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සියලු ම හිමිකම් ඇවිරිණි / முழுப் பதிப்புரிமையுடையது / All Rights Reserved]

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Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka
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கல்விப் பொதுத் தராதர் பத்திர (உயர் தர)ப் பரீட்சை, 2016 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2016

සංයුක්ත ගණිතය I இணைந்த கணிதம் I **Combined Mathematics I**	**10 E I**	පැය තුනයි மூன்று மணித்தியாலம் ***Three hours***

Index Number							

Instructions:

* *This question paper consists of two parts;*
  Part A (Questions 1 - 10) and **Part B** (Questions 11 - 17).
* **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(10) Combined Mathematics I		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
	Percentage	

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

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Part A

1. Using the **Principle of Mathematical Induction**, prove that $\sum_{r=1}^{n} r(r+1) = \frac{n}{3}(n+1)(n+2)$ for all $n \in \mathbb{Z}^+$.

2. Sketch the graphs of $y = |x| + 1$ and $y = 2|x - 1|$ in the same diagram. **Hence or otherwise**, find all real values of x satisfying the inequality $|x| + 1 > 2|x - 1|$.

[see page three

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3. Sketch on the same Argand diagram, the loci of points representing complex numbers z satisfying

(i) $|z - i| = 1$, (ii) $\text{Arg}(z - i) = \frac{\pi}{6}$

and find the complex number represented by the point of intersection of these loci in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $0 < \theta < \frac{\pi}{2}$.

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4. How many different numbers with five digits can be made from the digits 1, 2, 3, 4 and 5, if each digit is used only once?

How many of these numbers

(i) are even numbers?

(ii) have the digits 3 and 4 next to each other?

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5. Let $\alpha > 0$. Find the value of α such that $\lim_{x \to 0} \frac{1 - \cos(\alpha x)}{\sqrt{4 + x^2} - \sqrt{4 - x^2}} = 16$.

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6. Show that the area of the region enclosed by the curves $y = x^2$ and $y = 2x - x^2$ is $\frac{1}{3}$ square units.

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7. A curve C is given by the parametric equations $x = 3\sin^2\frac{\theta}{2}$, $y = \sin^3\theta$ for $0 < \theta < \frac{\pi}{4}$. Show that $\frac{dy}{dx} = \sin 2\theta$.

If the gradient of the tangent at a point P on C is $\frac{\sqrt{3}}{2}$, find the value of the parameter θ corresponding to P.

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8. Let l be the straight line that passes through the origin and the point of intersection of the straight lines $2x + 3y - k = 0$ and $x - y + 1 = 0$, where k ($\neq 0$) is a constant. Find the equation of l in terms of k.

It is given that the two points (1, 1) and (3, 4) are on the same side of l. Show that $k < 18$.

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9. Let $A \equiv (1, 2)$, $B \equiv (-5, 4)$ and S be the circle with AB as a diameter. Find the equations of

(i) the circle S, and

(ii) the circle with centre $(1, 1)$ which intersects S orthogonally.

10. Solve the equation $\cos x + \cos 2x + \cos 3x = \sin x + \sin 2x + \sin 3x$ for $0 \leq x \leq \frac{\pi}{2}$.

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[see page seven

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අධ්‍යයන පොදු සහතික පෙළ (උසස් පෙළ) විභාගය, 2016 අගෝස්තු
கல்விப் பொதுத் தராதர் பத்திர (உயர் தர)ப் பரீட்சை, 2016 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2016

සංයුක්ත ගණිතය I இணைந்த கணிதம் I **Combined Mathematics I**	**10 E I**

PART B

* Answer **five** questions only.

11. (*a*) Let $a, b, c \in \mathbb{R}$ such that $a \neq 0$ and $a + b + c \neq 0$, and let $f(x) = ax^2 + bx + c$.

Show that 1 **is not** a root of the equation $f(x) = 0$.

Let α and β be the roots of $f(x) = 0$.

