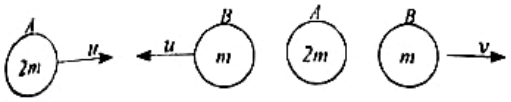
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Paper II

Part A

01.



Applying $\underline{I} = \Delta(m\underline{v})$ for the System

$\rightarrow 0 = [2m(0) + mv] - [2mu - mu]$ M-05

$\Rightarrow mv = mu$

$\Rightarrow v = u$ M-05

Newton's Law of restitution;

$v - 0 = e(u + u)$ M-05

$u = 2ue$

$e = \frac{1}{2}$ M-05

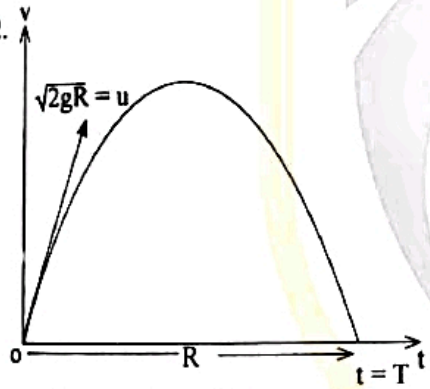
$\rightarrow$ Impulse on B = Change in L. M of B

$= mv - (-mu)$

$= mu + mu$

$= 2mu$ M-05 T.M-25

02.



Time of flight, T is given

by applying $\uparrow s = ut + \frac{1}{2}at^2$

$0 = u\sin\alpha\, T - \frac{g}{2}T^2$ M-05

$0 = T(u\sin\alpha - \frac{g}{2}T)$

$T = \frac{2u\sin\alpha}{g}$

$\rightarrow$ $s = ut$

$R = u\cos\alpha . T$

$R = u\cos\alpha \frac{2u\sin\alpha}{g}$ M-05

$R = \frac{u^2}{g} . 2\cos\alpha \sin\alpha$

$R = \frac{2gR}{g}\sin 2\alpha$

$\sin 2\alpha = \frac{1}{2} = \sin\frac{\pi}{6}$ M-05

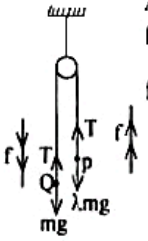
$2\alpha = \frac{\pi}{6}; \frac{5\pi}{6}$

Two Possible angles of projection are

$\alpha_1 = \frac{\pi}{12}$ and $\alpha_2 = \frac{5\pi}{12}$ M-05

and $\alpha_2 - \alpha_1 = \frac{\pi}{12}(5-1) = \frac{\pi}{3}$ M-05 T.M-25

03.



Apply $\underline{F} = m\underline{a}$

for P: $\downarrow mg - T = m\frac{g}{2}$ —— (01) M-05

for Q: $\uparrow T - \lambda mg = \lambda m.\frac{g}{2}$ —— (02) M-05

(01) + (02) $mg(1-\lambda) = \frac{mg}{2}(1+\lambda)$ M-05

$2 - 2\lambda = 1 + \lambda$

$3\lambda = 1$

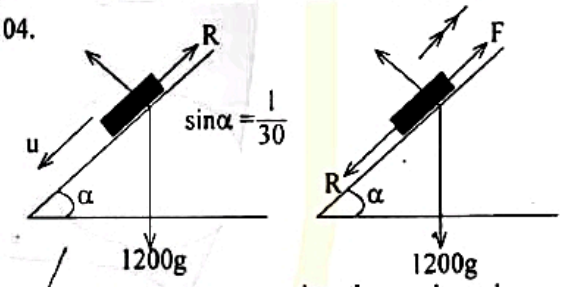
$\lambda = \frac{1}{3}$ M-05

Time t taken by Q to reach its maximum height is given by

$0 = v - gt \Rightarrow o = v - gt$

$\Rightarrow t = \frac{v}{g}$ M-05 T.M-25

04.



$1200g\sin\alpha - R = 0$ when the car desends, under resistance R only

$\Rightarrow R = 1200(10)(\frac{1}{30}) = 400N$ M-05

Let F be the tractive force, When the car ascends. M-05

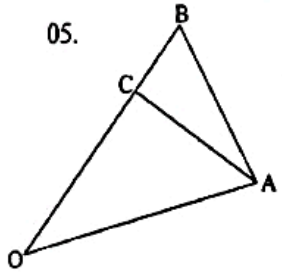
$F - R - 1200g\sin\alpha = 1200(\frac{1}{6}) \Rightarrow F - 400 - 400 = 200$ M-05

$\Rightarrow F = 1000 = \frac{P}{v}$ where P is the prower at speed v

Hence, Power $P = FV = 15(1000)$ watts M-05

$P = 15$ KW M-05 T.M-25

05.



