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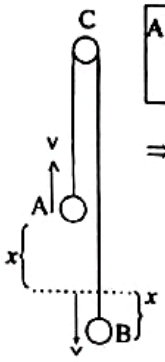
G.C.E. (Advanced Level) Examination - August 2015
Combined Mathematics - II
Scheme of Marking

Part A

(01) Using k.E + P.E = constant

$\frac{1}{2}mv^2 + \frac{1}{2}(2m)v^2 - mg(l-x) - 2mg(l+x) =$ Constant

= Initial value = 0 - 3 mgl (15)



Aliter : Conservation of energy ⇒ (15)

$2mgx - mgx = \frac{1}{2}mv^2 + \frac{1}{2}(2m)v^2$

$\Rightarrow \frac{1}{2}3mv^2 = (2mg - mg)x$

$v^2 = \frac{2gx}{3}$ (05)

Differentiating w.r.t.x ;

$2v\frac{dv}{dx} = \frac{2g}{3}$

Acceleration of the system $= \frac{g}{3}$ (05)

Aliter

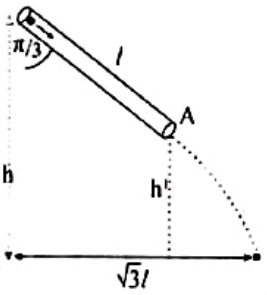
Differentiating w.r.t. time t

$2v\frac{dv}{dt} = \frac{2g}{3}\left(\frac{dx}{dt}\right)$

Acceleration $= \frac{dv}{dt} = \frac{g}{3}$ (05)

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(02) For motion of particle with constant acceleration from O to A :



$v^2 = u^2 + 2as$

$v^2 = 2.g\cos\pi/3.\ l$ (05)

$v^2 = gl$; $v = \sqrt{gl}$

For motion from A to B

$\rightarrow s = ut + \frac{1}{2}at^2$

$\sqrt{3}l - l\sin\pi/3 = t\sqrt{gl}\sin\pi/3$ (05)

$\frac{\sqrt{3}}{2}l\frac{1}{\sqrt{gl}} - \frac{\sqrt{3}}{2}t$

$t = \sqrt{l/g}$ (05)

$h' = \frac{1}{2}\sqrt{gl}$ (05) $\sqrt{\frac{l}{g}} + \frac{1}{2}g.\frac{l}{g} = l$

$\because h = l + l\cos\pi/3 = \frac{3l}{2}$ (05)

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(03) → u

P O Q

→v P O → w Q

K. E. before impact

$= T_0 = \frac{1}{2}mu^2$

K.E. after impact

$= T_1 = \frac{1}{2}m(v^2 + w^2)$

Conservation of momentum : (05)

$mu = mv + mw$ — (1) (05)

$u = v + w$

Newton's law of restitution

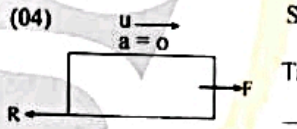
$eu = w - v$ — (2) (05)

Ratio of K.E $= \frac{T_1}{T_0} = \frac{v^2+w^2}{u^2} = \left[\frac{(v+w)^2 + (w-v)^2}{2u^2}\right]$ (05)

$= \left(\frac{u^2 + e^2u^2}{2u^2}\right)$ (05)

$= \frac{1}{2}(1 + e^2)$ 25

(04)



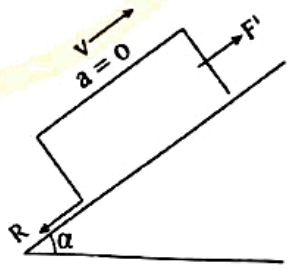
Since the power = HKW

Tractive force $F = \frac{1000H}{u}$ N (05)

→ F = ma

$\frac{1000H}{u} - R = 0$

$R = \frac{1000H}{u}$ N (05)



F = ma

$F' - R - 1000Mg\sin\alpha = 0$ (05)

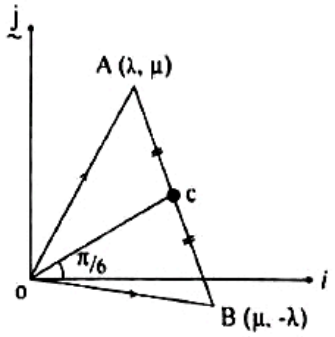
$\frac{2000H}{v} = \frac{1000H}{u} + 1000Mg\sin\alpha$ (05)

$\frac{2H}{v} = \frac{H + Mgu\sin\alpha}{u}$ (05)

$\rightarrow v = \frac{2Hu}{H + Mgu\sin\alpha}$ ms^{-1}

25

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$\vec{OA} = \lambda i + \mu j$

$\vec{OB} = \mu i - \lambda j$

$\vec{OA} \cdot \vec{OB} = \lambda\mu - \mu\lambda = 0$ for $\lambda, \mu \in \mathbb{R}$ (05)

$\Rightarrow A\hat{O}B = \pi/2$

$\vec{OC} = \vec{OA} + \vec{AC} = \lambda i + \mu j + \vec{AC}$

$= \lambda i + \mu j + \frac{1}{2}\{(\mu i - \lambda j) - (\lambda i + \mu j)\}$

$= \frac{1}{2}(\lambda + \mu) i + \frac{1}{2}(\mu - \lambda) j$ (05)

$\vec{OC} \cdot i = 2 \cos \pi/6 = \sqrt{3} = \frac{1}{2}(\lambda + \mu)$ (05)

