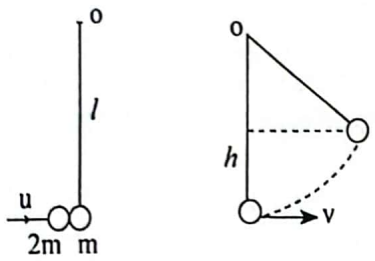
G.C.E. (Advanced Level) Examination - August 2016
Combined Mathematics - II
Distribution of Marks

Part A

(01)



Let V be the velocity with which the composite Particle begins to move.

Apply $\underline{I} = \Delta(M\underline{V})$ for the system

$\rightarrow O = 3mv - 2mu$ (05)

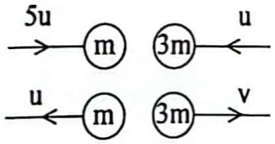
$\Rightarrow V = \frac{2u}{3}$ (05)

By the Conservation of Energy,

$(3mg)h = \frac{1}{2}(3m)V^2$, where h is the required height. (10)

$\therefore h = \frac{V^2}{2g} = \frac{4u^2}{9(2g)} = \frac{4gl}{18g} = \frac{2l}{9}$ (05) [25]

(02)



Apply $\underline{I} = \Delta(M\underline{V})$ for the system :

$\rightarrow O = (3mv - mu) - (5mu - 3mu)$ (05)

$\Rightarrow 3mv = 3mu$

$\Rightarrow v = u$ —— ① (05)

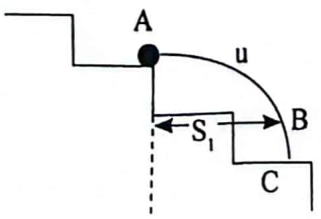
By newton's law of restitution :

$v + u = e(5u + u)$ (10)

① $\Rightarrow 2u = 6eu$

$\therefore e = \frac{1}{3}$ (05) [25]

(03)



For the motion of P, apply $S = ut + \frac{1}{2}at^2$

↓ from A to B : $a = \frac{1}{2}g t_1^2$, where t_1 is the time taken to reach the level of the 1st step below. (05)

$\therefore t_1 = \sqrt{\frac{2a}{g}}$

Let S_1 be the horizontal distance moved in time t_1

→ From A to B : $S_1 = u \times t_1 = \frac{3}{2}\sqrt{ga} \times \sqrt{\frac{2a}{g}}$ (05)

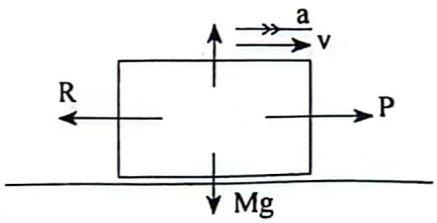
$= \frac{3}{\sqrt{2}}a > 2a$ (05)

Thus p will not hit the 1st step below A.

Time taken from A to C is $t_2 = \sqrt{\frac{2(2a)}{g}}$ (05)

→ $S = ut_2 = \frac{3}{2}\sqrt{ga} \cdot 2\sqrt{\frac{a}{g}} = 3a$ (05) [25]

(04)



Let PN be the tractive force

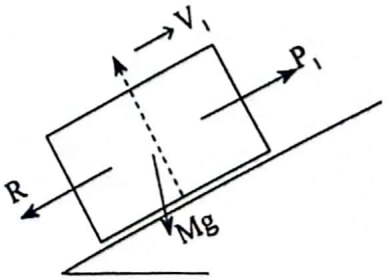
Apply $\underline{F} = m\underline{a}$ →:

$P - R = Ma$ —— ① (05)

Let HW be the power of its engine

Then $H = P \times V$ (05)

$= (R + Ma)V$ (by ①) (05)



↗ $\underline{F} = m\underline{a}$

$P_1 - R - Mg\sin\alpha = 0$ —— ② (05)

Also $H = P_1 V_1$

$\therefore V_1 = \frac{H}{P_1} = \frac{(R + Ma)V}{(R + Mg\sin\alpha)}$ (by ②) (05) [25]

(05) (i) Magnitudes of vectors

$|\underset{\sim}{a}| = \sqrt{3^2 + 4^2} = 5$

$|\underset{\sim}{b}| = \sqrt{4^2 + 3^2} = 5$ } (05)

(ii) $\underset{\sim}{a} \cdot \underset{\sim}{c} = 3\alpha + 4(1 - \alpha) = 4 - \alpha$ (05)

$\underset{\sim}{b} \cdot \underset{\sim}{c} = 4\alpha + 3(1 - \alpha) = 3 + \alpha$ (05)

Let θ be the angle between $\underset{\sim}{a}$ and $\underset{\sim}{c}$.

Then $\underset{\sim}{a} \cdot \underset{\sim}{c} = |\underset{\sim}{a}||\underset{\sim}{c}| \cos\theta$ and

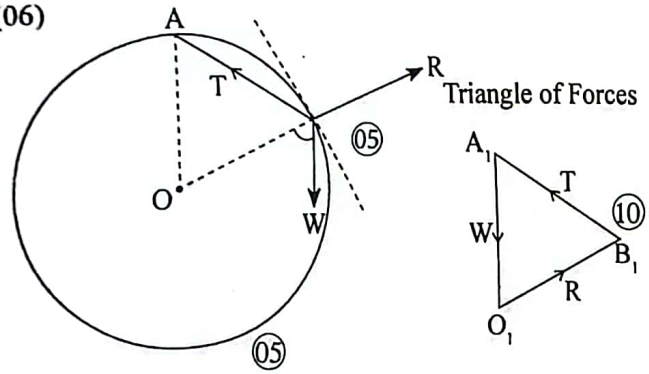
$\underset{\sim}{b} \cdot \underset{\sim}{c} = |\underset{\sim}{b}||\underset{\sim}{c}| \cos\theta$ (05)

since $|\underset{\sim}{a}| = |\underset{\sim}{b}|$, we have $\underset{\sim}{a} \cdot \underset{\sim}{c} = \underset{\sim}{b} \cdot \underset{\sim}{c}$