$\underline{OA} = 3\underline{i}, \underline{\vec{OB}} = 2\underline{i} + 3\underline{j}$

Then $\vec{OC} = \lambda\vec{OB} = \lambda(2\underline{i} + 3\underline{j})$

Where λ is such that M-05

$\vec{OC}$, is perpendicular to

$\Rightarrow \lambda(2\underline{i} + 3\underline{j}).\{-\lambda(2\underline{i} + 3\underline{j}) + 3\underline{j} = 0$ M-05 M-05

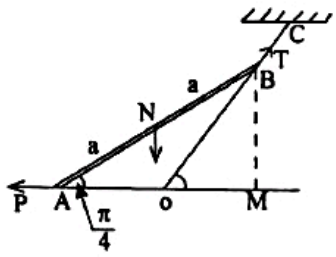
$6 - 13\lambda = 0 \Rightarrow \lambda = \frac{6}{13}$ M-05

Hence $\vec{OC} = \frac{12}{13}\underline{i} + \frac{18}{13}\underline{j}$ M-05

T.M-25

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06.



BMO

$BM = \frac{2a}{\sqrt{2}}$; $OM = \frac{a}{\sqrt{2}}$ M-05

$\tan\theta = \frac{BM}{OM} = \frac{\frac{2a}{\sqrt{2}}}{\frac{a}{\sqrt{2}}}$

$\tan\theta = 2$ M-05

$\uparrow T\sin\theta - W = 0$ M-05

$T = \frac{W}{\sin\theta} = \frac{W\sqrt{5}}{2}$ M-05 M-05

$\left(\because \sin\theta = \frac{2}{\sqrt{5}}\right)$

Aliter

$\uparrow T\sin\theta - W = 0 \longrightarrow (1) \Rightarrow T\sin\theta = W \longrightarrow (1)$ M-05

$\rightarrow T\cos\theta - P = 0 \longrightarrow (2) \Rightarrow T\cos\theta = P \longrightarrow (2)$ M-05

B w.a $\cos 45 - P.2a\sin 45 = 0$. $\frac{(1)}{(2)}$ $\tan\theta = \frac{W}{W}.2$ M-05

$P = \frac{W}{2}$ M-05 $\tan\theta = 2$

$T = \frac{W}{\sin\theta} = \frac{\sqrt{5}W}{2}$ M-05

07. Probabilities of events

$P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, $P(A \cap B) = \frac{1}{6}$

$P(A \cap B') + P(A \cap B) = P(A)$

$P(A \cap B') = P(A) - P(A \cap B) = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$ M-05

Thus

$P(A/B') = \frac{P(A \cap B')}{P(B')} = \frac{P(A \cap B')}{1 - P(B)} = \frac{\frac{1}{6}}{\frac{3}{4}} = \frac{2}{9}$ M-05

$P(A' \cap B') = P[(A \cup B')] = 1 - P(A \cup B)$

$= 1 - P(A) - P(B) - P(A \cap B)$ M-05

$= 1 - \frac{1}{3} - \frac{1}{4} + \frac{1}{6} = \frac{7}{12}$ M-05

$P(B'/A') = \frac{P(A' \cap B')}{P(A')} = \frac{\frac{7}{12}}{1 - \frac{1}{3}} = \frac{7}{8}$ M-05 T.M-25

08. All Red : $\frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} = \frac{1}{35}$ M-05

M-05

(i) All Black: Not Possible

(ii) RBRB: $\frac{4}{7} \times \frac{3}{6} \times \frac{3}{5} \times \frac{2}{4} = \frac{3}{35}$ M-05

BRBR: $\frac{3}{7} \times \frac{4}{6} \times \frac{2}{5} \times \frac{3}{4} = \frac{3}{35}$ M-05

$\therefore$ Answer $= \frac{6}{35}$ M-05

T.M-25

09. Let the mode be 2a

So, let the given integers be ; b, c, a, 2a, 2a M-05

mean ; Mode = 6 : 10

$\therefore \frac{10(b+c+5a)}{5}$ M-05 $= 6 \times 2a$ M-05

$\Rightarrow b + C = a$ M-05 $\therefore$ Given integers are M-05 1, 2, 3, 6, 6 T.M-25

Aliter

$x_1, x_2, x_3, x_4, x_5 < 8$

Mean : Mode : Mid = 6: 10 : 15

Mean $= \frac{x_1 + x_2 + x_3 + 2x_4}{5} \longrightarrow (1)$ M-08

Mode $= x_4$

Mid $= x_3$

$\frac{\text{Mode}}{\text{Mid}} = \frac{x_4}{x_3} = \frac{10}{5} = 2$ M-05

$x_4 = 2x_3 \longrightarrow (2)$

$\frac{\text{Mean}}{\text{Mode}} = \frac{x_1 + x_2 + x_3 + 2x_4}{5x_4} = \frac{6}{10}$

