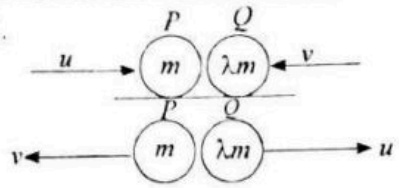
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Paper II

Part A

1.



Apply $\underline{I} = \Delta(M\underline{v}) \longrightarrow$ for the system :

$0 = (\lambda mu - mv) - (mu - \lambda mv)$ (05)

$\Rightarrow 0 = (\lambda - 1)u + (\lambda - 1)v$

$\Rightarrow 0 = (\lambda - 1)(u + v)$

$\Rightarrow \lambda = 1$ (05)

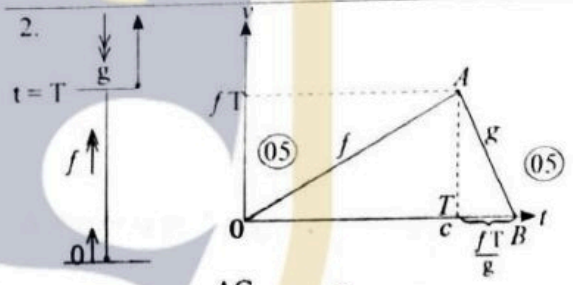
Let e be the coefficient of restitution between P and Q. Then by Newton's law of restituion

$(u + v) = e(u + v)$ (10)

$\therefore e = 1$ (05)

[25]

2.



$f = \frac{AC}{T}$ and $g = \frac{AC}{BC} \Rightarrow = \frac{f}{g}T$ (05)

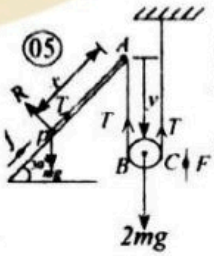
The maximum height = The area of the triangle

$OAB = \frac{1}{2}\left(T + \frac{fT}{g}\right) \times fT$ (05) + (05)

$= \frac{fT^2}{2g}(f + g)$

[25]

3. $x + 2y =$ constant $\Rightarrow \dot{x} + 2\dot{y} = 0 \Rightarrow 2\dot{y} = -\dot{x}$ (05)



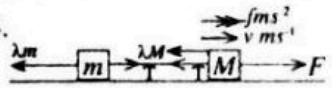
With f and F as shown in the figure, $f = 2F$ (05)

$E = ma \downarrow$ for P : $T - mg\sin 30^\circ = mf$ (05)

$E = ma \downarrow$ for 2mg: $2mg - 2T = 2mF$ (05)

[25]

4.



(05)

Tractive force $F = \frac{1000P}{v}$ N ——— (1) (05)

$E = ma \longrightarrow$ for M: $F - \lambda M - T = Mf$ ——— (2) (05)

$E = ma \longrightarrow$ for m: $T - \lambda m = mf$ ——— (3) (05)

Now ①, ② and ③ $\Rightarrow \frac{1000P}{v} - \lambda M - T = Mf$

$\Rightarrow \frac{1000P}{v} - \lambda M - T = \frac{M}{m}(T - \lambda M)$

$\Rightarrow T = \frac{1000mP}{(M+m)V}$ N (05)

25

5. Scalar Product : $\overrightarrow{OA} \cdot \overrightarrow{OB} = (-\underline{i} + 2\underline{j}) \cdot (2\alpha\underline{i} + \alpha\underline{j})$

$= -2\alpha + 2\alpha = 0$ (05)

$\therefore A\hat{O}B = \frac{\pi}{2}$ (05)

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$

$= \overrightarrow{OA} + \overrightarrow{OB}$

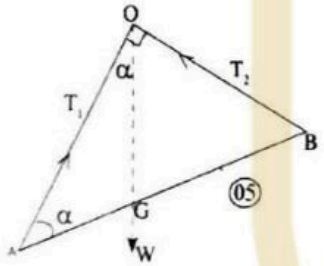
$= (-1 + 2\alpha)\underline{i} + (2+\alpha)\underline{i}$ (05)

$\overrightarrow{OC}$ lies along the $y - \alpha x$ is $\Rightarrow 1 - 2\alpha = 0$ (05)

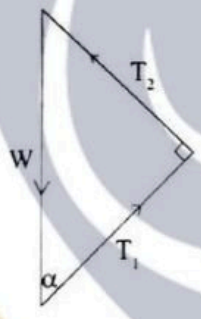
$\Rightarrow \alpha = \frac{1}{2}$ (05)

25

6.



