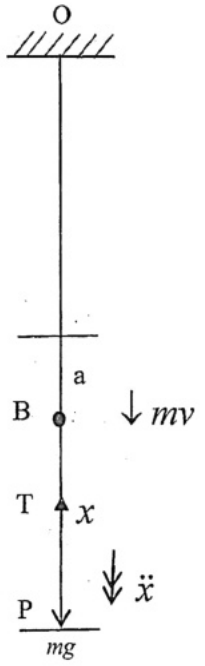
Now, suppose that $v=3\sqrt{ag}$.

Find the velocity with which P hits the floor.

The coefficient of restitution between P and the floor is e. If $e<\frac{1}{\sqrt{2}}$, show that the particle P will not reach O.

If it is given that $e=\frac{1}{2}$, find the velocity of P when the string becomes slack for the first time.

Find the total time taken by P to come to instantaneous rest for the first time, from the instant it was given the impulse at B.



At the equilibrium position,

$2mg\cdot\frac{x}{2a}=mg.$ (5)

$\therefore\ x=a.$ (5)

$\therefore$ The extension of the string is a. **10**

Apply $\underline{F}=m\underline{a}$:

$\downarrow m\ddot{x}=mg-2mg\frac{(a+x)}{2a}$ (15) / 6

$\ddot{x}=-\frac{g}{a}x$ (5)

$\therefore\ \ddot{x}+\omega^2 x=0,\ where\ \omega=\sqrt{\frac{g}{a}}.$

20

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$\dot{x} = v \quad when \quad x = 0$

$\therefore v^2 = \omega^2\left(c^2 - 0\right)$ (5)

$\therefore v = c\omega$

$\therefore c = \frac{v}{\omega}$ (5)

if $v > \sqrt{ag}$, then $c > \sqrt{ag} \cdot \sqrt{\frac{a}{g}} = a$ (10)

and hence the particle hits the floor. **20**

Let, $\dot{x} = u$, where $x = a$.

Then

$u^2 = \frac{g}{a}\left(9a^2 - a^2\right) = 8ag$, since $c = \frac{v}{w} = 3a.$ (5) (10)

$\therefore u = \sqrt{8ag}.$ (5)

20

The velocity just after P hit the floor $= eu \uparrow$. (5)

$\therefore \dot{x} = eu$, when $x = a$.

By symmetry of S.H.M. about the center, $\dot{x} = eu$, when $x = -a$. (15)

For the motion of P under gravity,

Apply $v^2 = u^2 + 2as$:

$\uparrow \; 0 = v_1^2 - 2gs$ (5)

$\therefore s = \frac{8e^2ag}{2g} = 4e^2a$ (5)

If $e < \frac{1}{\sqrt{2}}$, then $s < 2a$ and P will not reach O. (10)

40

When $e = \frac{1}{2}$, $v_1 = \sqrt{8e^2ag} = \sqrt{2ag}$ (10)

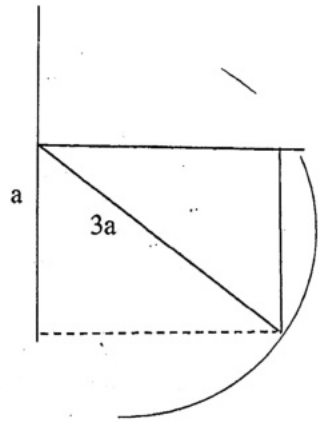
10

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Time taken to hit the floor, $T_1 = \dfrac{\sin^{-1}\left(\frac{1}{3}\right)}{\sqrt{\frac{g}{a}}}$ (10)

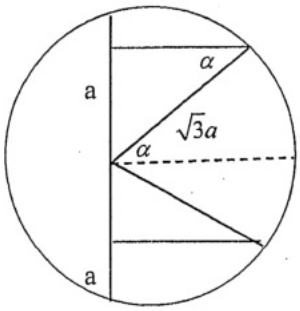
$= \sqrt{\dfrac{a}{g}}\sin^{-1}\left(\dfrac{1}{3}\right)$



Let $e = \dfrac{1}{2}$. Then $C_1 = \sqrt{3}a$.