$x_1 + x_2 + x_3 + 2x_4 = 3x_4$

$x_1 + x_2 + x_3 = x_4 \longrightarrow (3)$ M-05

(2) and (3), $x_1 + x_2 + x_3 = 2x_3$

$x_1 + x_2 = x_3$

x_1	x_2	x_3	x_4	x_5
1	2	3	6	6

M-10

10. $\mu = 28$, $\sigma_1 = 4$

Data Altered : $35 \rightarrow 25$ (−10)

$21 \rightarrow 31$ (+10)

$\therefore$ Sum is not altered, remains the same

$\therefore \mu$ = retained as 28 M-05

Earlier $\Sigma x_i^2 = 20 \times \sigma_1^2 + 20\mu^2 = 20(4^2 + 28^2)$ M-05

New Σx_i^2 = Earlier $\Sigma x_i^2 - 35^2 - 21^2 + 25^2 + 31^2$ M-05

$= 20(4^2 + 28^2) - 8 \times 10$ M-05

New $\sigma^2 = \frac{20(28^2 + 4^2) - 8 \times 10}{20}$

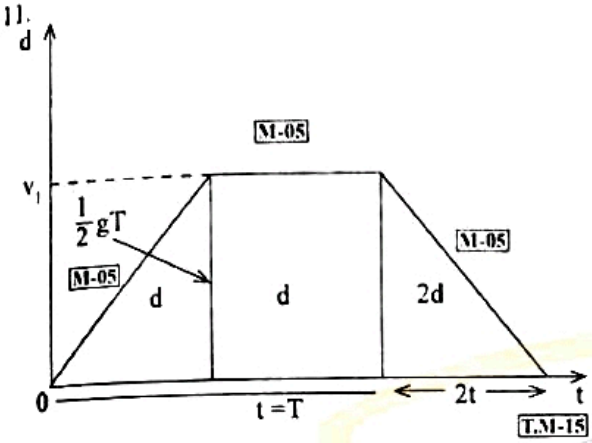
$= \frac{20 \times 16 - 20 \times 4}{20}$

$= 12$

$\sigma = \sqrt{12}$ M-05 T.M-25

Part B

11.



$d = \frac{1}{2} \times \frac{1}{2} gT \times T$

Also $2d = \frac{1}{2} \times \frac{1}{2} gT \times 2T$ M-05

$= \left(\frac{1}{2} g\right) T$ Time with constant $= \frac{T}{2}$ M-05

M-05

Time with Moving time with constant retar dation = 2T M-05

$T^2 = \frac{4d}{g} \Rightarrow T = \sqrt{\frac{d}{g}}$ M-05

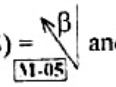
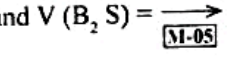
Total time $\frac{7T}{2} = 7\sqrt{\frac{d}{g}}$ M-05

Retardation $f = \frac{\frac{1}{2} gT}{2T} = \frac{1}{4} g$ M-05

T.M-35

(b) $\underline{v} = (S, E) = u\uparrow$

$\underline{v} = (B_i, E) = V$ for $i = 1, 2$

$\underline{v} = (B_1, S) =$  and $V(B_2, S) =$ 

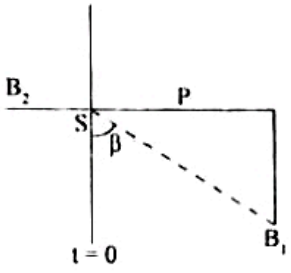
M-05 M-05

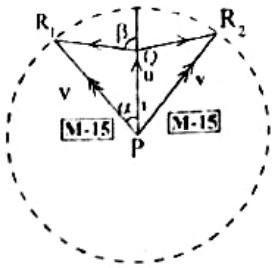
$\underline{v} = (B_i, E) = \underline{v}(B_i, S) + \underline{v}(S, E)$ M-05

$= \underline{v}(S, E) + \underline{v}(B_i, S)$ M-05

$= \overrightarrow{PQ} + \overrightarrow{QR_i}$

$= \overrightarrow{PR_i}$ (for $i = 1,2$)





from triangle PQR_1 using since formula

$\frac{v}{\sin\beta} = \frac{u}{\sin(\beta - \alpha)}$ M-05

$\sin(\beta - \alpha) = \frac{u \sin\beta}{v}$

$(\beta - \alpha) = \sin^{-1}\left(\frac{u \sin\beta}{v}\right)$

$\alpha = \beta - \sin^{-1}\left(\frac{u \sin\beta}{v}\right) \longrightarrow (1)$ M-05

$\therefore$ Path of B_1 relative to earth, makes a given by (1) angle west of north similarly path of B_1 relative of earth. marks γ Angle γ East of North

where $\gamma = \cos^{-1}\left(\frac{u}{v}\right)$ M-05 T.M-65

(ii) Take $\beta = \frac{\pi}{3}$ and $v = \sqrt{3}\,u$

Then

$\alpha = \frac{\pi}{3} - \sin^{-1}\left[\frac{u\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}u}\right] = \frac{\pi}{3} - \sin^{-1}\left(\frac{1}{2}\right)$