(05)



Since $A\hat{O}B = \frac{\pi}{2}$, AB is a diameter of the circle through A, O and B and G is the centre of it.

AG = OG

$\Rightarrow A\hat{O}G = O\hat{A}G = \alpha$ (05)

Resolving along AO and BO (05)

$T_1 = W\cos\alpha$ (05)

$T_2 = W\sin\alpha$ (05)

25

7. Since $A' \cup B' = (A \cap B)'$, We have $P[(A \cap B)'] = 1 - P(A \cap B)$

$\therefore P(A \cap B) = 1 - \frac{5}{6} = \frac{1}{6}$ (05)

Now $P(B/A) = \frac{P(B \cap A)}{P(A)} \Rightarrow P(A) = \frac{\frac{1}{6}}{\frac{1}{4}} = \frac{2}{3}$ (05) (05)

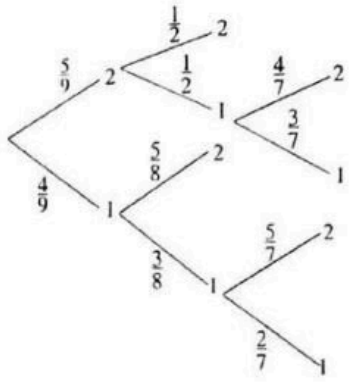
Also, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (05)

$\Rightarrow \frac{4}{5} = \frac{2}{3} + P(B) - \frac{1}{6}$

$\Rightarrow P(B) = \frac{4}{5} - \frac{1}{2} = \frac{3}{10}$ (05)

25

8.



(i) Answer $= \frac{5}{9} \times \frac{1}{2} = \frac{5}{18}$ (05)

(05)

(ii) Answer $= \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} = \frac{1}{21}$

(10) (05) 25

9. Modes are a and b (05)

It is given that $a + b = x$ and $ab = y$

Since the mean is $\frac{2}{7}$, We have $\frac{2a + 2b + x + y}{6} = \frac{7}{2}$ (05)

$\therefore 3a + 3b + ab = 21$ ——— (1) (05)

(1) $\Rightarrow a\,b$ is divisible by 3 and hence $ab \geq 3$

Again, (1) $\Rightarrow a + b \leq 6$ (05)

Since $1 < a < b$ we must have

$a = 2 \quad b = 3$ (05)

25

10. Mean = 10 $\Rightarrow \frac{\sum_{i=1}^{10} x_i}{10} = 10$ (05)

Variance = 9 $\Rightarrow \frac{\sum_{i=1}^{10} x_i^2}{10} - 10^2 = 9 \Rightarrow \sum_{i=1}^{10} x_i^2 = 1090$ (05)

The mean of the first nine numbers = 10

$\Rightarrow x_{10} = 10$ (05)

$\therefore \sum_{i=1}^{9} x_i^2 = 990$ (05)

The variance of the first nine numbers

$= \frac{990}{9} - 10^2 = 10$ (05)

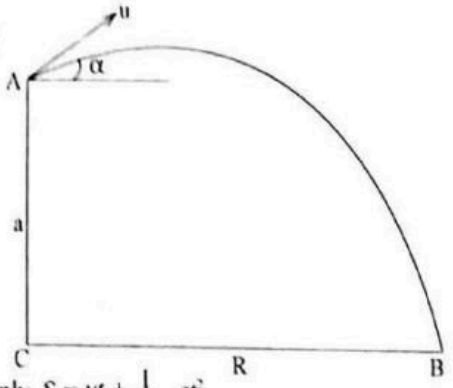
25

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Part B

11. (a)



Apply $S = ut + \frac{1}{2}at^2$

→ From A to B : $R = u\cos\frac{\pi}{4}\,.\,t = \frac{ut}{\sqrt{2}}$ ——① (05)