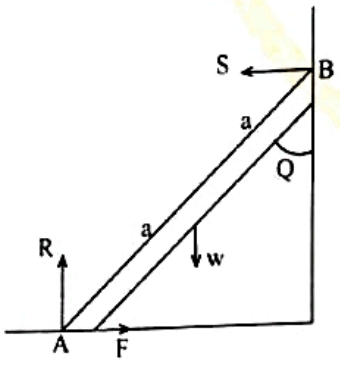
$\vec{OC} \cdot j = 2 \sin \pi/6 = 1 = \frac{1}{2}(\lambda - \mu)$ (05)

Subtracting and adding →

$\lambda = \sqrt{3} - 1, \quad \mu = \sqrt{3} + 1$ (05)

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(06)



Resolving

→ F = S (05)

↑ R = W (05)

Taking moments about A ;

↺ S. 2a cosθ = w. asinθ (05)

since F = S.

$F = \frac{1}{2} w \tan\theta \leq \mu R$ (05)

$\frac{1}{2} w \tan\theta \leq \mu w \Rightarrow \mu \geq \frac{1}{2} \tan\theta$ (05)

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(07) Given probabilities : $P(A) = \frac{1}{4}$, $p(B) = \frac{1}{2}$ and $p(A \cup B \cup C) = \frac{3}{4}$

Since A, B, and C are independent events

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A).P(B) - P(B).P(C) - P(C).P(A) + P(A).P(B).P(C)$ (10)

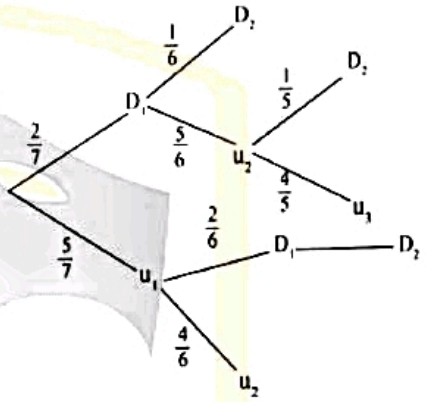
$\frac{3}{4} = \frac{1}{4} + \frac{1}{2} + P(C) - \frac{1}{8} - \left(\frac{1}{2} + \frac{1}{4}\right) P(C) + \frac{1}{8} P(C)$ (05)

$\frac{1}{8} = P(C)\left[1 + \frac{1}{8} - \frac{3}{4}\right] = P(C)\left(\frac{3}{8}\right) \quad \therefore p(C) = 1/3$ (05)

(5)

25

(08) Out of 7 electric bulbs, 2 are defective and 5 are usable



$P(D_1) = \frac{2}{7}$ (05)

$P(D_1 D_2) = \frac{2}{7} \cdot \frac{1}{6} = \frac{1}{21}$ (05)

$P(D_1 U_2 D_2) = \frac{2}{7} \cdot \frac{5}{6} \cdot \frac{1}{5} = \frac{1}{21}$ (05)

$P(u_1 D_1 D_2) = \frac{5}{7} \cdot \frac{2}{6} \cdot \frac{1}{5} = \frac{1}{21}$ (05)

Test ; 1st 2nd 3rd

Probability that just two tests are required

$= P(D_1 D_2) = \frac{1}{21}$

Probability that just three tests are required.

$= P(D_1 U_2 D_2) + P(U_1 D_1 D_2)$

$= \frac{1}{21} + \frac{1}{21} = \frac{2}{21}$ (5)

25

(09) Seven numbers ; 1 2 4 x y 11 13

(In ascending order)

Mean = $y \Rightarrow 1 + 2 + 4 + x + y + 11 + 13 = 7y$

$6y - x = 31$

Trials : suppose $x = 4$; $6y - 4 = 31$

$6y = 35$ No solution with integer y.

Suppose $x = 5$; $6y - 5 = 31$

$y = 6$ (05)

A/L සන්ඛිය
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Alter

$4 \le x \le 11$

$35 \le 6y \le 42$

Possible integer y :

$y = 6$ or $y = 7$

$y = 6 \Rightarrow x = 5$ (05)

$y = 7 \Rightarrow x = 42 - 31 = 11$ Contradiction (05)

Solution $x = 5, \quad y = 6 = \mu$

Variance $S^2 = \frac{1}{7}\sum_{i=1}^{7}(x_i - \mu)^2$ (05)

$= \frac{1}{7}\left[(-5)^2 + (-4)^2 + (-2)^2 + (-1)^2 + 0 + 5^2 + 7^2\right]$ (05)

$= \frac{1}{7}(25 + 16 + 4 + 1 + 25 + 49) = \frac{120}{7}$

25

(10)

Number x	1	2	3	4	5	6
Frequency f	α	9	γ	11	8	7

Given ; $\Sigma f = 50 = \alpha + \gamma + 35 \Rightarrow \alpha + \gamma = 15$ (05)

Also, mean

$= 3.66 \Rightarrow 50 \times 3.66 \Rightarrow 183 = 1.\alpha + 2.9 + 3.\gamma + 4.11 + 5.8 + 6.7$

$183 = \alpha + 3\gamma + 144 \Rightarrow \alpha + 3\gamma = 39$ (05)

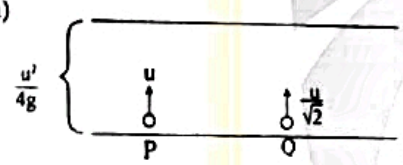
Solving : $\gamma = 12$ and $\alpha = 3$ (05) (05)