$\frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ M-05

$\therefore$ Then, vel B_1, S, V = U

M-05

B_1 Distance along relative path, the distance $= \frac{2p}{\sqrt{3}}$ M-05

For B_1, Time $t_1 = \frac{\frac{2p}{\sqrt{3}}}{u} = \frac{2p}{\sqrt{3}u}$ M-05

For B_2, Time $t_2 = \frac{q}{\sqrt{v^2 - u^2}} = \frac{q}{u\sqrt{3 - 1}} = \frac{q}{\sqrt{2}\,u}$ M-05

M-05

B_1 intercept S before B_2 if $t_1 < t_2$

i.e if $\frac{2p}{\sqrt{3}\,u} < \frac{q}{\sqrt{2}\,u}$

$\Rightarrow 2\sqrt{2}\,p < \sqrt{3}\,q$

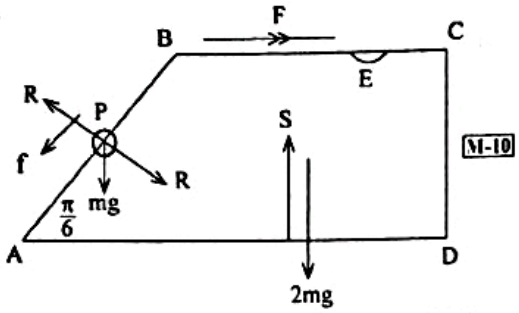
$\Rightarrow 8p^2 < 3q^2$ M-05

T.M-35

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12.



M-10

$\underline{a}$ (p, w) = $f\swarrow$ $\quad q$(W, E) = F $\longrightarrow$ M-05

$\underline{F} = m\underline{a}$

For System $\rightarrow$(P+W) $0 = m(-f\cos\frac{\pi}{6} + F) + 2mF$ M-15

$0 = -\frac{\sqrt{3}}{2}f + 3F \Rightarrow \frac{\sqrt{3}f}{6} = F$ M-05

For P$\swarrow$ $mg\cos\frac{\pi}{3} = m\left(f - F\cos\frac{\pi}{6}\right)$ M-10

$\frac{g}{2} = f - \frac{\sqrt{3}f}{6} \Rightarrow \frac{g}{2} = f - \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}f}{6}$ M-05

$\Rightarrow f = \frac{2g}{3}$ M-05

^{r}B, W = V

$\uparrow v^2 = u^2 + 2as$

$v^2 = u^2 - 2\left(\frac{2g}{3}\right)a$ M-05 $\qquad = \frac{3u^2 - 4ga}{3}$

$v^2 = \frac{3\frac{7ga}{3} - 4ga}{3}$

$v^2 = ga$

$v = \sqrt{ga}$ M-05 T.M-65

Motion of the particle relative to the Block after leaving the faceAB

q(P, W) $= q$(P, E) + q(E, W)

$= \downarrow g + \underline{o}$ ($\therefore$ Block will move with uniform velocity)

$= \downarrow g$ M-10

Let t be the time taken by the particle P to reach the upper face of the block again.

Then $\uparrow$ $o = v\sin\frac{\pi}{6}t - \frac{1}{2}gt^2$

$= \frac{v}{2}t - \frac{1}{2}gt^2$

$\Rightarrow t = \frac{v}{g} = \sqrt{\frac{a}{g}}$ M-05

Let R be the distance travelled by the particle along the upper face of the block during time t, relative to the block

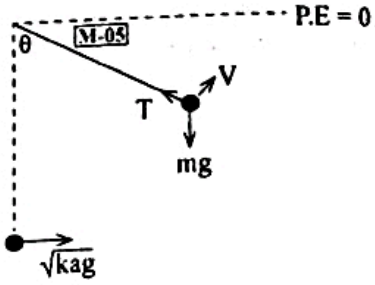
$R = V\cos\frac{\pi}{6}.t$ M-05

$R = V.\frac{\sqrt{3}}{2}.\sqrt{\frac{a}{g}} = \sqrt{ga}.\frac{\sqrt{3}}{2}.\sqrt{\frac{a}{g}}$ M-05

$\therefore R = \frac{\sqrt{3}a}{2}$ M-05

Hence the particle P will fall into the hole at T.M-30

(b)