$\therefore 4 - \alpha = 3 + \alpha$

$\alpha = 1/2$ // (05) 25

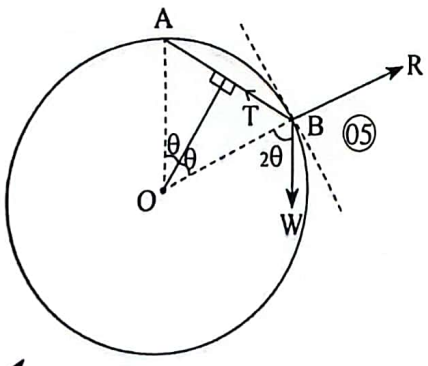
(06)

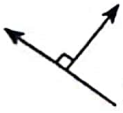


Triangle of Forces

$\frac{T}{AB} = \frac{W}{OA} \Rightarrow T = \frac{2Wl}{a}$ (05) (05)

Aliter I





Resolve in a direction perpendicular to OB

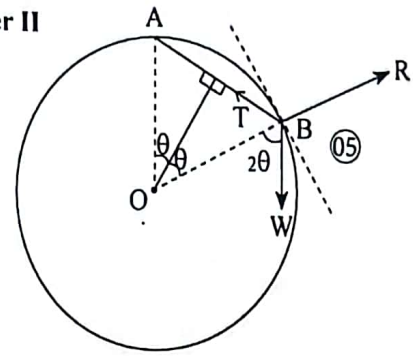
$T \cos\theta = W \sin 2\theta$ (10)

$T = W \frac{\sin 2\theta}{\cos\theta}$ (05)

$= 2w \sin\theta$

$= \frac{2wl}{a} (\because \sin\theta = 1/a)$ (05) 25

Aliter II



By Lami's Theorem,

$\frac{T}{\sin(\pi - 2\theta)} = \frac{W}{\sin(\pi/2 + \theta)}$ (10)

$T = W \frac{\sin 2\theta}{\cos\theta}$

$= \frac{2W \sin\theta \cancel{\cos\theta}}{\cancel{\cos\theta}}$ (05)

$= \frac{2Wl}{a} \left(\because \sin\theta = \frac{l}{a}\right)$ (05) 25

(07) For two events A and B,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ —— (1) (05)

This gives us $P(A \cup B) = \frac{3P}{2} - P(A \cap B)$

It is given that $P(A \cup B) - P(A \cap B) = \frac{2P}{3}$ —— (2) (05)

(1) and (2) $\Rightarrow \frac{3P}{2} - 2P(A \cap B) = \frac{2P}{3}$

$\Rightarrow P(A \cap B) = \frac{5P}{12}$ (05)

If A and B are independent, then $P(A \cap B) = P(A) P(B)$ (05)

$\Rightarrow \frac{5P}{12} = P \cdot \frac{P}{2}$

$\Rightarrow P = \frac{5}{6} (\because P > 0)$ (05) 25

(08) Probability that the first ball is white $= \frac{6}{n+6}$

Probabilty that the first ball is white and the second ball is black $= \frac{6}{n+6} \times \frac{n}{n+5}$ (10)

$\therefore \frac{6}{n+6} \cdot \frac{n}{n+5} = \frac{4}{15}$ (05)

$\Rightarrow 2n^2 - 23n + 60 = 0$ (05)

$(n - 4)(2n - 15) = 0$

$\Rightarrow n = 4$ ($\because$ n is a positive in teger) (05) 25

(09) Let x, y and z be distinct integers Less than 11 with a mean of 7.

Then $\frac{x+y+z}{3} = 7$ (05)

$\Rightarrow x + y + z = 21$ —— (1)

Since x y and z are distinct and the only mode is 3, at least one of the two in tegers additionally taken must be 3. Let the other be t.

Since the mean of the five integers is 5, We have

$\frac{x+y+z+t+3}{5} = 5$ (05)

$\Rightarrow 21 + 3 + t = 25$ (05)

$\Rightarrow t = 1$

Hence the integers are x, y, z, 3, 1 since the only mode 3, and x, y and z are distinct, exactly one of them must be 3 Let $z = 3$

Again (1) $\Rightarrow x + y = 18$ —— (2) (05)

Since x and y are integers less than 11, (2) given us ($x = 8$ and $y = 10$) or ($x = 10$ and $y = 8$) Hence, the five numbers are 1, 3, 3, 8 and 10. (05)

25

(10) Mean $\mu = 3 \Rightarrow \frac{1 + 2p + 3q + 20 + 10}{p + q + 8} = 3$ (05)

$\Rightarrow 2p + 3q + 31 = 3p + 3q + 24$

$\Rightarrow p = 7$ (05)

$\sigma^2 = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - \mu^2$ (05)

Variance

$= \frac{6}{5} \Rightarrow \frac{6}{5} = \frac{1.1^2 + 7.2^2 + q.3^2 + 5.4^2 + 2.5^2}{q + 15} - 3^2$ (05)

$\Rightarrow 51 (q + 15) = 5 (1 + 28 + 9q + 80 + 50)$

$\Rightarrow q = 5$ (05)

25

Aliter :

$\sigma^2 = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{\sum_{i=1}^{n} f_i}$ (05)

Variance $= \frac{6}{5} \Rightarrow \frac{6}{5}$

$= \frac{1(1-3)^2 + 7(2-3)^2 + q(3-3)^2 + 5(4-3)^2 + 2(5-3)^2}{1 + 7 + q + 5 + 2}$ (05)