Mode = 3, Median = 4 (05)

25

Part B

(11) (a)



(i) Particle P

$v^2 = u^2 - 2g\frac{u^2}{4g} = \frac{u^2}{2}$ (05)

Velocity of P just before it strikes the ceiling is $v = \frac{u}{\sqrt{2}}\uparrow$ (05)

Time T_1 is given by $\frac{u}{\sqrt{2}} = u - gT_1$ (05)

$\Rightarrow \quad T_1 = \frac{u}{g}\left(1 - \frac{1}{\sqrt{2}}\right)$ 15

Its velocity just after collision with the ceiling

$-e\,v \quad \frac{1}{\sqrt{2}}\cdot\frac{u}{\sqrt{2}} \quad \frac{u}{2}\downarrow$ (05)

Velocity wat the point of projection in return journey

$w^2 \quad \left(\frac{u}{2}\right)^2 + 2g\frac{u^2}{4g} \Rightarrow w \quad \frac{\sqrt{3}\,u}{2}$ (05) 10

(ii) Particle Q

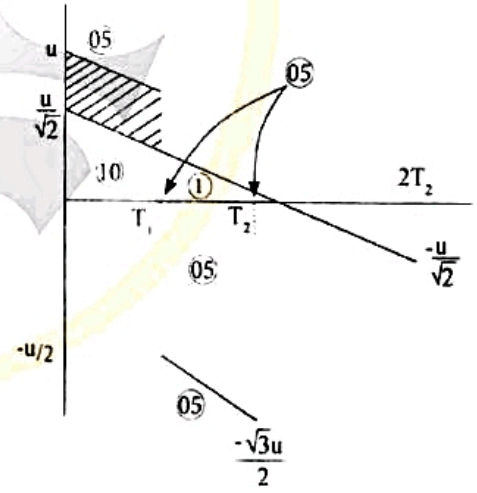
$\cdot \quad \left(\frac{u}{\sqrt{2}}\right)^2 \quad 2g\left(\frac{u^2}{4g}\right) \quad u$ (05) Particle Q reaches the ceiling with hero velocity

Time taken to reach the ceiling, T_2 is given by

$0 \quad \frac{u}{\sqrt{2}} - gT_2 \Rightarrow T_2 \quad \frac{u}{\sqrt{2}g}$ (05) 10

(iii) In the return (downward) motion Q reaches the point of projection with velocity $= \frac{u}{\sqrt{2}}\downarrow$ (05)

After a time $2T_2 = \frac{u}{g}\sqrt{2}$



30

(iv) Distance of Q below the ceiling at time T_1 = shaded area in the diagram.

$= (u - u/\sqrt{2})\,T_1$ (05)

$= \frac{u}{\sqrt{2}}(\sqrt{2} - 1) \times \frac{u}{\sqrt{2}g}(\sqrt{2} - 1)$ (05)

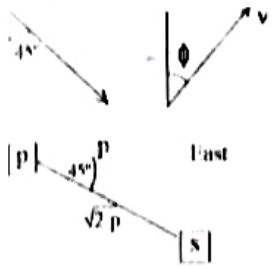
$= \frac{u^2}{2g}(\sqrt{2} - 1)^2$

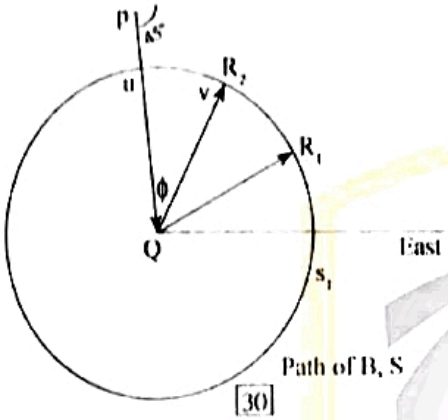
10

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(b) Port P, Ship S, Boat B

vel (B, S) = vel (B, P) + vel (P, S) 05





Path of B, S

30

Two possible directions for vel (B, P) each making angle θ with the relative path.

Given : $\frac{v}{u} = \sqrt{\frac{2}{3}}$

From PQR_1 or PQR_2

$\frac{v}{\sin 45} = \frac{u}{\sin\theta} \Rightarrow \sin\theta = \left(\frac{u}{v}\right)\frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2} \Rightarrow \theta = 60^\circ$ 05

05

$\phi = 15^\circ$

Relative to port (Earth) (i) velocity of B_1 makes angle 15° North of East 05

(i) Velocity of B_2 makes angle 15° East of North 05

60

$$T_2 - T_1 = \sqrt{2}\,P\left(\frac{1}{PR_2} - \frac{1}{PR_1}\right) = \frac{\sqrt{2}\,P}{PR_1\,PR_2}(PR_1 - PR_2) \quad 05$$

$$= \frac{\sqrt{2}\,pv}{\left(\frac{u}{\sqrt{2}} + \frac{v}{2}\right)\left(\frac{u}{\sqrt{2}} - \frac{v}{2}\right)} = \frac{\sqrt{2}\,pv}{\left(\frac{u^2}{2} - \frac{v^2}{4}\right)} \quad 05$$