Conservation of energy

$-mga + \frac{1}{2}m(kag) = -mga\cos\theta + \frac{1}{2}mv^2$ M-15

$\Rightarrow v^2 = -2ga + kga + 2ag\cos\theta$

$v^2 = (k-2)ag + 2ag\cos\theta$ M-05 T.M-25

$\nwarrow \underline{F} = m\underline{a}$

$T - mg\cos\theta = \frac{mv^2}{a}$ M-10

$\Rightarrow T - mg\cos\theta + \frac{m}{a}[(k-2)ag + 2ag\cos\theta]$

Tension : $T = (k-2)mg + 3mg\cos\theta$ M-05

Note that v and T both decrease as θ increase

$T = mg(3\cos\theta - 2 + k)$

M-05 T = 0 when $3\cos\theta - 2 + k = 0$

i.e $\cos\theta = \frac{2-k}{3}$ M-05

when $\cos\theta = \frac{2-k}{3}$

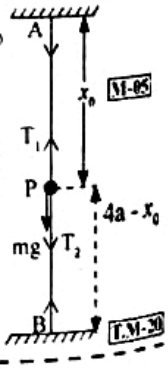
$V^2 = (k-2)ag + \frac{2ag(2-k)}{3}$

$= \frac{ag}{3}(k-2) > 0$ as, $K > 2$ M-05

Hence, String be comes slack when, $\theta = \alpha$

with $\cos\alpha = \frac{2-k}{3}$ $(2<k<5)$ T.M-30

13. At equilibrium Position, let $x = x_0$



Then $\uparrow$ $T_1 = T_2 + mg$ M-05

$\frac{mg}{a}(x_0 - a) = \frac{mg}{a}(4a - x_0 - a) + mg$ M-05

$x_0 - a = 3a - x_0 + a$

$\Rightarrow x_0 = \frac{5a}{2}$ M-05

Appling P $\downarrow$ $\underline{F} = m\underline{a}$ for P

$T_2' + mg - T_1' = m\ddot{x}$ M-05

$\frac{mg}{a}(4a - x - a) + mg - \frac{mg}{a}(x - a) = m\ddot{x}$ M-10

$\Rightarrow \ddot{x} = -\frac{2g}{a}(x - \frac{5a}{2})$

$\Rightarrow \ddot{x} + \frac{2g}{a}(x - \frac{5a}{2}) = 0$ M-05

Let, $x = x - \frac{5a}{2}$ and $w^2 = \frac{2g}{a}$

Then $\ddot{x} + w^2 x = 0$ M-05

Centre if S.H.M $x = \frac{5a}{2}$ M-05

$\dot{x}^2 = w^2(c^2 - x^2)$, Where C is the amplitude

$\dot{x} = 0$, When $x = \frac{-a}{2}$ M-05

$\dot{x}^2 = w^2(c^2 - \frac{a^2}{4}) = 0 \quad c = \frac{a}{2}$ M-10

$\therefore$ lowest $x = \frac{a}{2} \Rightarrow x = 3a$ M-05

After cutting the string PB

$\downarrow \underline{F} = m\underline{a}$

$mg - T = m\ddot{x}$

$mg - \frac{mg}{a}(x - a) = m\ddot{x}$ M-05

$\ddot{x} + \frac{g}{a}(x - 2a) = 0 \Rightarrow \ddot{Y} + \Omega^2 Y = 0$ Where $Y = x - 2a$

M-05 M-05

and $\Omega^2 = \frac{g}{a}$ M-05

Centres of new S.H.M $x = 2a$

$\dot{Y}^2 = \Omega^2(b^2 - y^2)$ M-05

Just after string PB is cut, $\dot{Y} = 0$ and $x = 3a$

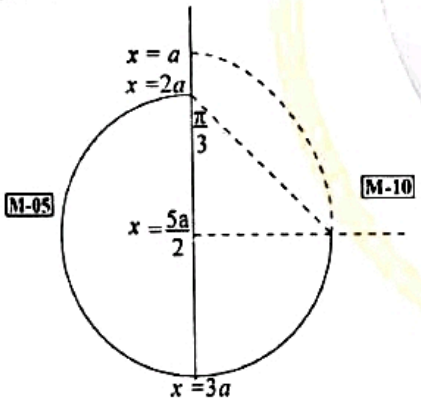
$\Rightarrow$ $Y = a$ M-05 M-05

Substituting $y = -a$ is the amplitude of the new S.H.M

$\therefore \dot{Y} = 0$ again when $Y = -a \Rightarrow x = a$ M-05

i.e particle reaches the high est problem $x = a$ M-05

T.M-45



M-05 M-10

x = Time takes to move downwards from $2a$ to $3a$

$\frac{\pi}{\omega} = \pi\sqrt{\frac{a}{2g}}$ M-05

Time takes to move upward from $x = 3a$ to

$\frac{5a}{2} = \frac{\pi}{3\omega} = \frac{\pi}{3}\sqrt{\frac{a}{g}}$ M-10

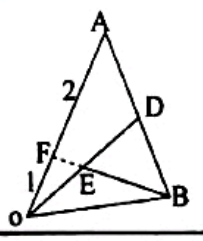
Total time $= \pi\sqrt{\frac{a}{2g}} + \frac{\pi}{3}\sqrt{\frac{a}{g}}$ M-05