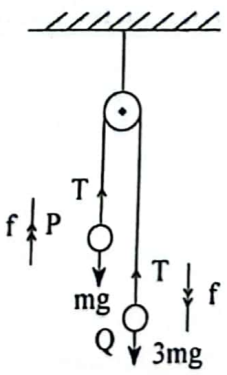
$\Rightarrow \frac{6}{5} = \frac{4 + 7 + 5 + 8}{15 + q}$

$\Rightarrow q = 5$ (05)

15

Part B

(11) (a)



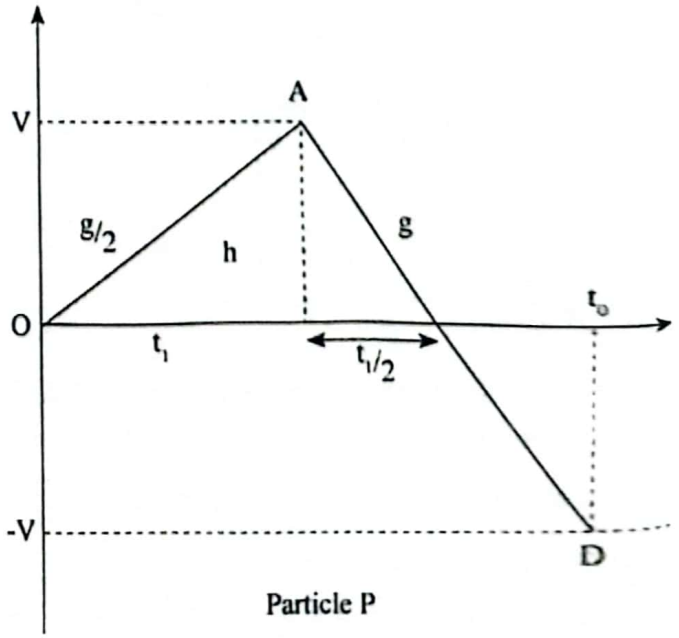
Apply $\underline{F} = m\underline{a}$;

For Q (3m) ↓ $3mg - T = 3mf$ (05)

For P (m) ↑ $\frac{T - mg = mf}{2mg = 4mf}$ (05)

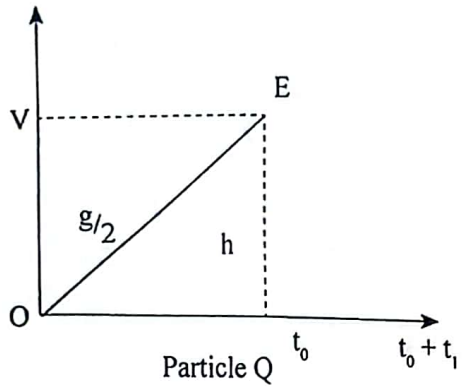
$\Rightarrow f = g/2$ (05)

15



Particle P

(15)



(10)

25

From the v - t graphs

Area under OA or OE $= \frac{1}{2} t_0 v = h$ —— (1) (05)

Gradient of OA or OE $= \frac{V}{t_0} = \frac{g}{2}$ —— (2) (05)

(01) × (02) $\Rightarrow \frac{1}{2} t_0 \frac{g t_0}{2} = h$

$\Rightarrow t_0 = \frac{4h}{g}$

$\Rightarrow t_0 = 2\sqrt{\frac{h}{g}}$ (05)

15

Also (2) $\Rightarrow V = \frac{g}{2} \cdot 2\sqrt{\frac{h}{g}} = \sqrt{gh}$ (05)

For its motion under gravity alone, time taken by P $= \frac{2V}{g}$

$\therefore t_1 = 2\sqrt{\frac{h}{g}}$ (05)

10

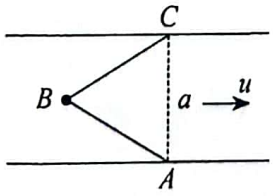
Maximum height reached by P $= \frac{1}{2} V \frac{t}{2} = \frac{h}{2}$ (05)

Total height above the level of floor

$= h + h + \frac{h}{2} = \frac{5h}{2}$ (05)

10

(b)



V/B, W = V

V(W, E) = $\vec{U}$

V/(B, E) = $\pi/6$ for AB and $\pi/6$ for BC (05)

V/(B, E) = V/(B, W) + V/(W, E)

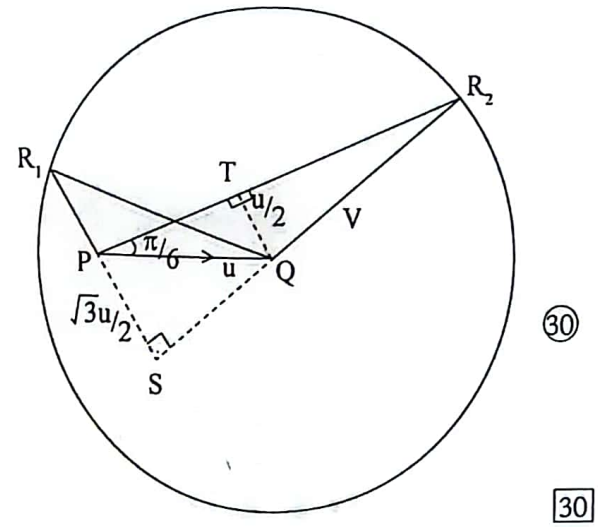
= V(W, Σ) + V(B, W) (05)

= $\vec{PQ} + \vec{QR_i}$ (05)

= $\vec{PR_i}$ for i = 1, 2 where AB//PR and BC//PR$_2$ respectively. (05)

20

(05)



(30)

30

For AB : Δ PQR$_1$

$R_1 S = \sqrt{V^2 - \frac{u^2}{4}}$

$PR_1 = R_1 S - PS$ (05)

$= \frac{1}{2}(\sqrt{4V^2 - u^2} - \sqrt{3}u)$ (05)