Time taken after that to reach the natural

Length $T_2 = \dfrac{2\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)}{\sqrt{\frac{g}{a}}}$. (10)

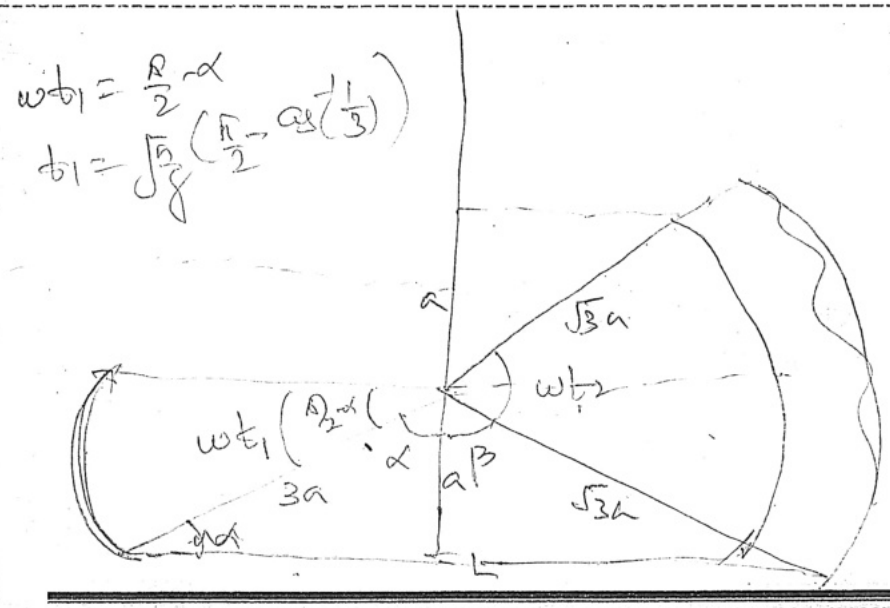


During is free flight : $\uparrow\ V = u + at$.

$\therefore$ Time in free flight $T_3 = \dfrac{\sqrt{2ag}}{g} = \sqrt{\dfrac{2a}{g}}$ (5)

Total time taken $T_1+T_2+T_3 = \sqrt{\dfrac{a}{g}}\left(\sin^{-1}\left(\dfrac{1}{3}\right)+2\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)+\sqrt{2}\right)$. (5)

30



Q13	10	20	20	20	40	10	30

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14. (*a*) Let the position vectors of four points A, B, C and D be $\mathbf{a}$, $\mathbf{b}$, $3\mathbf{a}$ and $4\mathbf{b}$, respectively with respect to a fixed origin O, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel vectors. E is the point of intersection of AD and BC. Using the triangle law of addition for the triangle OAE, show that $\overrightarrow{OE} = \mathbf{a} + \lambda(4\mathbf{b} - \mathbf{a})$ for $\lambda \in \mathbb{R}$.

Similarly, show also that $\overrightarrow{OE} = \mathbf{b} + \mu(3\mathbf{a} - \mathbf{b})$ for $\mu \in \mathbb{R}$.

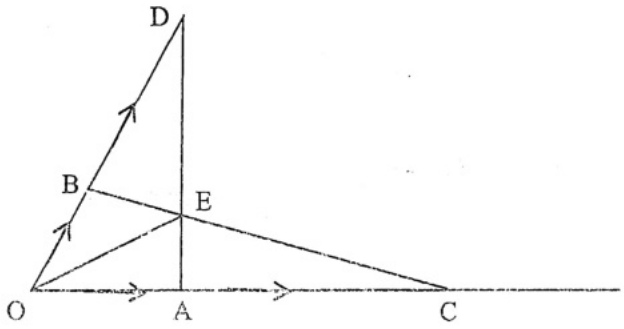
Hence, show that $\overrightarrow{OE} = \frac{1}{11}(9\mathbf{a} + 8\mathbf{b})$.

(*b*) Three forces $\alpha\mathbf{i} + 2\mathbf{j}$, $-3\mathbf{i} + \beta\mathbf{j}$ and $\mathbf{i} + 5\mathbf{j}$ act through the points with position vectors $\mathbf{i} + \mathbf{j}$, $3\mathbf{i} + \mathbf{j}$ and $2\mathbf{i} + 2\mathbf{j}$, respectively, where $\alpha, \beta \in \mathbb{R}$. It is given that this system of forces is equivalent to a couple. Find the values of α and β, and the moment of this couple.