$= \frac{\pi}{3}\sqrt{\frac{a}{2g}}(3+\sqrt{2})$ T.M-35

14. (a) $\overrightarrow{OA} = \underline{a}$, $OB = \underline{b}$

$\overrightarrow{OF} = \frac{1}{3}\underline{a}$

$\overrightarrow{OD} = \frac{1}{2}(\underline{a} + \underline{b})$ (05)



$\overrightarrow{OE} = \frac{1}{4}(\underline{a} + \underline{b})$

$\overrightarrow{BE} = \overrightarrow{OE} - \overrightarrow{OB} = \frac{1}{4}(\underline{a} + \underline{b}) - \underline{b} = \frac{1}{4}(\underline{a} - 3\underline{b})$ M-05

M-05

$\overrightarrow{BF} = \overrightarrow{OF} - \overrightarrow{OB} = \frac{1}{3}\underline{a} - \underline{b} = \frac{1}{3}(\underline{a} - 3\underline{b})$ M-05

$\Rightarrow 4\overrightarrow{BE} = 3\overrightarrow{BF}$

Hence B, E, F are collinear and BE : EF = 3:1 M-05

M-05 T.M-30

$\overrightarrow{DF} = \overrightarrow{OF} - \overrightarrow{OD} = \frac{1}{3}\underline{a} - \frac{1}{2}(\underline{a} + \underline{b}) = -\frac{1}{6}(\underline{a} + 3\underline{b})$ M-05

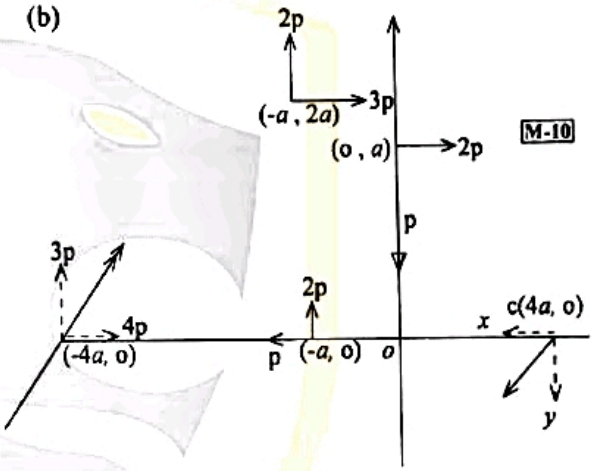
$\overrightarrow{BF}.\overrightarrow{DF} = \frac{1}{3}(\underline{a} - 3\underline{b}) \times \frac{1}{6}(-\underline{a} - 3\underline{b})$ M-05

$= -\frac{1}{18}(|\underline{a}|^2 - 9|\underline{b}|^2) = 0$, When $(|\underline{a}| = 3|\underline{b}|)$ M-05

$\therefore \overrightarrow{BF} \perp \overrightarrow{DF}$ since they are non - z ero M-05

T.M-20

(b)



Taking moments about o M-10

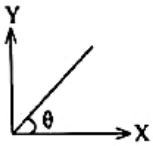
$G = 2Pa + 3P.2a + 2Pa + 2Pa = 12Pa$ Nm; *in the clockwise sense*

Re solving$\rightarrow$ $x = 3P + 2P - P = 4P$ M-05

$\uparrow Y = 2P + 2P - P = 3P$ M-05

Magnitude R of the resultant is given by

$R = \sqrt{x^2 + y^2} = P\sqrt{4^2 + 3^2} = 5PN$



Direction line of action makes $\angle\theta$ with x - ax is,

where $\tan\theta = \frac{y}{x} = \frac{3}{4}$ M-05

if the line of action of the resultant meets the x -ax is at the point (-b, 0), where (b>0) then

$\circlearrowright yb = 3Pb = 12Pa \Rightarrow b = 4a$ M-05

M-05

Equation of line of action of the resultant is

$y - 0 = \frac{3}{4}(x + 4a)$ M-05

$4y = 3x + 12a \Rightarrow 4y - 3x = 12a$ M-10 M-60

Now system becomes equivalent to a couple,only is a force (-4P, -3P) is applied at a point (C, 0), where $3P(C+4a) = 24Pa\circlearrowright$ M-10 M-05

$\Rightarrow C = 4a$ M-05

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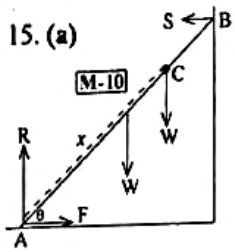
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Magnitude of extra force = 5 PN and its direction makes an angle $\tan^{-1}\left(\frac{-3p}{-4p}\right) = \tan^{-1}\left(\frac{3}{4}\right)$ [M-05], with the negative ox - $ax$is, Line of action of the additional force is $y - 0 = \frac{3}{4}(x - 4a)$ [M-05]

$\Rightarrow 4y - 3x + 12a = 0$ [M-10] [T.M-40]