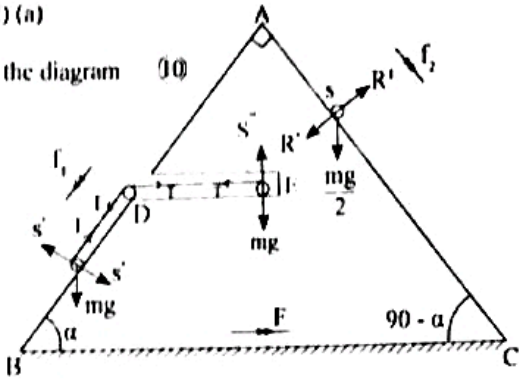
05

$$= \frac{\sqrt{2}\,p\frac{\sqrt{2}}{\sqrt{3}}u}{\frac{u^2}{2}\left(1 - \frac{1}{3}\right)} = \frac{2\sqrt{3}p}{u}$$

15

(12) (a)

for the diagram 10



Applying Newton's second law

For particle P along ED : ← $T = m(f_1 - F)$ — (1) 10

For particle Q along DB : α

$mg\sin\alpha - T = m(f_1 - F\cos\alpha)$ — (2) 10

For particle S along AC : α

$\frac{m}{2}g\cos\alpha = \frac{m}{2}(f_2 + F\sin\alpha)$ — (3) 10

For the system along BC : →

$O = MF + m(F - f_1) + m(F - f_1\cos\alpha) + \frac{m}{2}(F + f_2\sin\alpha)$ — (4)

15 55

(1) + (2) $m\,g\sin\alpha = 2f_1 - F(1+\cos\alpha)$

$$f_1 = \frac{g\sin\alpha + F(1+\cos\alpha)}{2} \quad 05$$

From (3),

$f_2 = g\cos\alpha - F\sin\alpha$ 05

(4) $\Rightarrow O = F\left\{M + \frac{5m}{2}\right\} - mf_1(1+\cos\alpha) + \frac{m}{2}f_2\sin\alpha$

$O = \frac{F}{2}(2M+5m) - \frac{m}{2}(1+\cos\alpha)\{g\sin\alpha + F(1+\cos\alpha)\}$

$+ \frac{m}{2}\sin\alpha(g\cos\alpha - F\sin\alpha)$ 10

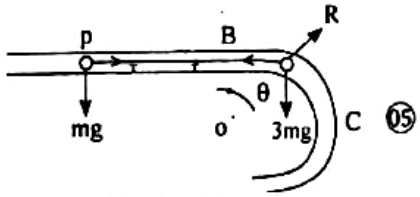
$mg\sin\alpha = F\{2M + 5m - m(1+\cos\alpha)^2 - m\sin^2\alpha\}$

$= F\{2M + 3m - 2m\cos\alpha\}$ 05

$$\Rightarrow F = \frac{mg\sin\alpha}{2M + 3m - 2m\cos\alpha}$$

25

(b)



05

Applying K. E + P.E = Constant :

$\frac{1}{2}m(a\dot\theta)^2 + \frac{3m}{2}(a\theta)^2 - 3mga(1-\cos\theta) = 0$ 05

10 10

$$\dot\theta^2 = \frac{3g}{2a}(1-\cos\theta) \quad 05$$

35

Differentiating w. r. t. θ

$$2\dot{\theta}\frac{d\dot{\theta}}{d\theta} = \frac{3g}{2a}\sin\theta \quad (05)$$

$$\Rightarrow a\ddot{\theta} = \frac{3g}{4}\sin\theta$$

$\therefore$ the acceleration of particle P $= a\ddot{\theta} = \frac{3g}{4}\sin\theta$ (05)

[10]

Applying Newton's second law for particles P and Q:

For the particle P along PB

$$\rightarrow T = ma\ddot{\theta} \quad (05)$$

$$\Rightarrow T = m\frac{3g}{4}\sin\theta \quad (05)$$

= Tension in the string.

For the particle Q along QO



$$3mg\cos\theta - R = 3ma\dot{\theta}^2 \quad (05)$$

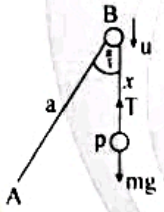
$$R = 3mg\cos\theta - 3ma\cdot\frac{3g}{2a}(1-\cos\theta) \quad (05)$$

$$= 3mg\cos\theta - \frac{9mg}{2} + \frac{9mg}{2}\cos\theta$$

$$= \frac{3mg}{2}(5\cos\theta - 3) \quad (05)$$

[25]

(13)



Apply F= ma to particle P.

$$\downarrow mg - T = m\ddot{x} \quad (05)$$

$T = 2mg\left(\frac{x}{a}\right)$, $\because$ modulus of elasticity $= 2mg$ (05)

Eliminating T and dividing by m

$$g - \ddot{x} = \frac{2g}{a}x \quad (05) \text{ or } \ddot{x} + \frac{2g}{a}(x - a/2) = 0 \quad (05)$$

$\ddot{x} + \omega^2 x = 0$, where $x = x - a/2$ and $\omega^2 = \frac{2g}{a}$ (05)

[25]

Centre C of SHM is

$x = 0 \Rightarrow x = \frac{a}{2}$ = BC (05)

Assumed solution of the SHM equation is

$\dot{x}^2 = \omega^2(A^2 - x^2)$,

where A is the amplitude of the motion.