Show that $(\alpha - 1)(\beta - 1) = \frac{1}{a}(a + b + c)$ and that the quadratic equation with $\frac{1}{\alpha - 1}$ and $\frac{1}{\beta - 1}$ as the roots is given by $g(x) = 0$, where $g(x) = (a + b + c)x^2 + (2a + b)x + a$.

Now, let $a > 0$ and $a + b + c > 0$.

Show that the minimum value m_1 of $f(x)$ is given by $m_1 = -\frac{\Delta}{4a}$, where $\Delta = b^2 - 4ac$.

Let m_2 be the minimum value of $g(x)$. Deduce that $(a + b + c)m_2 = am_1$.

Hence, show that $f(x) \geq 0$ for all $x \in \mathbb{R}$ if and only if $g(x) \geq 0$ for all $x \in \mathbb{R}$.

(*b*) Let $p(x) = x^3 + 2x^2 + 3x - 1$ and $q(x) = x^2 + 3x + 6$. Using the remainder theorem, find the remainder when $p(x)$ is divided by $(x - 1)$ and the remainder when $q(x)$ is divided by $(x - 2)$.

Verify that $p(x) = (x - 1)\,q(x) + 5$, and find the remainder when $p(x)$ is divided by $(x - 1)(x - 2)$.

12. (*a*) Let $n \in \mathbb{Z}^+$. State, in the usual notation, the binomial expansion for $(1 + x)^n$.

Show, in the usual notation, that $\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n - r}{r + 1}$ for $r = 0, 1, 2, \ldots, n - 1$.

The coefficients of x^r, x^{r+1} and x^{r+2} taken in that order, in the binomial expansion of $(1 + x)^n$ are in the ratios 1 : 2 : 3. In this case, show that $n = 14$ and $r = 4$.

(*b*) Let $U_r = \frac{10r + 9}{(2r - 3)(2r - 1)(2r + 1)}$ and $f(r) = r(Ar + B)$ for $r \in \mathbb{Z}^+$, where A and B are real constants.

Find the values of constants A and B such that

$$U_r = \frac{f(r)}{(2r - 3)(2r - 1)} - \frac{f(r + 1)}{(2r - 1)(2r + 1)} \text{ for } r \in \mathbb{Z}^+.$$

Show that $\sum_{r=1}^{n} U_r = -3 - \frac{(n + 1)(2n + 3)}{(4n^2 - 1)}$ for $n \in \mathbb{Z}^+$.

Show further that the infinite series $\sum_{r=1}^{\infty} U_r$ is convergent and find its sum.

[see page eight

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13. (*a*) Let $\mathbf{A} = \begin{pmatrix} -4 & -6 \\ 3 & 5 \end{pmatrix}$, $\mathbf{X} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\mathbf{Y} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

Find real constants λ and μ such that $\mathbf{AX} = \lambda\mathbf{X}$ and $\mathbf{AY} = \mu\mathbf{Y}$.

Let $\mathbf{P} = \begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix}$. Find $\mathbf{P}^{-1}$ and $\mathbf{AP}$, and show that $\mathbf{P}^{-1}\mathbf{AP} = \mathbf{D}$, where $\mathbf{D} = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$.

(*b*) In an Argand diagram, the point A represents the complex number $2+i$. The point B is such that $OB = 2(OA)$ and $A\hat{O}B = \frac{\pi}{4}$, where O is the origin and $A\hat{O}B$ is measured counter-clockwise from OA. Find the complex number represented by the point B.

Also, find the complex number represented by the point C such that $OACB$ is a parallelogram.

(*c*) Let $z \in \mathbb{C}$ and $w = \frac{2}{1+i} + \frac{5z}{2+i}$. It is given that $\operatorname{Im} w = -1$ and $|w - 1 + i| = 5$.
Show that $z = \pm(2 + i)$.

14. (*a*) Let $f(x) = \frac{(x-3)^2}{x^2-1}$ for $x \neq \pm 1$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x) = \frac{2(x-3)(3x-1)}{(x^2-1)^2}$.