↑ From A to B : $-a = u\sin\frac{\pi}{4}\,t - \frac{1}{2}gt^2$ ——② (10)

① and ② $\Rightarrow -a = R - \frac{1}{2}g\frac{2R^2}{u^2}$ (05)

$\Rightarrow gR^2 - u^2R - u^2a = 0$ (05) [25]

$\therefore R = \frac{u^2 \pm \sqrt{u^4 + 4u^2ag}}{2g}$ (05)

$R = \frac{u^2 + \sqrt{u^4 + 4agu^2}}{2g}$ $(\because R > 0)$ (05) (05) [15]

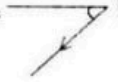
Syppose that $u^2 > \frac{4}{3}ga$

Then $R > \frac{\frac{4}{3}ga + \sqrt{\frac{16}{9}g^2a^2 + \frac{16}{3}g^2a^2}}{2g} = \frac{\frac{4}{3}ga + \frac{8}{3}ga}{2g} = 2a$ (05) (05)

$\Rightarrow P > 2a$ ∴ The stone will not fall into the pond [10]

(b) $\underline{v}(S, E) = \longrightarrow u$ (05)

$\underline{v}(B, E) = v$ (05)

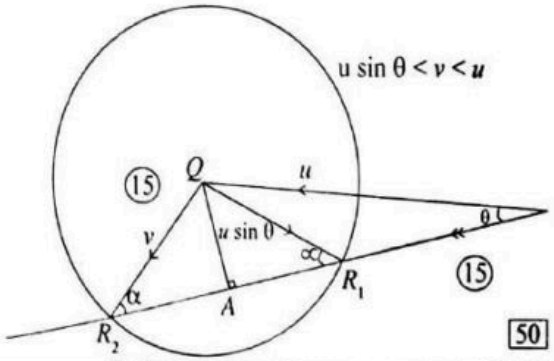
$\underline{v}(B, S) =$  (05)

$\underline{v}(B, S) = \underline{v}(B, E) + \underline{v}(E, S)$ (05)

$= \underline{v}(E, S) + \underline{v}(B, E)$

$= \overrightarrow{PQ} + \overrightarrow{QR}$

$= \overrightarrow{PR}$



[50]

Required angel = $R_1\hat{Q}R_2$ (05)

$= \pi - 2\alpha$, where $Q\hat{R}_2R_1 = \alpha$ (05)

$\sin\alpha = \frac{QA}{QR_2} = \frac{u\sin\theta}{v}$ (05)

$\therefore \alpha = \sin^{-1}\left(\frac{u\sin\theta}{v}\right)$ [15]

$t_1 + t_2 = \frac{1}{PR_1}(05) + \frac{1}{PR_2} = \frac{l(PR_1 + PR_2)}{PR_1 \cdot PR_2}$

$PR_1 = PA - AR_1$

$= u\cos\theta - \sqrt{v^2 - u^2\sin^2\theta}$ (10)

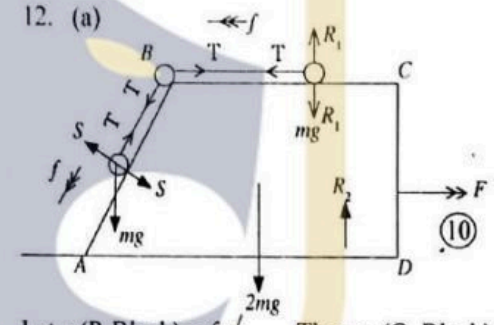
$PR_2 = PA + AR_2$

$= u\cos\theta + \sqrt{v^2 - u^2\sin^2\theta}$ (10)

$\therefore t_1 + t_2 = \frac{2ul\cos\theta}{u^2\cos^2\theta - (v^2 - u^2\sin^2\theta)}$ (05)

$= \frac{2lu\cos\theta}{u^2 - v^2}$ (05) $(\because \cos^2\theta + \sin^2\theta = 1)$ [35]

12. (a)



(10)