Initially, when x = o, $x = -a/2$ and (05)

$v = u = \sqrt{\frac{5ga}{8}} = u$ (05)

Substituting into the given form of solution.

$$\frac{5ga}{8} = \frac{2g}{a}\left[A^2 - (-a/2)^2\right] \quad (05)$$

$$\frac{5a}{16} + \frac{a^2}{4} = A^2 = \frac{9a^2}{16}$$

Since A is positive, $A = \frac{3a}{4}$ (05)

The amplitude of the SHM $= \frac{3a}{4}$

Maximum extension of the string $\Rightarrow \dot{x} = 0$ (05)

occurs when,

$$x = A \rightarrow x - a/2 = \frac{3a}{4} \rightarrow x = \frac{5a}{4} \quad (05)$$

[35]

$\dot{x}^2 = \omega^2(A^2 - x^2)$, where $A = \frac{3a}{4}$

Speed V of the particle, while passing the centre C (x = 0) is given by

$$V^2 = \omega^2 A^2 = \frac{2g}{a}\cdot\frac{9a^2}{16} \Rightarrow v = 3\sqrt{\frac{ga}{8}} \quad (05)$$

(05)

Also, $u^2 = \frac{5ga}{8}$

$$\therefore \left(\frac{v}{u}\right)^2 = \frac{9ga}{8}\cdot\frac{8}{5ga} \quad (05)$$

$$\Rightarrow v = \frac{3u}{\sqrt{5}} \quad (05)$$

[20]

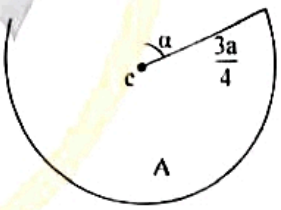
Let α be the acute angle, $\alpha = \cos^{-1}(2/3)$ (05)

Time t_0 taken by the particle to move from B to C is given by

$$\omega t_0 = \pi/2 - \alpha \Rightarrow t_0 = \frac{1}{\omega}(\pi/2 - \alpha)$$

(05) (05)

i.e $t_0 = \sqrt{\frac{a}{2g}}\left\{\pi/2 - \cos^{-1}(2/3)\right\}$ (05)



For the diagram (15)

[35]

Time t_1 taken by the particle to reach its lowest position is given by

$$\omega t_1 = \pi - \alpha \Rightarrow t_1 = \frac{1}{\omega}(\pi - \alpha)$$

(05) (05)

i. e $t_1 = \sqrt{\frac{a}{2g}}\left\{\pi - \cos^{-1}\left(\frac{2}{3}\right)\right\}$ (05)

[15]

Maximum extention $= \frac{5a}{4}$

Maximum tension is, $T_{max} = 2mg\left\{\frac{5a/4}{a}\right\} = \frac{5mg}{2}$

(05)

Maximum magnitude of the force on the

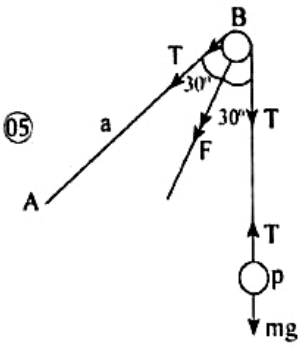
Peg $= 2T_{max}\cos(\pi/6)$

$= T_{max}\sqrt{3} = \left(\frac{5mg}{2}\right)$ (05)

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For the diagram (05)



Aliter [20]

Assume a solution in the form $x = x - a/2$

$x - a/2 = \alpha\cos\omega t + \beta\sin\omega t$ — ① where $\omega^2 = \frac{2g}{a}$.

Differentiating $\dot{x} = \dot{x} = -\alpha\omega\sin\omega t + \beta\omega\cos\omega t$ — ② (05)

The constants α and β are given by the initial conditions;

$x = 0$ and $\dot{x} = u = \sqrt{\frac{5ga}{8}}$ when $t = 0$ (05) (05)

$\Rightarrow -a/2 = \alpha$ and $u = \beta\omega$, *i.e.* $\beta = \frac{u}{\omega}$ (05)

Substitution in (i) and (ii) give $x = a/2\,(1-\cos\omega t) + \frac{u}{\omega}\sin\omega t$ (05)

and $\rightarrow \dot{x} = \frac{a\omega}{2}\sin\omega t + u\cos\omega t$. (05) [30]

Centre C of the SHM is given by $x = 0$; i.e. $x = a/2$ (05)

Particle passes C at time t_0 given by

$0 = -\frac{a}{2}\cos\omega t_0 + \frac{u}{\omega}\sin\omega t_0$

i. e. $\tan\omega t_0 = \frac{a\omega}{2u} = \frac{2}{\sqrt{5}}$. since $\left\{\frac{a\omega}{2u}\right\}^2 = \frac{2ga}{5ga/2}$

Let the acute angle $\phi = \tan^{-1}(2/\sqrt{5}) = \pi/2 - \cos^{-1}(2/3)$

Hence the required time $t_0 = \sqrt{\frac{a}{2g}}\left\{\pi/2 - \cos^{-1}(2/3)\right\}$



Speed V of the particle as it passes C is given by.

$v = \frac{a\omega}{2}\sin\omega t_0 + u\cos\omega t_0 = u\{\tan\omega t_0 + \cos\omega t_0\}$ (05) (05)

$= u\sec\omega t_0 = \frac{3u}{\sqrt{5}}$ (05) [15]

Time t_1 from peg B to lowest position E is = time from B to C + time from C to E (05)

$t_1 = t_0 + \frac{1}{\omega}\cdot\pi/2$ (05)

$t_1 = \sqrt{\frac{a}{2g}}\left\{\pi - \cos^{-1}(2/3)\right\}$ (05) [15]

Maximum extension, occurring when $t = t_1$ is obtained as

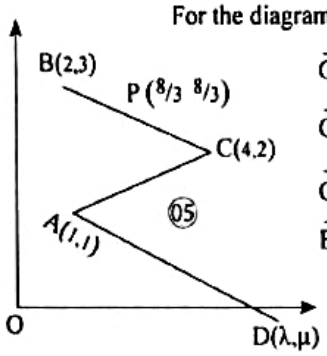
$x_1 = a/2\,(1-\cos\omega t_1) + \frac{u}{\omega}\sin\omega t_1 = \frac{a}{2}\left(1 + \frac{2}{3}\right) + \frac{\sqrt{5}a}{4}\cdot\frac{\sqrt{5}}{3}$ (05) (05)

$= \frac{5a}{4}$ (05)

Amplitude of the SHM $= \frac{5a}{4} - \frac{a}{2} = \frac{3a}{4}$ (05) [20]

(14) Trape Zium ABCD with AD // BC and PD ⊥ AC.