For BC : Δ PQR$_2$

$PR_2 = PT + TR_2$

$= \frac{\sqrt{3}u}{2} + \sqrt{V^2 - \frac{u^2}{4}}$ (05)

$= \frac{1}{2}(\sqrt{4V^2 - u^2} + \sqrt{3}u)$ (05)

$PR_1 \, PR_2 = \frac{1}{4}(4V^2 - u^2 - 3u^2)$

Total Time $= \frac{a}{PR_1} + \frac{a}{PR_2}$

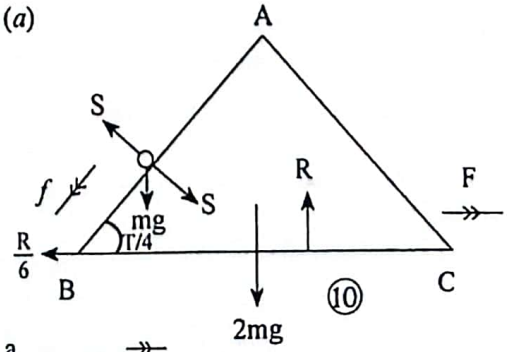
$= \frac{a(PR_1 + PR_2)}{PR_1 . PR_2}$

$= \frac{a\sqrt{4V^2 - u^2}}{(V^2 - u^2)}$ (05)

$PR_1 + PR_2 = \sqrt{4V^2 - u^2}$
$PR_1 . PR_2 = V^2 - u^2$

25

(12) (a)

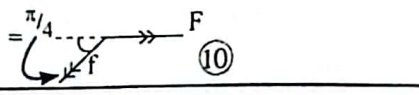


(10)

$\underset{\sim}{a}(2m, F) = \vec{F}$

$\underset{\sim}{a}(m, 2m) = \swarrow f$

$\underset{\sim}{a}(m, E) = \underset{\sim}{a}(m, 2m) + \underset{\sim}{a}(2m, E)$



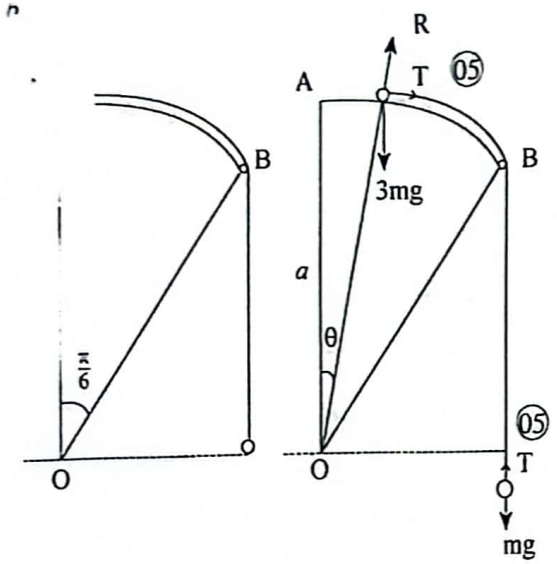
(10)

Apply $\underset{\sim}{F} = m\underset{\sim}{a}$

(i) For the particle $\swarrow$ $mg\frac{\sqrt{2}}{2} = m\left(f - F\left(\frac{\sqrt{2}}{2}\right)\right)$ (15)

(ii) For the system $\rightarrow$ $\frac{-R}{6} = 2mF \div m\left(F - \frac{1}{\sqrt{2}}\right)$ (15)

(For the system $\uparrow$ $R - 3mg = \frac{-mf}{\sqrt{2}}$ (10) [60]



(05) (05)

By conservation of Mechanical Energy

$3mga = 3mga\cos\theta - mga\theta + \frac{1}{2}(3m)(a\dot{\theta})^2 + \frac{1}{2}(m)(a\dot{\theta})^2$ (05) (10) (10)

$2a\dot{\theta}^2 = 3g(1 - \cos\theta) + g\theta$ (05) [40]

Applying $\underset{\sim}{F} = m\underset{\sim}{a}$

For P $\searrow$ $T + 3mg\sin\theta = 3mf$ —— ① (15)

For Q $\downarrow$ $mg - T = mf$ —— ② (10)

By ① and ②

$3mg - 3T = T + 3mg\sin\theta$

$4T = 3mg(1 - \sin\theta)$

$T = \frac{3mg}{4}(1 - \sin\theta)$ (05) [30]

Appling $\underset{\sim}{F} = m\underset{\sim}{a}$, for P

$\swarrow$ $3mg\cos\theta - R = 3ma\dot{\theta}^2$ (10)

$R = 3mg\cos\theta - \frac{3m}{2}[3g(1 - \cos\theta) + g\theta]$ (10)

$= \frac{3mg}{2}(2\cos\theta - 3 + 3\cos\theta - \theta)$

$= \frac{3mg}{2}(5\cos\theta - \theta - 3)$ [20]

Note : P will not leave the surface for $0 < \theta < \pi/6$

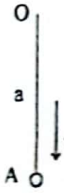
$R_{/\theta = 0} = 3mg > 0$

$\frac{dR}{d\theta} = \frac{3mg}{2}(-5\sin\theta - 1) < 0$ for $0 < \theta < \pi/6$

$R_{/\theta = \pi/6} = \frac{3mg}{2}\left(\frac{5\sqrt{3}}{2} - \pi/6 - 3\right) > 0$

(13) For motion of P under gravity alone

from O to A :



$\downarrow V^2 = 2ga \Rightarrow V = \sqrt{2ag}$ (05)

Tension in the string : $T = \frac{4mg(x - a)}{a}$ (05)

$\downarrow \underset{\sim}{F} = m\underset{\sim}{a}$: $-T + mg = m\ddot{x}$ (05)

Eliminating T : $-4mg\frac{(x - a)}{a} + mg = m\ddot{x}$ (05)