Let $\underline{a}$ (P, Block) = f ↓ Then $\underline{a}$ (Q, Block) = f ←

Also let , $\underline{f}$ (Block, E) = F →

Apply $\underline{F} = m\underline{a}$

for the system → $0 = 2mF + m(F - f) + m(F - f\cos\alpha)$

$\Rightarrow 0 = 4F - f - \frac{3f}{5}$ (10)

$\therefore f = \frac{5F}{2}$ ——① (10)

For the particle P ↓ $mg\sin\alpha - T = m(f - F\cos\alpha)$ (10) ——②

For the particle Q ← $T = m(f - F)$ (05) ——③ (10)

② + ③ $\Rightarrow mg \times \frac{4}{5} = m(f - F) + m(f - F \times \frac{3}{5})$

$\Rightarrow 4g = 5f - 5F + 5f - 3F$

$\Rightarrow 4g = 10f - 8F$ (05)

Now (1) $\Rightarrow 4g = 25F - 8F$

$\Rightarrow F = \frac{4}{17}g$ (05)

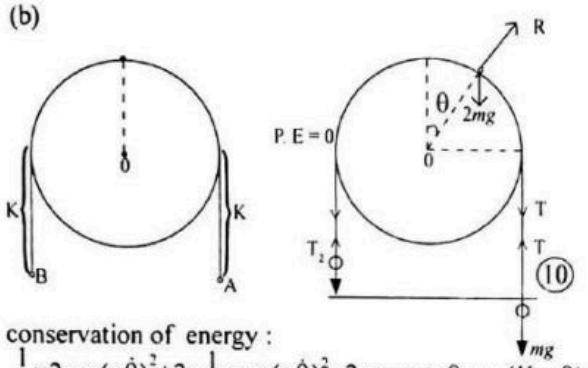
(1) $\Rightarrow f = \frac{10g}{17}$ (05) [70]

Apply $S = ut + \frac{1}{2}at^2$ ↓

For the motion of (P, B) : $a = 0 + \frac{1}{2}ft^2$ (05)

$\therefore t = \sqrt{\frac{2a}{\frac{10g}{17}}} = \sqrt{\frac{17a}{5g}}$ (05) [10]

(b)



(10)

conservation of energy :

$\frac{1}{2}\times 2m\times(a\dot\theta)^2+2\times\frac{1}{2}\times m\times(a\dot\theta)^2+2mga\cos\theta-mg(K-a\theta)$

$-mg(K-a\theta) = -2mgk+2mga$

$\Longrightarrow 2a\dot\theta^2 = -2g\cos\theta + 2g$ (10) (25)

$\dot\theta^2 = \frac{g}{a}(1-\cos\theta)$

PE 10
KE 10
Equation 05

45

Apply $\underline{F} = m\underline{a}$:

For C ↗ ; $R - 2mg\cos\theta = -2m\,a\dot\theta^2$ (10)

$\Rightarrow R = 2mg\cos\theta - 2mg(1-\cos\theta)$

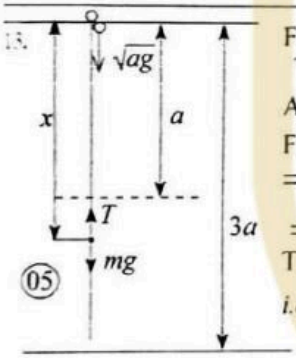
$= 2mg(2\cos\theta - 1)$ (05)

$\Rightarrow$ R decrease as θ increases, and R = 0 (05)

When $\cos\theta = \frac{1}{2}$

∴ C leaves the sphere when $\theta = \frac{\pi}{3}$ (05) 25

13.



For $a \le x < 3a$:

$T = \frac{mg}{a}(x-a)$ (05)

Apply $\underline{F} = m\underline{a}$:

For $m\downarrow$: $mg - T = m\ddot{x}$ (10)

$\Rightarrow mg - \frac{mg}{a}(x-a) = m\ddot{x}$

$\Rightarrow \ddot{x} + \frac{g}{a}(x-2a) = 0$ (05)

The centre is given by $\bar{x} = 0$

i.e $x = 2a$ (05)

So the centre is at the point C, Where C is vertically below O with OC = 2a (05) 35

Conservation of Energy :

$\frac{1}{2}m(ga) = \frac{1}{2}m\dot{x}^2 - mgx + \frac{1}{2}mg\frac{(x-a)^2}{a}$ (20)