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15. (a)



[M-10]

A for the rod AB

$\Rightarrow S.2a \sin\theta = w(a\cos\theta + x\cos\theta)$ [M-15]

$\Rightarrow S.a.\frac{6}{5} = w(a+x)\frac{4}{5}$

$\Rightarrow S = \frac{2w(a+x)}{3a}$ [M-05]

$\tan\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$ [M-05]

$\rightarrow F = S = \frac{2w(a+x)}{3a}$ [M-05]

$\uparrow R = 2w$ [M-05]

[M-05] $F \le \mu R$ and $\mu = \frac{5}{6}$

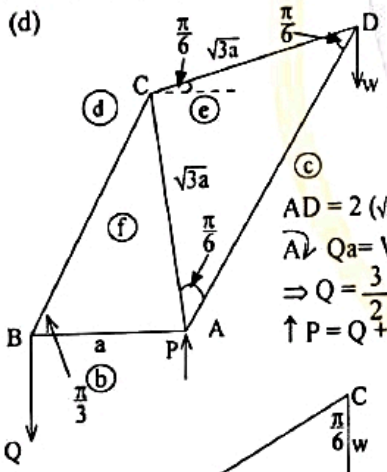
$\Rightarrow \frac{2w(a+x)}{3a} \le \frac{5}{6} \cdot 2w$ [M-05]

$\Rightarrow a + x \le \frac{5a}{2}$

$\Rightarrow x \le \frac{3a}{2}$ [M-05]

[T.M-60]

(d)

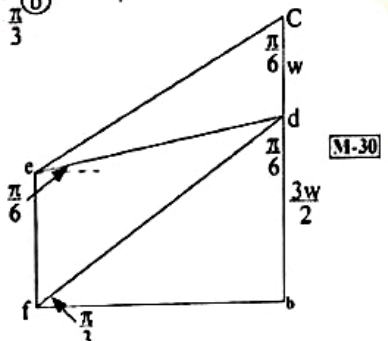


$AD = 2(\sqrt{3}a\cos 30^\circ) = 3a$

A $\circlearrowleft$ $Qa = W.AD\cos 60$

$\Rightarrow Q = \frac{3}{2} - W$ [M-10]

$\uparrow P = Q + W \rightarrow P = \frac{5}{2}W$

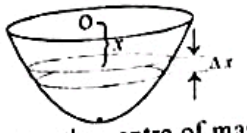


[M-30]

Rod	Tension	Thrust	
AB	-	$\frac{\sqrt{3}w}{2}$	[M-10]
BC	$\sqrt{3}w$	-	[M-10]
AC	-	W	[M-10]
CD	W	-	[M-10]
AD	-	$\sqrt{3}w$	[M-10]

[T.M-90]

16.



[M-05]

By symmetry, the centre of mass G lies on OA Let $OG = \bar{x}$ and P be the density. then

$\Delta m = \pi(a^2 - x^2)\Delta x.\rho$ and

$$\bar{x} = \frac{\int_0^a \pi(a^2-x^2)\rho x\,dx}{\int_0^a \pi(a^2-x^2)\rho\,dx}$$ [M-15]

$$= \frac{\int_0^a (a^2x - x^3)dx}{\int_0^a \pi(a^2-x^2)dx} = \frac{\left[a^2\frac{x^2}{2} - \frac{x^4}{2}\right]_0^a}{\left[a^2x - \frac{x^3}{3}\right]_0^a}$$ [M-05] [M-05] [M-05]

$$\bar{x} = \frac{\left(\frac{a^4}{2} - \frac{a^4}{4}\right)}{\left(a^3 - \frac{a^3}{3}\right)} = \frac{3}{8}a$$ [M-05]

Hence the center of mass is at a distance $\frac{3}{8}a$ from 0

[T.M-40]

Object	Mass	Distance from 0
	$\frac{2}{3}\lambda\pi a^3\rho$ [M-05]	$\frac{11}{8}a$ [M-05]
	$\pi a^3\rho$ [M-05]	$\frac{1}{2}a$ [M-05]
	$\frac{2}{3}\lambda a^3\rho\pi$ [M-05]	$\frac{3}{8}a$ [M-05]
	$\left(\frac{2}{3}\lambda + \frac{1}{3}\right)a^3\rho\pi$ [M-05]	$\bar{x}$

By symmerty, the cente of mass lies on the axis of symmetry [M-05]

$$\frac{1}{3}(2\lambda+1)\pi a^3\rho\,\bar{x} = \frac{11}{8}a\times\frac{2}{3}\pi a^3\lambda\rho + \frac{a}{2}\times\pi a^3\rho - \frac{3}{8}a\frac{2}{3}\pi a^3\rho$$ [M-25]