For the diagram (05)



$\vec{OA} = \underset{\sim}{i} + \underset{\sim}{j}$

$\vec{OB} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$

$\vec{OC} = 4\underset{\sim}{i} + 2\underset{\sim}{j}$

$\vec{BC} = \vec{BO} + \vec{OC}$ (05)

$= -(2\underset{\sim}{i} - 3\underset{\sim}{j}) + (4\underset{\sim}{i} + 2\underset{\sim}{j})$

$= 2\underset{\sim}{i} - \underset{\sim}{j}$ (05)

$\vec{BP} = \frac{1}{3}\vec{BC} = \frac{1}{3}(2\underset{\sim}{i} - \underset{\sim}{j})$ (05) and so $\vec{OP} = \vec{OB} + \vec{BP}$

$= \frac{8}{3}(\underset{\sim}{i} + \underset{\sim}{j})$ (05)

$\vec{AD} // \vec{BC} \Rightarrow \frac{\lambda - 1}{2} = \frac{\mu - 1}{-1} \Rightarrow \lambda - 1 = 2(1-\mu)$ (05)

$\lambda + 2\mu = 3$ — ① (05)

$\vec{PD} \perp \vec{AC} \Rightarrow \vec{PD}.\vec{AC} = 0$, $\vec{AC} = 3\underset{\sim}{i} + \underset{\sim}{j}$ (05) (05)

$\Rightarrow \left\{\left(\frac{8}{3} - \lambda\right)\underset{\sim}{i} + \left(\frac{8}{3} - \mu\right)\underset{\sim}{j}\right\}.(3\underset{\sim}{i} + \underset{\sim}{j}) = 0$

$8 - 3\lambda + \frac{8}{3} - \mu = 0$ (05)

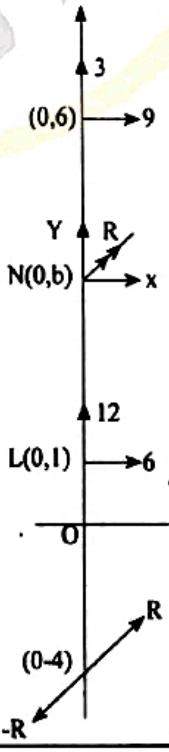
$9\lambda + 3\mu = 32$ — ② (05)

From ① $\Rightarrow 9\lambda + 18\mu = 27$

$15\mu = -5 \Rightarrow \mu = -1/3$ and $\lambda = 3 + \frac{2}{3} = 11/3$ (05) [70]

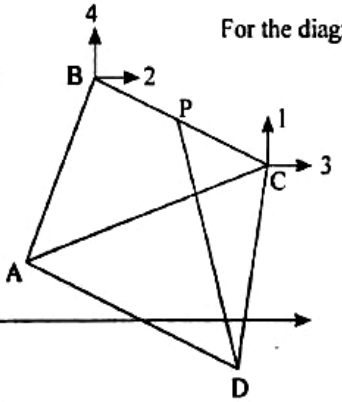
Hence $\vec{OD} = \frac{11}{3}\underset{\sim}{i} - \frac{1}{3}\underset{\sim}{j}$

System of forces



Point	Force
B(2,3)	$\underset{\sim}{F_1}$=(2,4)
C(4,2)	$\underset{\sim}{F_2}$=(3,1)
L(0,1)	$\underset{\sim}{F_3}$=(6,12)
M(0,6)	$\underset{\sim}{F_4}$=(9,3)

For the diagram (10)



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(i) Moment of F_1 and F_2 about O ↺ = 2.4 - 3.2 + 4.1-2.3 = 0 (05)

Moment of F_1 and F_2 about A ↺ = 1.4 - 2.2 + 3.1 - 1.3 = 0 (05)

Moment of F_1, F_2, F_3 and F_4 about O = Moment of F_3 and E_4 about 0 (05)

= 6.1 + 9.6 = 60 ↷ Nm (05) [30]

(ii) Resolving the system

→ X = 2 + 3 + 6 + 9 = 20 (05)

↑ Y = 4 + 1 + 12 + 3 = 20 (05)

(05)

Let N(o, b) be the point of intersection of the y-axis and the line of action of the resultant foerce (x,y) Then taking moments about the origin o,

↺O b.x = 60 ⇒ $b = \frac{60}{x} = \frac{60}{20} = 3$ (05) (05) [25]

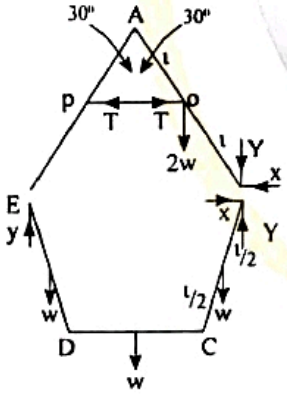
∴ Point N has coordinates (0, 3).