$ga = \dot{x}^2 - 2gx - \frac{g}{a}(x^2 - 2ax + a^2)$

$\dot{x}^2 = 2gx - \frac{g}{a}x^2 + 2gx$

$\dot{x}^2 = \frac{g}{a}(4ax - x^2)$ for $a \le x < 3a$ (05) 25

$X = x - 2a \Rightarrow \dot{X} = \dot{x}$ (05)

also $a \le x < 3a \Longleftrightarrow -a \le X < a$

$\dot{X}^2 = \frac{g}{a}\{4a(X+2a) - (X+2a)^2\}$ (05)

$= \frac{g}{a}\{4a^2 - X^2\}$ for $-a \le X < a$ (05)

∴ A = 2a (05) 20

Let $\downarrow v$ be the velocity of the particle just before impact

Then $v^2 = \frac{g}{a}(4a^2 - a^2) = 3ga$

$\therefore v = \sqrt{3ga}$ (05) (05) 10

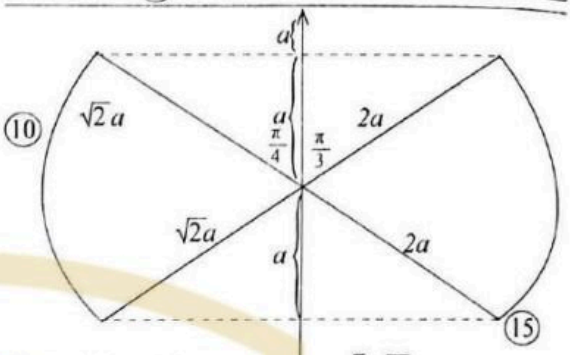
By Newton's law of restituion, velocity ↑ Just after impact $= \sqrt{ga}$ ($\because e = \frac{1}{\sqrt{3}}$) (10)

$X^2 = \frac{g}{a}(B^2 - X^2)$

when X = a

$ga = \frac{g}{a}(B^2 - a^2)$ (05)

$\Rightarrow B = \sqrt{2}\,a$ (05) 20



Since $\sqrt{\frac{g}{a}}\,t_1 = \frac{\pi}{3}$, we have $t_1 = \frac{\pi}{3}\sqrt{\frac{a}{g}}$ (05)

Since $\sqrt{\frac{g}{a}}\,t_2 = \frac{\pi}{2}$, we have $t_2 = \frac{\pi}{2}\sqrt{\frac{a}{g}}$ (05)

$\therefore t_1 + t_2 = \frac{5\pi}{6}\sqrt{\frac{a}{g}}$ (05) 40

14. (a) $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$ and $\overrightarrow{OC} = \underline{c}$

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = -\overrightarrow{OC} - \overrightarrow{OA}$ (05)

$= \underline{c} - \underline{a} = (1-\lambda)\underline{a} + \lambda\underline{b} - \underline{a}$

$= \lambda(\underline{b} - \underline{a})$ (05) (05)

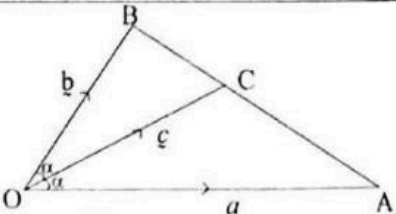
$\overrightarrow{CB} = \underline{b} - \underline{c} = \underline{b} - (1-\lambda)\underline{a} - \lambda\underline{b}$ (05)

$= (1-\lambda)(\underline{b} - \underline{a})$ (05) 25

$\overrightarrow{AC} = \frac{\lambda}{1-\lambda}\overrightarrow{CB}$ (05)

∴ C lies on AB and $\frac{AC}{CB} = \frac{\lambda}{1-\lambda}$ (05)

i.e. $AC : CB = \lambda : (1-\lambda)$ (05) 15



$B\hat{O}C = A\hat{O}C$

$\underline{a}\cdot\underline{c} = |\underline{a}|\,|\underline{c}|\cos\alpha$ (05)

$\underline{b}\cdot\underline{c} = |\underline{b}|\,|\underline{c}|\cos\alpha$ (05)