$\Rightarrow \ddot{x} + \frac{4g}{a}(x - a) = \frac{4g}{a}\cdot\frac{a}{4}$

$\Rightarrow \ddot{x} + \frac{4g}{a}\left(x - \frac{5a}{4}\right) = 0$ —— ① (05)

Write $x = x - \frac{5a}{4} \Rightarrow \dot{x} = \dot{x}$ and $\ddot{x} = \ddot{x}$ (05)

Then ① be comes $\ddot{x} + \frac{4g}{a}x = 0$

Hence $\ddot{x} + \omega^2 x = 0$; (05)

Where $\omega = 2\sqrt{\frac{g}{a}}$ $(\because \omega > 0)$ (05)

$\Rightarrow \dot{x}^2 = \omega^2(c^2 - x^2)$ —— ②

$\dot{x} = \sqrt{2ga}$ when $x = a \Rightarrow \dot{x}^2 = 2ga$, when $x = \frac{-a}{4}$ (05) (05)

Then ② $\Rightarrow 2ga = \frac{4g}{a}\left[c^2 - \frac{(-a)^2}{4}\right]$ (05)

$\Rightarrow a^2 = 2c^2 - \frac{a^2}{8} \Rightarrow c^2 = \frac{9a^2}{16}$

$\Rightarrow c = \frac{3a}{4}$ $(\because c > 0)$ (05)

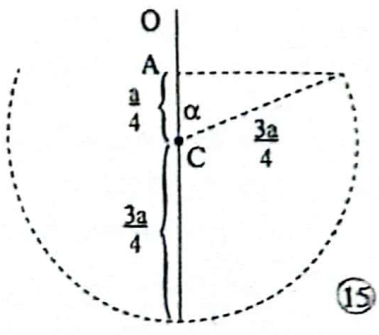
Centre is given by $x = 0$; *i. ex* $= \frac{5a}{4}$ (05)

$AL = \frac{a}{4} + \frac{3a}{4} = a$ (05)

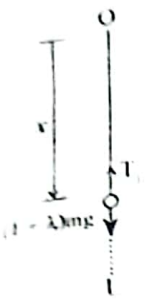
$\cos\alpha = \frac{1}{3}$ (05)

Time taken from A to L $= \frac{\pi - \alpha}{\omega}$ (05)

$= \frac{1}{2}\sqrt{\frac{a}{g}}\left\{\pi - \cos^{-1}\left(\frac{1}{3}\right)\right\}$ (05)



(15)



$T_1 = \frac{4mg(x-a)}{a}$

∤ For composite particle $\underset{\sim}{F} = m\underset{\sim}{a}$:

$(1+\lambda)mg - T_1 = (1+\lambda)m\ddot{x}$ (10)

$(1+\lambda)mg - \frac{4mg}{a}(x-a) = (1+\lambda)m\ddot{x}$ (05)

$\ddot{x} + \frac{4g}{(1+\lambda)a}(x-a) - g = 0$ (05)

$\ddot{x} + \frac{4g}{(1+\lambda)a}\left\{x - a - (1+\lambda)\frac{a}{4}\right\} = 0$

$\ddot{x} + \frac{4g}{(1+\lambda)a}\left\{x - (5+\lambda)\frac{a}{4}\right\} = 0$ (05)

60

Centre C_1 : $x = OC_1 = (5+\lambda)\frac{a}{4}$

$C_1L = 2a - (5+\lambda)\frac{a}{4}$ (05)

$= (3-\lambda)\frac{a}{4}$ (05)



New amplitude $C_1 = (3-\lambda)\frac{a}{4} (> 0) \therefore \lambda < 3$

Complete Simple Harmonic Motion if and only if

$AC_1 \geq C_1$ (10)

$(5+\lambda)\frac{a}{4} - a \geq (3-\lambda)\frac{a}{4}$

$5 + \lambda - 4 \geq 3 - \lambda$

$\lambda \geq 1$ (05)

25

Aliter

Let $x = A\cos\omega t + B\sin\omega t$, Where A and B are constants to be determined.

$\Rightarrow \dot{x} = -A\omega\sin\omega t + B\omega\cos\omega t$ (05)

When $t = 0$ and $x = a$, $x = \frac{-a}{4}$ and $\dot{x} = V = \sqrt{2ga}$ (05)

(05)

$\therefore \frac{-a}{4}$ (05) $= A$ and $V = B\omega \Rightarrow B = \frac{V}{\omega}$ (05)

Solution : $x = \frac{-a}{4}\cos\omega t + \frac{V}{\omega}\sin\omega t$

Differentiating : $\dot{x} = \frac{a\omega}{4}\sin\omega t + V\cos\omega t$ (05)

Lowest point L is reached when $\dot{x} = 0$ (05)

For the first time at $t = t_1$ say

Then $\tan\omega t_1 = \frac{-4V}{a\omega}$ (05)

$\omega t_1 = \pi - \alpha$

$\tan\alpha = \frac{4V}{a\omega}$ (05)

where $0 < \alpha < \pi/2$

Centre C of SHS is such that $x = 5a/4$ or $AC = a/4$ (05)

$a/4 = C\cos\alpha = \frac{c(a\omega)}{\sqrt{16V^2 + a^2\omega^2}}$

$= C\frac{2\sqrt{ga}}{\sqrt{16 \times 2ga + 4ga}} = \frac{1}{3}C$ (05)

$\Rightarrow C = 3a/4$

Also from above

$\omega t_1 = \pi - \cos^{-1}(1/3)$ (05)

$t_1 = \frac{1}{\omega}[\pi - \cos^{-1}(1/3)]$

60

(14) (*a*) Let $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$

Then $\overrightarrow{OC} = \frac{1}{3}\overrightarrow{OB} = \frac{\underset{\sim}{b}}{3}$ and $\overrightarrow{OD} = \frac{1}{2}\overrightarrow{AB} = \frac{\underset{\sim}{b} - \underset{\sim}{a}}{2}$ (05)