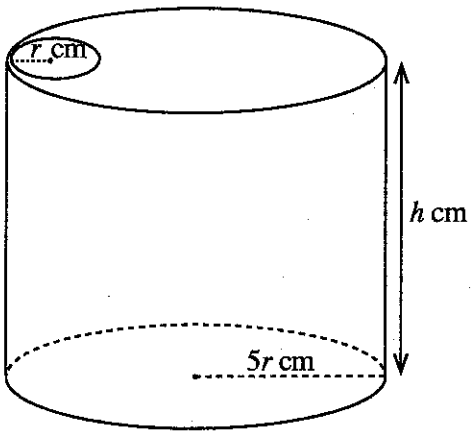
Write down the equations of the asymptotes of $y = f(x)$.

Find the coordinates of the point at which the horizontal asymptote intersects the curve $y = f(x)$.

Sketch the graph of $y = f(x)$ indicating the asymptotes and the turning points.

(*b*) A thin metal container, in the shape of a right circular cylinder of radius $5r$ cm and height h cm has a circular lid of radius $5r$ cm with a circular hole of radius r cm. (See the figure.) The volume of the container is given to be 245π cm^3. Show that the surface area S cm^2 of the container with the lid containing the hole is given by $S = 49\pi\left(r^2 + \frac{2}{r}\right)$ for $r > 0$.

Find the value of r such that S is minimum.



15. (*a*) (i) Find $\int \frac{dx}{\sqrt{3+2x-x^2}}$.

(ii) Find $\frac{d}{dx}\left(\sqrt{3+2x-x^2}\right)$ and **hence**, find $\int \frac{x-1}{\sqrt{3+2x-x^2}}\,dx$.

Using the above integrals, find $\int \frac{x+1}{\sqrt{3+2x-x^2}}\,dx$.

(*b*) Express $\frac{2x-1}{(x+1)(x^2+1)}$ in partial fractions and **hence**, find $\int \frac{(2x-1)}{(x+1)(x^2+1)}\,dx$.

(*c*) (i) Let $n \neq -1$. Using integration by parts, find $\int x^n (\ln x)\,dx$.

(ii) Evaluate $\int_1^3 \frac{\ln x}{x}\,dx$.

[see page nine

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16. (*a*) The equation of the diagonal AC of a rhombus $ABCD$ is $3x - y = 3$ and $B \equiv (3, 1)$. Also, the equation of CD is $x + ky = 4$, where k is a real constant. Find the value of k and the equation of BC.

(*b*) Sketch the circles, C_1 and C_2 given by the equations $x^2 + y^2 = 4$ and $(x-1)^2 + y^2 = 1$ respectively, indicating clearly their point of contact.

A circle C_3 touches C_1 internally and C_2 externally. Show that the centre of C_3 lies on the curve $8x^2 + 9y^2 - 8x - 16 = 0$.

17. (*a*) Write down the trigonometric identity for $\tan(\alpha + \beta)$ in terms of $\tan\alpha$ and $\tan\beta$.

Hence, obtain $\tan 2\theta$ in terms of $\tan\theta$, and show that $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$.

By substituting $\theta = \dfrac{5\pi}{12}$ in the last equation, verify that $\tan\dfrac{5\pi}{12}$ is a solution of $x^3 - 3x^2 - 3x + 1 = 0$.

Given further that $x^3 - 3x^2 - 3x + 1 = (x + 1)(x^2 - 4x + 1)$, deduce that $\tan\dfrac{5\pi}{12} = 2 + \sqrt{3}$.

(*b*) Show that $\tan^2\dfrac{A}{2} = \dfrac{1 - \cos A}{1 + \cos A}$ for $0 < A < \pi$.

In the usual notation, using the Cosine Rule for a triangle ABC, show that

$(a + b + c)(b + c - a)\tan^2\dfrac{A}{2} = (a + b - c)(a + c - b)$.

(*c*) Show that $\sin^{-1}\left(\dfrac{3}{5}\right) + \sin^{-1}\left(\dfrac{5}{13}\right) = \sin^{-1}\left(\dfrac{56}{65}\right)$.

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