$\Rightarrow \frac{\underline{a}\cdot\underline{c}}{|\underline{a}|} = \frac{\underline{b}\cdot\underline{c}}{|\underline{b}|}$ (05)

$\Rightarrow |\underline{b}|(\underline{a}\cdot\underline{c}) = |\underline{a}|(\underline{b}\cdot\underline{c})$ (05) 20

$\Rightarrow |\underline{b}|\{(1-\lambda)|\underline{a}|^2 + \lambda\,\underline{a}\cdot\underline{b}\} = |\underline{a}|\{(1-\lambda)\underline{a}\cdot\underline{b} + \lambda|\underline{a}|^2\}$ (05)

(05)

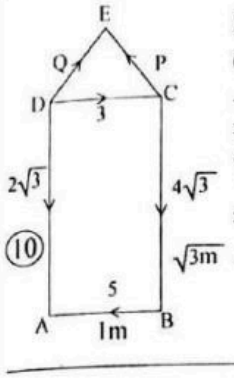
$(1-\lambda)|\underline{a}|\{|\underline{a}|\,|\underline{b}| - \underline{a}\cdot\underline{b}\} = \lambda|\underline{b}|\{|\underline{a}|\,|\underline{b}| - \underline{a}\cdot\underline{b}\}$

$(1-\lambda)|\underline{a}| = \lambda|\underline{b}| \Rightarrow \lambda = \frac{|\underline{a}|}{|\underline{a}| + |\underline{b}|}$ (05)

(∴ $\underline{a}$ *and* $\underline{b}$ are distinct and non - collinear) 15

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Since the system reduces to a couple,

$\rightarrow 3-5+Q\cos 60 - P\cos 60 = 0$ (05)

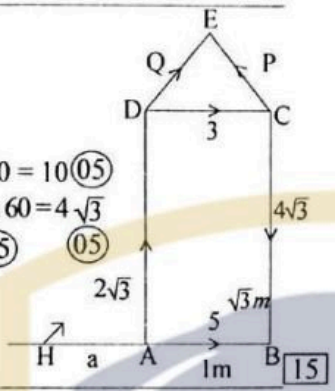
$\Rightarrow P - Q = -4$, and (05)

$\uparrow -2\sqrt{3} - 4\sqrt{3} + Q\sin 60 + P\sin 60 = 0$ (05)

$\Rightarrow P + Q = 12$ (05)

$\therefore P = 4$ and $Q = 8$ (05)

$\overleftarrow{E}$ Moment of the couple $= 7\sqrt{3}$ Nm (10) 45



$\rightarrow x = 5 + 3 + 8\cos 60 - 4\cos 60 = 10$ (05)

$\uparrow Y = 2\sqrt{3} - 4\sqrt{3} + 8\sin 60 + 4\sin 60 = 4\sqrt{3}$ (05)

$R = \sqrt{100+48} = 2\sqrt{37}$ (05) (05)

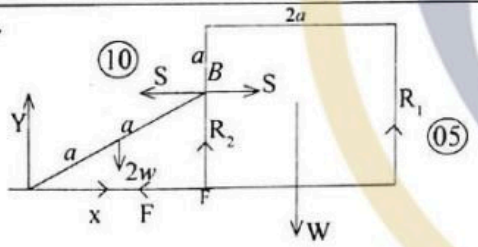
Let H be the point at which on BA Produced meets the line of action.

H $\circlearrowleft$ $-6\sqrt{3} + 2\sqrt{3}(1+a) + \sqrt{3}(3+4-2) = 0$ (10)

$-6a + 2 + 2a + 5 = 0$

$a = \frac{7}{4}m$ (05) 15

15.