Now, a new force $3\gamma\mathbf{i} + 4\gamma\mathbf{j}$ acting through the origin O is added to the above system of forces, where $\gamma > 0$. Show that the new system consisting of 4 forces is equivalent to a resultant force and, find its magnitude, direction and the equation of its line of action.

Next, it is given that when a force $p\mathbf{i} + q\mathbf{j}$ acting through the point with position vector $2\mathbf{i} + 3\mathbf{j}$ is added, this system consisting of 5 forces is in equilibrium. Find the values of γ, p and q.

(*a*)



By the triangle OAE,

$$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$$
$$= \underline{a} + \lambda\overrightarrow{AD} \quad (5)$$
$$= \underline{a} + \lambda\left(\overrightarrow{AO} + \overrightarrow{OD}\right) \quad (5)$$
$$= \underline{a} + \lambda\left(4\underline{b} - \underline{a}\right) \quad (5)$$

By the triangle OBE,

$$\overrightarrow{OE} = \overrightarrow{OB} + \overrightarrow{BE}$$
$$= \underline{b} + \mu\overrightarrow{BC} \quad (5)$$
$$= \underline{b} + \mu\left(\overrightarrow{BO} + \overrightarrow{OC}\right) \quad (5)$$
$$= \underline{b} + \mu\left(3\underline{a} - \underline{b}\right) \quad (5)$$

30

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$\therefore\ \underline{a}+\lambda(4\underline{b}-\underline{a})=\underline{b}+\mu(3\underline{a}-\underline{b})$ (5)

$(1-\lambda)\underline{a}+4\lambda\underline{b}=3\mu\underline{a}+(1-\mu)\underline{b}$ (5)

$\Rightarrow 1-\lambda=3\mu$ and $1-\mu=4\lambda$ (5)

$\therefore\ \lambda=\frac{2}{11}$ (5)

If not vector notation Less — 05

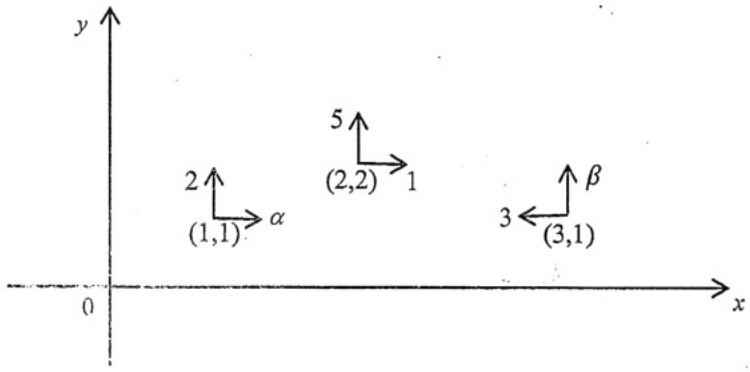
$\therefore \overrightarrow{OE}=a+\frac{2}{11}(4\underline{b}-\underline{a})$ (5)

$=\frac{1}{11}(9\underline{a}+8\underline{b})$. (5)

60

~~30~~ ~~60~~

(b)



Since the system is equivalent to a couple,

$\rightarrow X=0,\quad \uparrow Y=0$ and $G\neq 0$.

$X=\alpha-3+1=0.$ (5)

$\Rightarrow \alpha=2$ (5)

$Y=2+\beta+5=0.$ (5)

$\Rightarrow \beta=-7$ (5)

20

$O\circlearrowleft\ G=2(1)-2(1)+3(1)-7(3)+5(2)-1(2)$ (5)

$=3-21+10-2$

$=13-23$

$=-10.$ (5)

10

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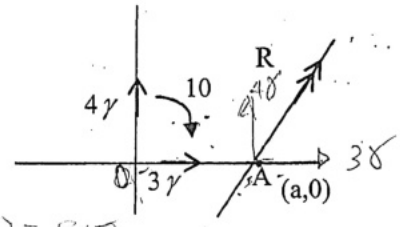
$R^2 = 9\gamma^2 + 16\gamma^2$ (5)

$= 25\gamma^2$

$\therefore R = 5\gamma.$ (5)

$\tan\theta = \frac{4\gamma}{3\gamma} = \frac{4}{3}$ (5)