(05)

$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$ $\quad\quad$ $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$= -\underset{\sim}{a} + \frac{\underset{\sim}{b}}{2} - \frac{\underset{\sim}{a}}{2}$ $\quad\quad$ $= -\underset{\sim}{a} + \frac{\underset{\sim}{b}}{3}$ —— ② (05)

$= \frac{3}{2}\left(-\underset{\sim}{a} + \frac{\underset{\sim}{b}}{3}\right)$ —— ① (05)

By ① and ② $\overrightarrow{AD} = \frac{3}{2}\overrightarrow{AC}$ (05)

25

$\overrightarrow{PC} = \overrightarrow{PO} + \overrightarrow{OC}$

$= \overrightarrow{PA} + \overrightarrow{AO} + \overrightarrow{OC}$

$= -\lambda\overrightarrow{AB} - \overrightarrow{OA} + \overrightarrow{OC}$

$= -\lambda(\underset{\sim}{b} - \underset{\sim}{a}) - \underset{\sim}{a} + \underset{\sim}{c}$ (05)

$= (\lambda - 1)\underset{\sim}{a} - \lambda\underset{\sim}{b} + \frac{\underset{\sim}{b}}{3}$ (05)

$= (\lambda - 1)\underset{\sim}{a} + \frac{1}{3}(1 - 3\lambda)\underset{\sim}{b}$ —— ③ (05)

$\overrightarrow{CQ} = \overrightarrow{CO} + \overrightarrow{OQ}$

$= -\overrightarrow{OC} + (1-\lambda)\overrightarrow{OD}$ (05)

$= \frac{-\underset{\sim}{b}}{3} + (1-\lambda)\frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a})$ (05)

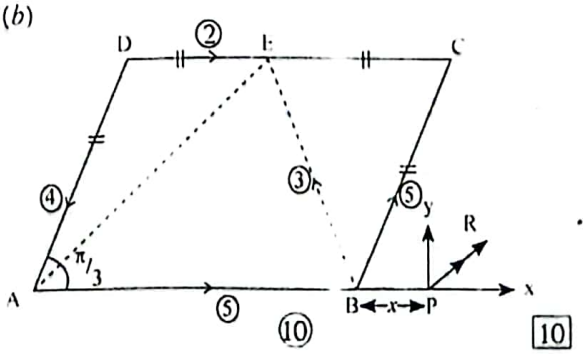
$= \frac{1}{2}\left[(\lambda - 1)\underset{\sim}{a} - \frac{3}{2}\underset{\sim}{b} + \underset{\sim}{b} - \lambda\underset{\sim}{b}\right]$

$= \frac{1}{2}\left[(\lambda - 1)\underset{\sim}{a} + \frac{1}{3}(1 - 3\lambda)\underset{\sim}{b}\right]$ ④ (05)

35

By ③ and ④ $\overrightarrow{PC} = 2\overrightarrow{CQ}$

(*b*)



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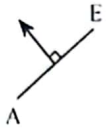
10

Resolving parallel to $\overrightarrow{AE}$:

$= -4 \cos \pi/6 + 2 \cos \pi/6 + 5 \cos \pi/6 + 5 \cos \pi/6$

$= 4\sqrt{3}$ N (05) (10)

Resolving



: $3 - 4 \sin \pi/6 + 5 \sin \pi/6 - 5 \sin \pi/6 - 2 \cos \pi/6$ (10)

$= 3 - 2 + 5/2 - 5/2 - 1$

$= 0$ (05)

Resultant R is of magnitude $4\sqrt{3}$N $\uparrow\uparrow$ to $\overrightarrow{AE}$ (05) (05)

40

OR $\longrightarrow x = 5 + 5/2 + 2 - 3/2 - 4/2 = 6$ N (10)

$\uparrow Y = \frac{\sqrt{3}}{2}(5 + 3) - 4\frac{\sqrt{3}}{2} = 2\sqrt{3}$ N (10)

$\frac{Y}{X} = \frac{2\sqrt{3}}{6} = \frac{1}{\sqrt{3}}$ (05)

(05)

Resultant R is of magnitude $2\sqrt{3}\ \sqrt{3+1}$

$= 4\sqrt{3}$ N

Its line of action makes an angle,

$\tan^{-1}(1/\sqrt{3}) = \pi/6$ with side AB, $\therefore$ it is parallel to AE (05) (05)

40

Let P be the point where resultant meets AB produced. Taking moments about

B↺

$yx = 4 \times 2 \sin \pi/3 - 2 \times 1 \sin \pi/3$ (10)

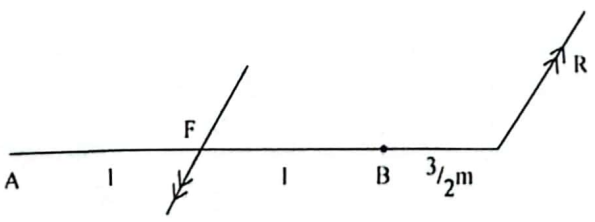
$2\sqrt{3x} = 3\sqrt{3}$

$x = 3/2$ m (05)

15

Line of action of R meets AB produced at $x = 3/2$ m from B

Additional force must be parrallel to $\overrightarrow{EA}$ (05)



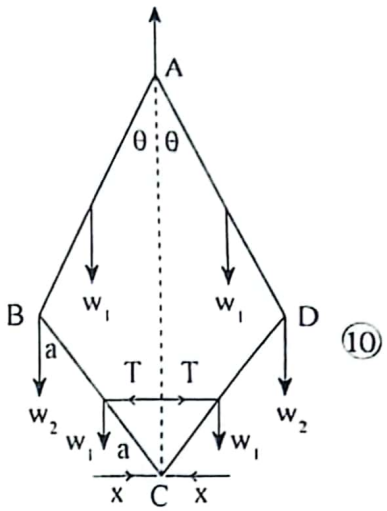
A↺ $R \times (2 + 3/2) \sin 30 = F.\ 1.\ \sin 30$ (15)

$4\sqrt{3} \times 7/2 = F$

$F = 14\sqrt{3}$ N (05)

25

(15) (*a*)



(10)

Symmetry (05)

Let 2a be the length of each heavy rod.