(iii) Introduce forces -R and R at (0,-4). Then the system ≡ a force R at point (0,-4) together with a couple of moment G = X (3 + 4) = 140 Nm ↻ (05) (05)

Line of action of single force R at (0,-4) is y = x - 4 (05)
The coordinates of D ($^{11}/_3$, $^{-1}/_3$) satisfy this equation as $^{-1}/_3 = {}^{11}/_3 - 4$ (05)

⇒ D lies on the lin e of action of the single force. (05) [25]

(15)



(10)

Resolving vertically for rods BC, CD, DE

↓ 2y = 3N ⇒ y = $^{3w}/_2$ (10)

Taking moments about C ↺ For CB

$-x.l\frac{\sqrt{3}}{2} + y.\,{}^{l}/_2 = w\,{}^{l}/_4$ (10)

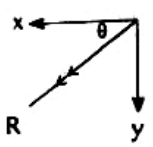
$x\sqrt{3} = \frac{3}{2}w - \frac{w}{2} = w$ (05)

Taking moments about A ↻ for the rod AB

$T.l\frac{\sqrt{3}}{2} = xl\sqrt{3} + yl + \frac{2wl}{2}$ (15)

$T.\frac{\sqrt{3}}{2} = w + \frac{3}{2}w + w = \frac{7}{2}w$ (05)

$T = \frac{7w}{\sqrt{3}}$ (05) [60]

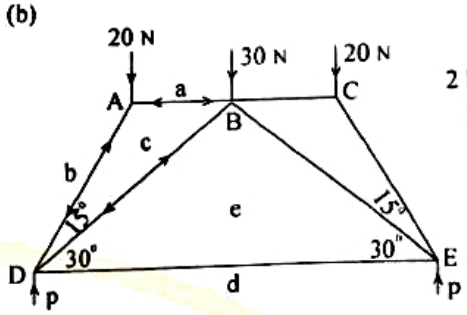


$R = \sqrt{x^2 + y^2} = w\sqrt{\frac{1}{3} + \frac{9}{4}} = \frac{\sqrt{31}}{12}$ (05)

$\tan\theta = \frac{Y}{X} = \frac{3w/2}{w/\sqrt{3}} = \frac{3\sqrt{3}}{2}$ (05)

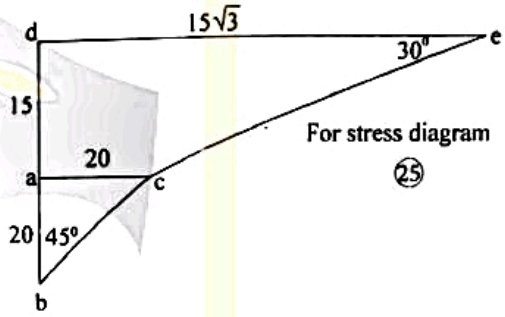
Reaction at B makes an angle $\theta = \tan^{-1}\left(\frac{3\sqrt{3}}{2}\right)$ with the lorizontal (05) [15]

(b)



2 P = 70

P = 35N (05)



For stress diagram (25)

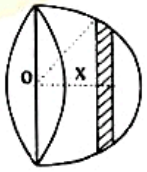
bc = $20\sqrt{2}$ N : Thrust in AD - (10)

ca = 20 N : Thrust in AB - (10)

de = $20 + 15\sqrt{3}$ N : Tension DE - (15)

ec = 30 N : Thrust in DB - (10) [75]

(16) Uniform solid hemisphere of radius a



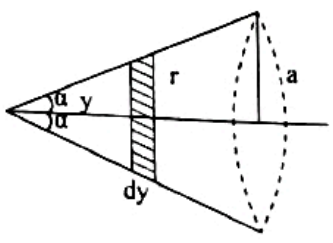
$\left(\frac{2}{3}\pi a^3\rho\right)\bar{x}_1 = \int_0^a x\rho\pi(a^2 - x^2)\,dx$ (05) (05)

$= \rho\pi\left[-\frac{(a^2 - x^2)^2}{4}\right]_0^a$ (05)

$= \frac{\rho\pi a^4}{4}$ (05)

$\bar{x}_1 = \frac{3}{8}a$ (05) [25]

Uniform solid cone of base radius a and height h. CM lies on the axis of symmetry at a distance $\bar{y}_1$, from the vertex v, where.



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$\left(\frac{1}{3}\pi a^2\rho h\right) y_1 = \int_0^h y.\rho\pi\left(\frac{ay}{h}\right)^2 dy$ (05) (05)

$\left(\frac{\tan\alpha = a/h}{r = y\tan\alpha}\right)$ (05)

$= \frac{\rho\pi a^2}{h^2}\left[\frac{y^4}{4}\right]_0^h$ (05)

$\Rightarrow y_1 = \frac{3h}{4}$ (05)

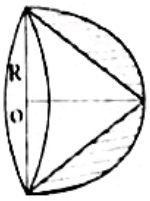
Distance of CM from the centre of the base $= \frac{h}{4}$ [25]

Remainning solid R

Mass of solid R $= \frac{2}{3}\pi a^3\rho - \frac{1}{3}\pi a^2.a\rho$ (05)

$M - M/2 = M/2$ (05)

Distance $\bar{x}$ of CM of R from the centre O is given by (05)



$$\bar{x} = \frac{M\frac{3}{8}a - \frac{M}{2}\cdot\frac{a}{4}}{M/2} = \left(\frac{3}{4} - \frac{1}{4}\right)a = \frac{a}{2}$$ (05)