(10) (05)

For the block

$\uparrow R_1 + R_2 = W$ (10)

For the block

$\rightarrow F = S$ (05)

For AB, A $\circlearrowleft$ $S \times a - 2W \cdot \frac{\sqrt{3}a}{2} = 0$ (10)

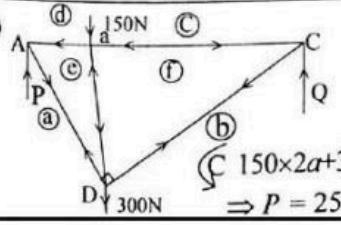
$\therefore S = \sqrt{3}\ W$ (05)

$\mu \geq \frac{|F|}{(R_1 + R_2)}$ (10)

$\mu \geq \frac{\sqrt{3}w}{w}$

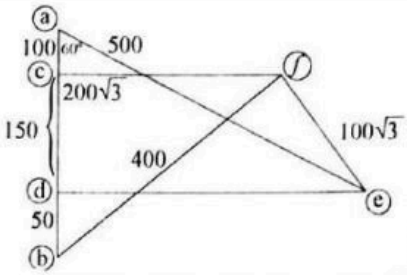
$\mu \geq \sqrt{3}$ (05) 60

(b)



$\circlearrowleft$ $150\times 2a + 300\left(2a - \frac{a}{2}\right) - P.3a = 0$

$\Rightarrow P = 250N$ (05) (05) 10



Three joints: (30)

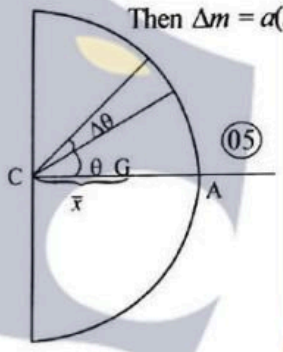
Rod	Tention	Thrust
AB		$250\sqrt{3}$ N (10)
BC		$200\sqrt{3}$ N (10)
CD	400N (10)	
DA	500 N (10)	
DB		$100\sqrt{3}$ N (10)

80

16. By Symmetry, center of mass, G, lies on CA and $OG = \bar{x}$ (05)

Let ρ be the mass per unit length

Then $\Delta m = a(\Delta\theta)\rho$ and $\bar{x} = \frac{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} a\rho\, a\cos\theta\, d\theta}{\pi a\rho}$ (10)



(05)

$\bar{x} = \frac{a}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta\, d\theta$ (05)

$= \frac{a}{\pi} \times \sin\theta \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$ (05)

$= \frac{a}{\pi}\left[2\sin\frac{\pi}{2}\right]$

$= \frac{2a}{\pi}$ (05)

Hence the centre of mass is at a distance $\frac{2a}{\pi}$ from C. 35

Object	mass	Vertical distance from p↓ to the center of mass
PR	$a\rho$	$\frac{a}{4}$ (05)
PQ	$a\rho$ (10)	$\frac{a}{4}$ (05)
ST	$2a\rho$	$\frac{a}{2}$ (05)
SUT	$\pi ak\rho$ (05)	$\frac{a}{2} + \frac{2a}{\pi}$ (05)
Combined Object	$(4+\pi k)a\rho$ (05)	$\bar{x}_1$

By Symmetry, centre of mass of L lies on the line joining P and 0 (05)

by the definition of centre of mass

$(4a\rho + \pi ak\rho)\ \bar{x}_1 = 2a\rho \times \frac{a}{4} + 2a\rho \times \frac{a}{2} + \pi ak\rho\left(\frac{a}{2} + \frac{2a}{\pi}\right)$ (15)

$\Rightarrow (4 + \pi k)\ \bar{x}_1 = \frac{a}{2} + a + \frac{\pi ak}{2} + 2ak$ (05)

$\Rightarrow \bar{x}_1 = \left(\frac{\pi k + 4k + 3}{\pi k + 4}\right)\frac{a}{2}$ (05) 70

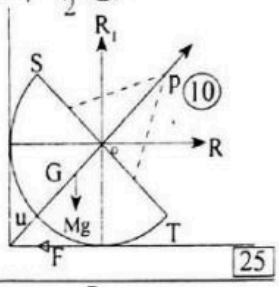
For the wire frame L to be in equilibirium in the given position, we must have $\bar{x}_1 > \frac{a}{2}$ (05)

i.e $\left(\frac{\pi k+4k+3}{\pi k+4}\right)\frac{a}{2} > \frac{a}{2}$ (05)

$\Longleftrightarrow \pi k+4k+3 > \pi k+4$

$k > \frac{1}{4}$ (05)