B↺ for BC : $X \cdot 2a \cos\theta - w_1\, a \sin\theta - T\, a \cos\theta = 0$ (10)

$\Rightarrow 2x - T = w_1 \tan\theta$ —— ① (05)

A↺ for AB and BC :

$X \cdot 4a \cos\theta + 2w_1\, a \sin\theta + w_2\, 2a \sin\theta - T \times 3a \cos\theta = 0$ (20)

$\Rightarrow 4x - 3T = -2(w_1 + w_2) \tan\theta$ —— ② (05)

① × 2 - ②

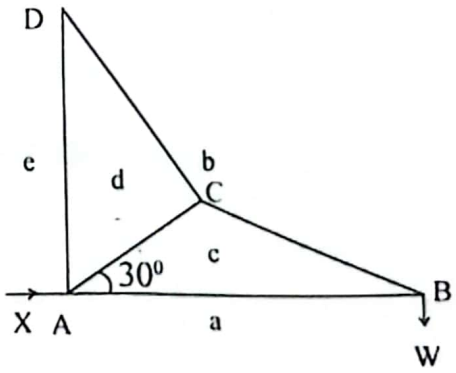
$\Rightarrow T = 2w_1 \tan\theta + (2w_1 + w_2) \tan\theta$

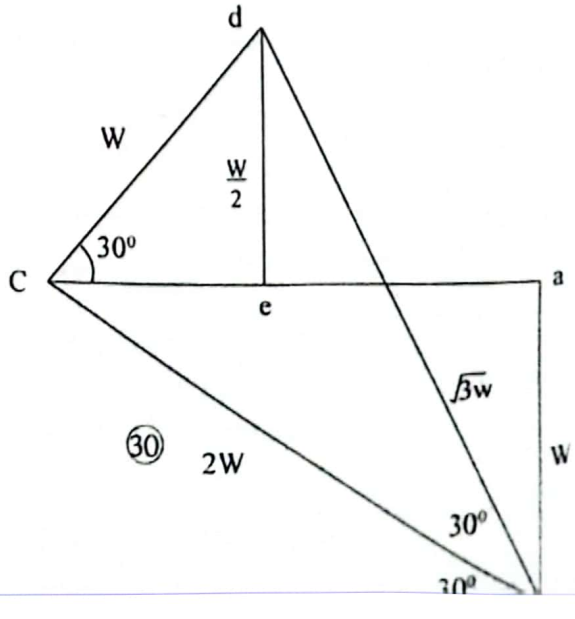
$= (4w_1 + 2w_2) \tan\theta$

$= 2(2w_1 + w_2) \tan\theta$ (10)

65

(*b*)

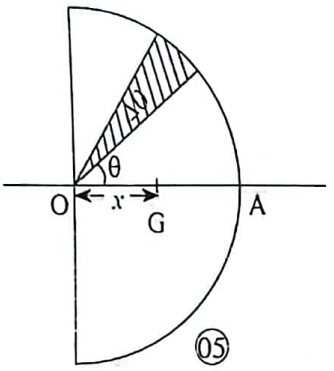




Rod	Magnitude	Tension / Trust
BC (bc)	2 W (05)	Tension (05)
AB (ca)	$\sqrt{3}$ W (05)	Thrust (05)
CD (bd)	$\sqrt{3}$ W (05)	Tension (05)
AC (dc)	W (05)	Tension (05)
AD (de)	$W/2$ (05)	Thrust (05)
X (ea) = $\frac{\sqrt{3}\ W}{2}$ (05)		

85

(6)



05

By symmtry C. M. lies on OA

Let σ be the mass per unit area.

Then $\Delta m = \frac{1}{2} r^2 (\Delta\theta)\sigma$

$$\bar{x} = \frac{\int_{-\pi/2}^{\pi/2} \frac{1}{2} r^2 \sigma \cdot \frac{2}{3} r \cos\theta \, d\theta}{\pi/2 \ r^2 \sigma}$$

$$= \frac{2r}{3\pi} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta$$

$$= \frac{2r}{3\pi} \sin\theta \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{2r}{3\pi} \left[2 \sin \pi/2\right]$$

$$= \frac{4r}{3\pi}$$

Hence the CM is at a distance $\frac{4r}{3\pi}$ from O.

Object	Mass	Distance from RS to the CM →
D C A B	$2a^2\sigma$	3a (10)
R Q S P	$4a^2\sigma$	a (10)
$\pi/2$	$\frac{1}{2} \cdot \pi \cdot \frac{a^2}{4}\sigma$	$\frac{4}{3\pi} \cdot \frac{a}{2} = \frac{2a}{3\pi}$ (10)
	$(6 - \pi/8)\, a^2\sigma$	$\bar{x}_1$ (05)

By definition of centre of mass

$$\frac{a^2\sigma(48-\pi)}{8}\bar{x}_1 = 2a^2\sigma \times 3a + 4a^2\sigma \ a - \frac{\pi a^2}{8} \ \sigma \frac{2a}{3\pi}$$ (10)