[25]

Let OG = $\bar{x}$, where G the centre of mass of the composite solid S

$M\bar{X} = \frac{M}{2}\left(\frac{a}{2}\right) - \frac{M}{2}\left(\frac{a}{4}\right) \Rightarrow \bar{X} = (a/8)$ (05) (05) (15)

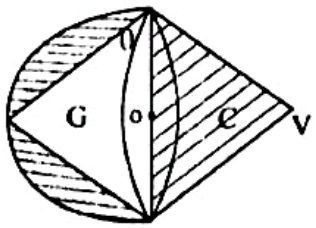
[25]

(a) (i) When suspended from point P, G is vertically below P. Angle θ between PO and the vertical is given by

$\tan\theta = \frac{a/8}{a} = 1/8$ (10)

(ii) In order to make OV horizontal, so that O is below P.

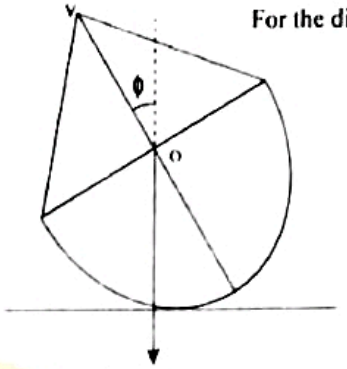
$O\hat{}\ mga = Mg(a/8) \Rightarrow m = \frac{M}{8}$ (05) (10)



[25]

(b) R = (M + m)g, for all ϕ $-\pi/2 \le \phi \le \pi/2$ (05) (10)

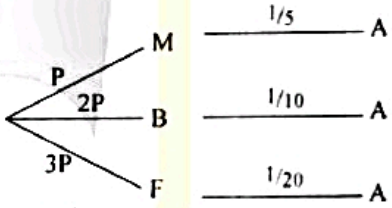
It can rest with its ov axis inclined at any acute angle to the vertical. (05)



For the diagram. (05)

[25]

(17).(a) M = Travelling by Motorcycle
B = Travelling by Bicycle
F = Travelling on Foot
A = Accident occuring



Since the three events M, B and F are mutually exclusive and exhanstive,

$P(M) + P(B) + P(F) = 1$ (05)

$\Rightarrow P + 2P + 3P = 1$ i.e. $P = 1/6$ (05) [10]

Now $P(A) = P(M\cap A) + P(B\cap A) + P(F\cap A)$ (05)

$= P(A|M).P(M) + P(A\backslash B).P(B) + P(A\backslash F).P(F)$ (05)

$= \frac{1}{6}\cdot\frac{1}{5} + \frac{2}{6}\cdot\frac{1}{10} + \frac{3}{6}\cdot\frac{1}{20}$ (15)

$= \frac{1}{30} + \frac{1}{30} + \frac{1}{40} = \frac{4+4+3}{120} = \frac{11}{120}$ (05)

[30]

(i) $P(M|A) = \frac{P(M\cap A)}{P(A)} = \frac{1/30}{11/120} = \frac{4}{11}$ (05) (05)

(ii) $P(B\backslash A) = \frac{P(B\cap A)}{P(A)} = \frac{1/30}{11/120} = \frac{4}{11}$ (05)

(iii) $P(F\backslash A) = \frac{P(F\cap A)}{P(A)} = \frac{1/40}{11/120} = \frac{3}{11}$ (05)

Least probability of an accident is when the man travels on foot, since other probabilities are higher (05)

∴ Safest mode of transportation is walking (05)

[30]

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(b) Length x of the road measured in metres

x	99.8	99.9	100.0	100.1	100.2	100.3	100.4
Frequency (f)	5	7	12	33	25	15	3

Transformation

$y = \frac{x - \bar{x}_0}{d} = \frac{x - 100.1}{0.1}$ (05)

extended table

y	-3	-2	-1	0	1	2	3	(10)
y^2	9	4	1	0	1	4	9	(05)

$\Sigma fy = -15 - 14 - 12 + 0 + 25 + 30 + 9 = -41 + 64 = 23$ (05)

Mean of y :

$\bar{y} = \frac{1}{100} \Sigma fy = 0.23$ (05)

$x = \bar{x}_0 + dy \Rightarrow \bar{x} = \bar{x}_0 + d\bar{y}$ (05)

$\therefore$ the mean of x is $\bar{x} = 100.1 + (0.1)\,0.23 = 100.123$ (05)

40

Variance of y is

$S_y^2 = \frac{1}{100} \Sigma fy^2 - \bar{y}^2$ (05) and

$\Sigma fy^2 = 45 + 28 + 12 + 0 + 25 + 60 + 27 = 85 + 85 + 27 = 197$ (05)

$\frac{1}{100} \Sigma fy^2 = 1.97$, (05) $\bar{y}^2 = (0.23)^2 = 0.0529$ (05)

$\therefore$ Variance of y = 1.97 - 0.0529 = 1.917 (05)

Linear relation $x = dy + \bar{x}_0$

var $(x) = d^2$ var (Y), d = 0.1

$S_x^2 = d^2 S_y^2$ (05)

$\therefore$ var $(x) = (0.1)^2\, 1.917$

Taking square root of var (x)

Standard Deviation $= S_x = (0.1)\sqrt{1.917}$ (05)

$S_x = 0.1385$; $(\because \sqrt{1.917} \approx 1.385)$

Standard Deviation of x is 0.1385 m $\approx$ 0.139 m (05)

40