$\therefore \theta = \tan^{-1}\left(\frac{4}{3}\right)$ (5)

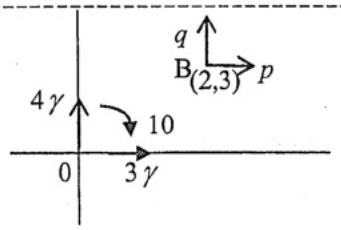


A) $4\gamma a + 10 = 0$

$\therefore a = \frac{-5}{2\gamma}$ (5)

The equation the line of action $4x - 3y + \frac{10}{\gamma} = 0.$ (5)

30



$\rightarrow p + 3\gamma = 0$ (5)

$\therefore p = -3\gamma$

$\uparrow q + 4\gamma = 0$ (5)

$\therefore q = -4\gamma$

B) $(3\gamma \times 3) - (4\gamma \times 2) - 10 = 0$ (5)

$\therefore \gamma = 10.$ (5)

$\therefore p = -30$ (5) and $q = -40$ (5)

30

Aliter

O) $q(2) - 3p - 4r(x) = 0$ (5)

$2q - 3p - 4r\left(\frac{5}{2r}\right) = 0$ (5)

$2q - 3p - 10 = 0$

O) $\uparrow q + 4\gamma = 0 \Rightarrow q = -4\gamma$ (5)

$\rightarrow p + 3\gamma = 0 \Rightarrow p = -3\gamma$ (5)

$2(-4\gamma) - 3(-3\gamma) = 10$

$-8\gamma + 9\gamma = 10$

$\gamma = 10$

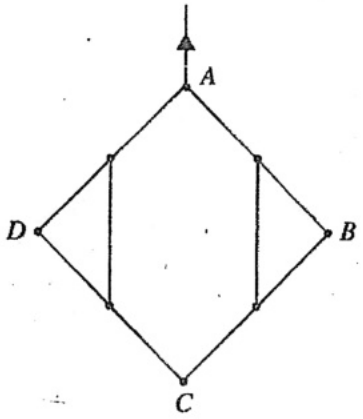
$p = -30$ (5) & $q = -40$ (5)

30

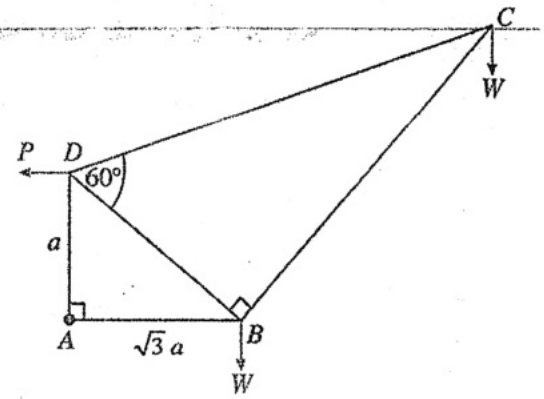
Q14 | 20 | 20 | 10 | 30 | 30 |

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15. (*a*) Four uniform rods AB, BC, CD and DA each of length $2a$ and weight W are smoothly jointed at their ends A, B, C and D. The midpoints of AB and BC are joined by a light inextensible string of length a. Similarly, midpoints of AD and DC are also joined by a light inextensible string of length a. The system is suspended in a vertical plane from the point A and stays in equilibrium as shown in the figure. Find the tensions in the strings and the reaction exerted on AB by BC at the joint B.



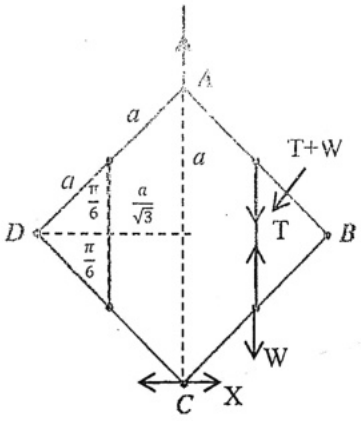
(*b*) The framework shown in the figure consists of five light rods AB, BC, CD, DA and DB smoothly jointed at their ends. It is given that $AD = a$, $AB = \sqrt{3}\,a$, $B\hat{A}D = 90°$, $C\hat{B}D = 90°$ and $B\hat{D}C = 60°$. At each of the joints B and C, a load W is suspended and the framework is smoothly hinged at A to a fixed point and kept in equilibrium in a vertical plane with AB horizontal by a horizontal force P applied to it at the joint D.