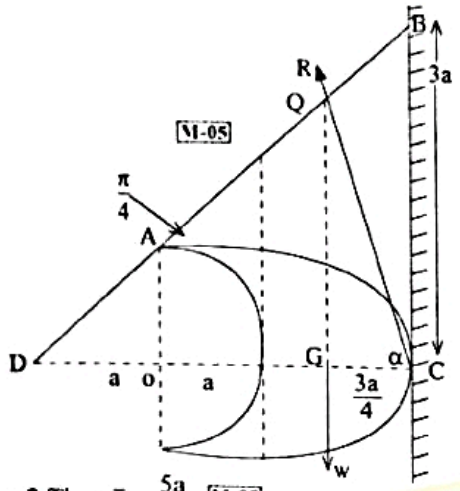
$$\frac{1}{3}(2\lambda+1)\bar{x} = \frac{11a}{8}\times\frac{2\lambda}{3} + \frac{a}{2} - \frac{3a}{8}\times\frac{2}{3}$$

$$= \frac{11\lambda}{12}a + \frac{a}{2} - \frac{a}{4} = \frac{1}{12}(11\lambda+3)a$$

$$\bar{x} = \frac{(11\lambda+3)a}{4(2\lambda+1)}$$ [M-10]

[T.M-75]

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Let $\lambda = 2$ Then $\bar{x} = \frac{5a}{4}$ M-05

For equilibirium $\mu \geq \tan\alpha = \frac{QG}{GC} = \frac{\frac{9a}{4}}{\frac{3a}{4}} = 3$ M-10 M-05 M-05

$\therefore \mu \geq 3$ M-05

T.M-35

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17. X = Male selected for this job

A = Obtained an A grade for the aptitude test

(i) $P(X) = \frac{3}{5}\times\frac{3}{5} + \frac{2}{5}\times\frac{1}{10}\times\frac{3}{10} = \frac{93}{250}$ M-10 M-10 M-10

T.M-30

(ii) $P(A/X) = \frac{P(X \cap A)}{P(X)} = \frac{\frac{3}{5}\times\frac{3}{5}}{\frac{93}{250}} = \frac{30}{31}$ M-10 M-10 M-10

T.M-30

M-05 M-05 M-05

Range of values	f	Mid Point y	y^2	fy	fy^2
-2 - 0	30	-1	1	30	30
0 - 2	40	1	1	40	40
2 - 4	15	3	9	45	135
4 - 6	10	5	25	50	250
6 - 8	5	7	49	35	245
	$\Sigma f = 100$			$\Sigma fy = 140$	$\Sigma fy^2 = 700$

M-05 M-05

Mean $= \mu_y = \frac{\Sigma fy}{\Sigma f} = \frac{140}{100} = \frac{7}{5}$ M-05 M-05

Standard deviation : $\sigma_y^2 = \frac{\Sigma fy^2}{\Sigma f} - \mu_y^2 = \frac{700}{100} - \frac{49}{25}$ M-05

$\sigma_y = \frac{\sqrt{504}}{10} \approx 2.24$ M-05

T.M-45

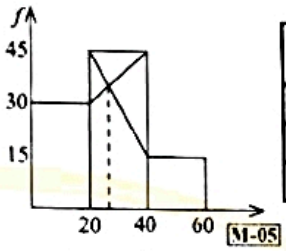
$y = \frac{x - 20}{10} \Rightarrow x = 10y + 20$

Hence $\mu = 10\,\mu_y + 20 = 10\left(\frac{7}{5}\right) + 20 = 34$ M-05 M-05

$\sigma = 10\,\sigma_y \approx 10\,(2.24) \approx 22.4$ M-05 M-05

T.M-20

To find the mode M :



M-05

range of y	range of x	frequency
-2 - 0	0 - 20	30
0 - 2	20 - 40	40
2 - 4	40 - 60	15

$\frac{d}{40-30} = \frac{20-d}{40-15} \Rightarrow d = \frac{40}{7} \Rightarrow M = 20 + \frac{40}{7} \approx 25.71$

M-05 M-05 M-05

$K = \frac{\mu - M}{\sigma} = \frac{34 - 25.71}{22.4} \approx 0.37$ M-05

T.M-25

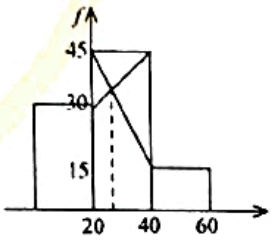
Aliter

$M = L_{M_0} + C\left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) = 20 + 20\left(\frac{10}{10+25}\right) \approx 25.71$

M-10 M-05 M-05

T.M-20

Aliter: finding the mode using transformed data



M-05

$\frac{d}{40-30} = \frac{2-d}{40-15} \Rightarrow d = \frac{4}{7}$ M-05

$\Rightarrow$ Mode of the transformed data $= 0 + \frac{4}{7}$ M-05

$\therefore M = 10\left(\frac{4}{7}\right) + 20 \approx 25.71$ M-05

T.M-20