Let K = 1

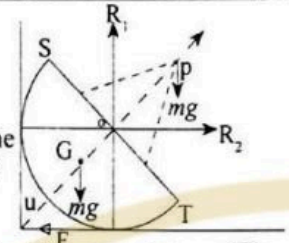


(10)

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Then $\bar{x}_1 = \left(\frac{\pi+7}{\pi+4}\right)\frac{a}{2}$

Let $\bar{x}_2$ be the distance to the new centre of mass from P



Then $[(4ap + \pi ap) + m]\,\bar{x}_2 = (4ap + \pi ap)\,\bar{x}_1$ (05)

$\Longleftrightarrow [(4ap + \pi ap) + m]\,\bar{x}_2 = (4ap + \pi ap)\left(\frac{\pi+7}{\pi+4}\right)\frac{a}{2}$

$\Longleftrightarrow [(4ap + \pi ap) + m]\,\bar{x}_2 = ap\,(\pi+7)\frac{a}{2}$

$\Longleftrightarrow \bar{x}_2 = \frac{ap\,(\pi+7)}{[(4ap + \pi ap) + m]} \times \frac{a}{2}$ (05)

To maintain equilibrium in the above position we must have $\bar{x}_2 > \frac{a}{2}$ (05)

i.e $\frac{ap\,(\pi+7)}{[(4ap + \pi ap) + m]} \quad \frac{a}{2} > \frac{a}{2}$

$\Longleftrightarrow ap\,(\pi+7) > 4ap + \pi ap + m$ (05)

$\Longleftrightarrow m < 3\,ap$

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17. (a) Let x = First ball drawn is white and the Second ball drawn is black. By the Law of Total Probability.

(05)

$P(x) = P(x|A)\,P(A) + P(x|B)\,P(B) + P(x|C)\,P(C)$ —①

$P(x|A) = \frac{4}{6} \times \frac{2}{5} = \frac{4}{15}$ (10)

$P(x|B) = \frac{2}{6} \times \frac{4}{5} = \frac{4}{15}$ (10)

$P(x|C) = \frac{m}{(2m+1)} \cdot \frac{m+1}{2m} = \frac{(m+1)}{2(2m+1)}$ (10)

Also $P(A) = P(B) = P(C) = \frac{1}{3}$ (05)

Since $P(x) = \frac{5}{18}$

(1) $\Rightarrow \frac{5}{18} = \frac{4}{15}\times\frac{1}{3} + \frac{4}{15}\times\frac{1}{3} + \frac{m+1}{2(2m+1)} \times \frac{1}{3}$ (10)

$\Rightarrow \frac{5}{6} - \frac{8}{15} = \frac{m+1}{2(2m+1)}$ (05)

$\Rightarrow 3\,(2m+1) = 5(m+1)$

$\Rightarrow m = 2$ (05)

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$m = 2 \Rightarrow P(x|C) = \frac{3}{10}$ (05)

By Baye's Theorem

$P(C|x) = \frac{P(x|C)\,P(C')}{P(x)}$ (05)

$= \frac{\frac{3}{10} \times \frac{1}{3}}{\frac{5}{18}}$ (05)

$= \frac{9}{25}$ (05)

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(05) (05) (05)

Marks range	f	Mid Point x	x^2	fx	fx^2
0-2	15	1	1	15	15
2-4	25	3	9	75	225
4-6	40	5	25	200	1000
6-8	15	7	49	105	735
8-10	5	9	81	45	405
	$\Sigma f = 100$			$\Sigma fx = 440$	$\Sigma fx^2 = 238$

(05) (05)

$\mu = \frac{\Sigma fx}{\Sigma f} = \frac{440}{100} = 4.4$ (05)

$\sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \mu^2}$ (05)

$= \sqrt{\frac{2380}{100} - \left(\frac{44}{10}\right)^2}$ (05)

$= \sqrt{23.8 - 19.36}$ (05)

$= \sqrt{4.44}$

$= 2.11$ (05)

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$M = 4 + \frac{10}{40} \times 2$ (05)

$= 4.5$ (05)

$K = \frac{3(4.4 - 4.5)}{2.11}$ (05)

$= -\frac{0.3}{2.11}$

$= -0.14$ (05)

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