(i) Find the value of P.

(ii) Draw the stress diagram using Bow's notation for the joints C, B and D. Hence, find the stresses in the rods, stating whether they are tensions or thrusts.

(*a*)



By symmetry, the reaction at C from DC on CB is horizontal.

(5)

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For ABC,

$\curvearrowright A$: $X \cdot 2a - 2W \cdot \frac{a\sqrt{3}}{2} = 0$ (5)

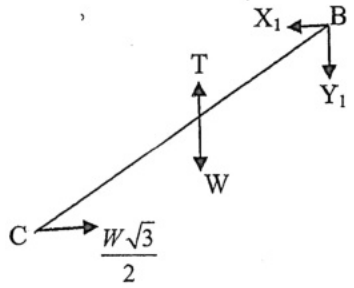
$$X = \frac{\sqrt{3}W}{2} \quad (5)$$

For BC:

$\curvearrowright B$: $\frac{W\sqrt{3}}{2} \cdot a + W \cdot \frac{a\sqrt{3}}{2} - T \cdot \frac{a\sqrt{3}}{2} = 0$ (10)

$T = 2W.$ (5)

For BC:



$\rightarrow \quad X_1 = \frac{W\sqrt{3}}{2}$; (5)

$\uparrow T - W - Y_1 = 0.$ (5)

$\therefore Y_1 = W$ (5)

$$R = \sqrt{\left(\frac{\sqrt{3}W}{2}\right)^2 + W^2}$$

$$= \frac{\sqrt{7}w}{2} \quad (5)$$

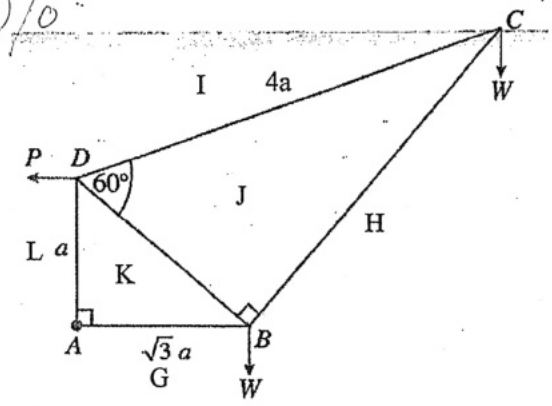
$$\tan\theta = \frac{Y_1}{X_1} = \frac{W}{\frac{W\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

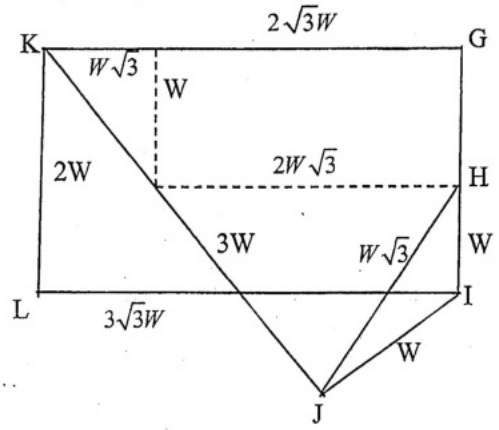
$\therefore \theta = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (5)

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(b) A ↻ $P \times a - W \times \sqrt{3}a - W \times 2\sqrt{3}a = 0$ (10) /10

$\therefore\ P = 3\sqrt{3}W.$ (5)





joint C: (10)

joint D: (10)

joint B: (10)

Rod	Stress	Tension	Magnitude	
AB	✓	-	$3\sqrt{3}W$	(10)
BC	✓	-	$\sqrt{3}W$	(10)
CD	-	✓	W	(10)
BD	-	✓	5W	(10)
AD	✓	-	2W	(10)