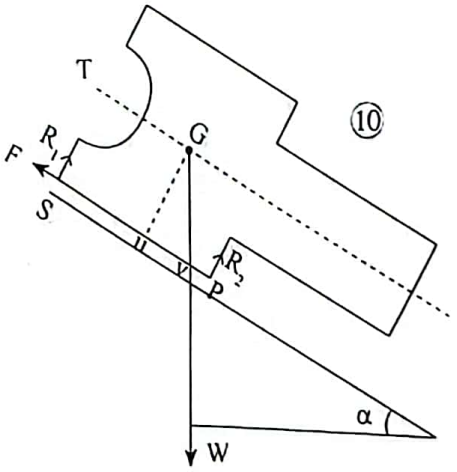
$$\frac{(48-\pi)}{8}\bar{x}_1 = \left(10 - \frac{1}{2}\right)a$$

$$\frac{(48-\pi)}{8}\bar{x}_1 = \frac{119}{12}a$$

$$\therefore \ \bar{x}_1 = \frac{238}{3(48-\pi)}a$$ (05)

$= ka$

50



10

Since, PS is in contact with the plane, we must have UV < UP (10)

i.e. $a \tan\alpha < 2a - ka$

$\Rightarrow \tan\alpha < (2 - k)$ (10) (Note that $k < 2$)

$R_1 + R_2 = W \cos\alpha$ (10)

$F = w \sin\alpha$ (05)

Since L does not slide, we must have $\mu \geq \frac{F}{R_1 + R_2}$ (10)

This gives us $\mu \geq \tan\alpha$ (05)

60

(17) (*a*) In a single toss of the die A, the probability P(n) of obtaining a face with number n is given below.

n	1	2	3	4	5
P(n)	$^1/_6$	$^1/_6$	$^1/_3$	$^1/_6$	$^1/_6$

Let X_i be the number obtained in the i^{th} toss for $i = 1, 2$

Then $P(x_1 + x_2 = 6) = P(x_1 = 1 \text{ and } x_2 = 5) + P(x_1 = 5 \text{ and } x_2 = 1) + P(x_1 = 2 \text{ and } x_2 = 4) + P(x_1 = 4 \text{ and } x_2 = 2) + P(x_1 = 3 \text{ and } x_2 = 3)$

$= 4 \times {}^1/_6 \times {}^1/_6 + {}^1/_3 \times {}^1/_3$ (15)

$= \frac{2}{9}$ (05) [20]

For the die B, replace X_1 by Y_1

n	2	3	4	5
P(n)	$^1/_3$	$^1/_6$	$^1/_{36}$	$^1/_6$

Then $P(y_1 + y_2 = 6) = 2 \times {}^1/_3 \times {}^1/_3 + {}^1/_6 \times {}^1/_6$

$= \frac{1}{4}$ (05) (15) [20]

By Bayes' Theorem

$P(A \mid \text{sum} = 6) =$

$$\frac{P(\text{sum} = 6/A)\,P(A)}{P(\text{sum} = 6/A)\,P(A) + P(\text{sum} = 6/B)\,P(B)}$$ (10)

$$= \frac{2/9 \times 1/2}{2/9 \times 1/2 + 1/4 \times 1/2} = \frac{8}{17}$$ (05) (10) (05) [30]

(*b*) $\mu_1 = \frac{\sum_{i=1}^{n} x_i}{n}$ $\mu_2 = \frac{\sum_{j=1}^{m} y_j}{m}$ (05)

now $\mu_3 = \frac{\sum_{i=1}^{n} x_i + \sum_{j=1}^{m} y_j}{n + m}$ (05)

$= \frac{n\mu_1 + m\mu_2}{m + n}$ (05) [15]

$\sum_{i=1}^{n} (x_i - \mu_3)^2 = \sum_{i=1}^{n} (x_i - \mu_1 + \mu_1 - \mu_3)^2$ (05)

$= \sum_{i=1}^{n} (x_i - \mu_1 - d_1)^2$

$= \sum_{i=1}^{n} \{(x_i - \mu_1)^2 + 2d_1 (x_i - \mu_1) + d_1^2\}$ (05)

$= \sum_{i=1}^{n} (x_i - \mu_1)^2 - 2d_1 \sum_{i=1}^{n} (x_i - \mu_1) + \sum d^2$ (05)

$= n\sigma_1^2 + n d_1^2$ [$\because \sum_{i=1}^{n} (x_i - \mu_1) = 0$ and (05) $\sigma_1^2 = \frac{\sum_{i=1}^{n} (x_i - \mu_1)^2}{n}$] (05)

$= n(\sigma_1^2 - d_1^2)$ (05) [30]

Similarly, $\sum_{i=1}^{n} (y_i - \mu_2)^2 = m(\delta_2^2 - d_2^2)$, where $d_2 = \mu_3 - \mu_2$ (05) [05]

$$\sigma_3^2 = \frac{\sum_{i=1}^{n} (x_i - \mu_3)^2 + \sum_{j=1}^{m} (y_j - \mu_3)^2}{m + n}$$ (05)

$$= \frac{n(\sigma_1^2 + d_1^2) + m(\sigma_2^2 + d_2^2)}{m + n}$$

$$= \frac{(n\sigma_1^2 + m\sigma_2^2) + n(d_1^2 + m d_2^2)}{m + n}$$ (05) [10]

For the first 100 :

$n = 100,\ \mu_1 = 2.3\ \sigma_1^2 = 0.8$

For the second 100 :

$m = 100,\ \mu_2 = 1.7,\ \sigma_2^2 = 0.5$

From the above, $\mu_1 = \frac{230 + 170}{200} = 2$ (05) (05)

Note that $d_1 = -0.3$ and $d_2 = 0.3$

$\sigma_3^2 = \frac{100}{200}[0.8 + 0.5 + (0.3)^2 + (0.3)^2]$ (05)

$= \frac{1}{2}[1.3 + 0.09 \times 2]$

$= \frac{1}{2}(1.3 + 0.18)$

$= \frac{1.48}{2}$

$\sigma_1 = 0.74$

$\sigma_1 = \sqrt{0.74}$ (05)