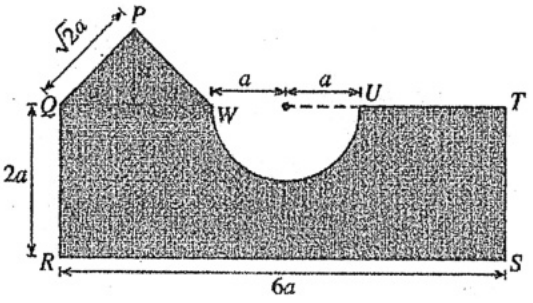
95

Q15 ⇒ 55 | 95

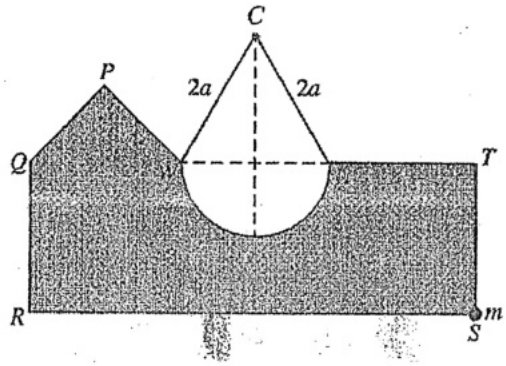
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16. Show that the centre of mass of a uniform semi-circular lamina of radius r and centre O is at a distance $\frac{4r}{3\pi}$ from O.

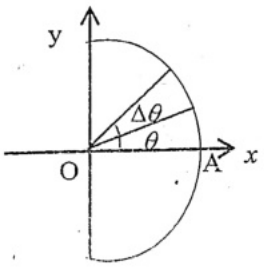
A plane lamina is made from a uniform thin sheet metal of surface density σ by removing a semi-circle of radius a from the rectangle $QRST$ and by adding an isosceles triangle PQW with equal side-lengths $\sqrt{2}a$ to it, as shown in the adjoining figure. $QR = 2a$, $RS = 6a$ and $QW = 2a$. The centre of mass of this lamina lies at a distance $\bar{x}$ from QR and $\bar{y}$ from RS. Show that $\bar{x} = \frac{(74-3\pi)a}{(26-\pi)}$ and $\bar{y} = \frac{2(15-\pi)a}{(26-\pi)}$.



The lamina with a particle of mass m fixed to it at S, hangs in equilibrium by a light inextensible string of length $4a$ whose ends are attached to U and W and passing over a small smooth fixed peg C with side RS horizontal as shown in the figure. Find the value of m and the tension of the string in terms of a and σ.



By Symmetry, $\bar{y} = 0$ (5)



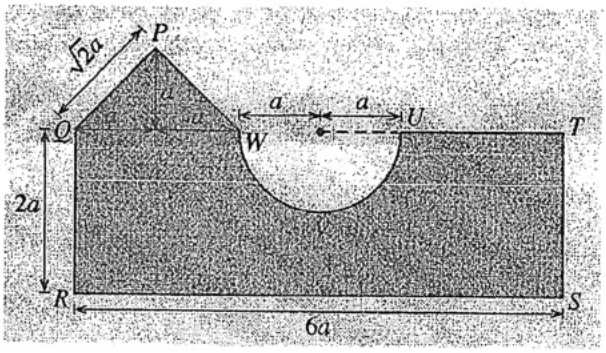
$$\Delta m = \frac{1}{2}r^2\Delta\theta\times\sigma$$

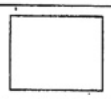
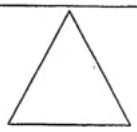
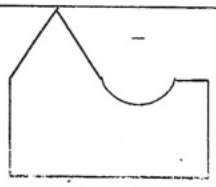
$$\bar{x} = \frac{\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}r^2\sigma\cdot\frac{2}{3}r\cos\theta d\theta}{\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}r^2\sigma\cdot d\theta} \quad (5)$$

$$= \frac{\frac{2}{3}r^3\sin\theta\Big|_{\frac{-\pi}{2}}^{\frac{\pi}{2}}}{\frac{1}{2}r^2\theta\Big|_{\frac{-\pi}{2}}^{\frac{\pi}{2}}} \quad (5) \quad (5)$$

$$= \frac{4r}{3\pi} \quad (5)$$

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Objects	Mass	Distance from QR	Distance from RS
	$12a^2\sigma$ (05)	$3a$ (05)	a (05)
	$\frac{1}{2}\pi a^2\sigma$ (05)	$3a$ (05)	$2a-\frac{4a}{3\pi}$ (05)
	$\frac{1}{2}(2a)a\sigma$ $=a^2\sigma$ (05)	a (05)	$2a+\frac{1}{3}a=\frac{7a}{3}$ (05)
	$12a-\frac{1}{2}\pi i+a^2\sigma$ (5) $\left(13-\frac{\pi}{2}\right)a^2\sigma$	$\bar{x}$	$\bar{y}$

(15)

$$\left(13-\frac{\pi}{2}\right)a^2\sigma\bar{x}=12a^2\sigma(3a)-\frac{1}{2}\pi a^2\sigma(3a)+a^2\sigma(a) \quad (15)\ /0$$

$$\Rightarrow \quad (26-\pi)a^2\sigma\bar{x}=72a^3\sigma 3\pi a^3\sigma+2a^3\sigma$$

$$\Rightarrow \quad \bar{x}=\frac{(74-3\pi)a}{(26-\pi)} \quad (5)$$

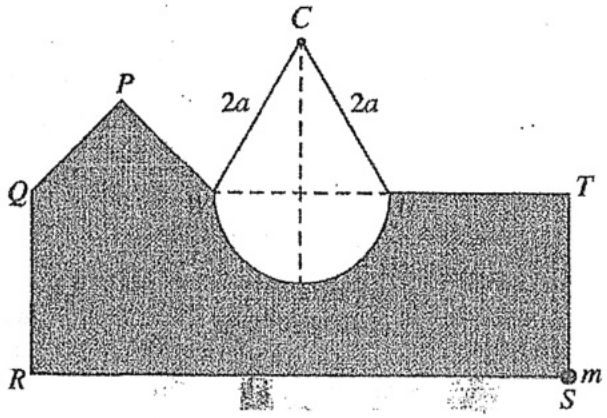
$$\left(13-\frac{\pi}{2}\right)a^2\sigma\bar{y}=12a^2\sigma(a)-\frac{1}{2}\pi a^2\sigma\left(2a-\frac{4a}{3\pi}\right)+a^2\sigma\left(\frac{7a}{3}\right) \quad (15)\ /0$$

$$\Rightarrow\left(\frac{26-\pi}{2}\right)a^2\sigma\bar{y}=12a^3\sigma-\pi a^3\sigma+\frac{2a^3\sigma}{3}+\frac{7a^3\sigma}{3} \quad (5)$$

$$=\frac{45a^3\sigma-3\pi a^3\sigma}{3}$$

$$\bar{y}=\frac{2(15-\pi)a}{(26-\pi)} \quad (5)$$

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C :

$$mg(3a) = \left(13 - \frac{\pi}{2}\right) a^2 \sigma g (3a - \bar{x}) \quad (10)$$

$$m = \frac{(26-\pi)}{6} a\,\sigma \left(3a - \frac{(74-3\pi)a}{26-\pi}\right) \quad (5)$$

$$= \frac{a^2\sigma}{2}(4a + 3\pi a - 3\pi a)$$

$$m = \frac{2a^2\sigma}{3}. \quad (5)$$

$$\uparrow \quad 2T\cos\frac{\pi}{6} = mg + \left(13 - \frac{\pi}{2}\right) a^2\sigma g \quad (5)$$

$$\Rightarrow \quad \sqrt{3}\,T = \frac{2}{3}a^2\sigma g + 13a^2\sigma g - \frac{\pi}{2}a^2\sigma g$$

$$= \frac{41a^2\sigma g}{3} - \frac{\pi a^2\sigma g}{2}$$

$$T = \frac{(82-3\pi)a^2\sigma g}{6\sqrt{3}} \quad (5)$$

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Q16 ⇒

30	90	30

17. (*a*) Four identical boxes B_1, B_2, B_3 and B_4, each contains 4 pens which are identical in all aspects except for their colour. Each box B_k contains k red pens and $4-k$ black pens for $k = 1, 2, 3, 4$. One of the four boxes is chosen at random and 2 pens are drawn from that box. Find the probability that

(i) the two pens drawn are red,

(ii) the pens are drawn from box B_4, given that the two pens drawn are red.

(*b*) The data sets $\{x_1, x_2, \ldots, x_n\}$ and $\{y_1, y_2, \ldots, y_m\}$ have the same mean, and their standard deviations are σ_x and σ_y, respectively. Show that the variance of the combined data set $\{x_1, \ldots, x_n, y_1, \ldots, y_m\}$ is given by $\dfrac{n\sigma_x^2 + m\sigma_y^2}{n+m}$.

Diameters of bolts produced at a factory is summarised in the following table:

Diameter (mm)	Number of bolts (in thousands)
2 – 6	2
6 – 10	5
10 – 14	8
14 – 18	4
18 – 22	1

Estimate the mean, the median and the variance of the distribution given above.

The diameters of another 40 000 bolts produced by a neighboring factory has the same mean, while the variance is 22.53 mm^2. Estimate the combined variance of the diameters of the bolts produced by both factories.

(*a*)

$$P(RR) = P(RR|B_1)P(B_1) + P(RR|B_2)P(B_2) + P(RR|B_3)P(B_3) + P(RR|B_4)P(B_4) \quad (10)$$

$$= 0 \cdot \frac{1}{4} + \frac{{}^2C_2}{{}^4C_2} \cdot \frac{1}{4} + \frac{{}^3C_2}{{}^4C_2} \cdot \frac{1}{4} + \frac{{}^4C_2}{{}^4C_2} \cdot \frac{1}{4} \quad (20)$$

$$= \frac{1}{4 \cdot {}^4C_2}[1+3+6]$$

$$= \frac{10}{24} = \frac{5}{12} \quad (5)$$

$$P(B_4|RR) = \frac{P(B_4|RR)P(B_4)}{P(RR)} \quad (10)$$

$$= \frac{1 \cdot \frac{1}{4}}{\frac{5}{12}} \quad (5) \quad (5)$$

$$= \frac{12}{20} = \frac{3}{5}. \quad (5)$$

60

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(*b*)

Let μ be the mean of each the data sets $\{x_1, x_2, ..., x_n\}$ and $\{y_1, y_2, ..., y_m\}$.

Then the mean of the combined data set is also μ. (5)

Set;

$$\sigma^2 = \frac{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{m} y_i^2}{n+m} - \mu^2 \quad (5)$$

$$= \left[\frac{\sum_{i=1}^{n} x_i^2 - n\mu^2}{n+m}\right] + \left[\frac{\sum_{i=1}^{m} y_i^2 - m\mu^2}{n+m}\right] \quad (5)$$

$$= \frac{1}{n+m}\left[n\left(\underbrace{\frac{\sum_{i=1}^{n} x_i^2}{n} - \mu^2}_{\sigma_x^2}\right) + m\left(\underbrace{\frac{\sum_{i=1}^{m} y_i^2}{m} - \mu^2}_{\sigma_y^2}\right)\right] \quad (5)$$

$$= \frac{n\sigma x^2 + m\sigma y^2}{n+m} \quad (5)$$

25

Diameter (mm)	$f(10^3)$	Mid. value x	xf	$x^2 f$
2 - 6	2	4	8	32
6 - 10	5	8	40	320
10 -14	8	12	96	1152
14 -18	4	16	64	1024
18 -22	1	20	20	400
	20		228	2928
	(5)		(10)	(10)

$$\text{Mean} = \frac{\sum xf}{\sum f} = \frac{228}{20} = 11.4\text{mm} \quad (5)$$ (5)

$$\text{Variance} = \frac{\sum x^2 f}{\sum f} - \mu^2 = \frac{2928}{20} - (11.4)^2 = 146.4 - 129.96$$ (5) (5)

$$= 16.44 \text{ mm}^2.$$

$$\text{Median} = 10 + \frac{(10-7)}{8} \times 4 \quad (5)$$

$$= 11.5 \text{ mm}.$$

f	x	$y = \frac{1}{4}x$	y^2	fy	fy^2
2	4	1	1	2	2
5	8	2	4	10	20
8	12	3	9	24	72
4	16	4	16	16	64
1	20	5	25	5	25
				57	183

$\bar{y} = \frac{\sum fy}{\sum f} = \frac{57}{20}$

$\bar{y} = \frac{1}{4}\bar{x} \Rightarrow \bar{x} = 4\bar{y}$

$= \frac{57}{5} = 11.4$

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(10)

$$\text{Combined Variance} = \frac{1}{20+40}\{20\sigma_1^2 + 40\sigma_2^2\} = \frac{1}{60}\{20\times 16.44 + 40\times 22.53\} \quad (5)$$

$$= 20.5 \text{ mm}^2.$$

65

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