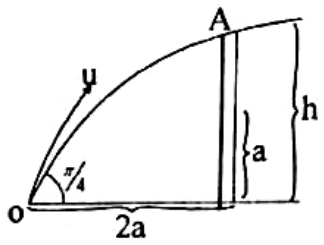
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G.C.E. (Advanced Level) Examination - August 2014
Combined Mathematics - II
Scheme of marking

Part A

(01)



Let $u > 2\sqrt{ga}$

For motion from O to A

$S = ut + \frac{1}{2}at^2$

$2a = u\cos\left(\frac{\pi}{4}\right)t$ (05)

$t = \frac{2\sqrt{2}a}{u}$

$\uparrow h = u\sin\frac{\pi}{4}t - \frac{1}{2}gt^2$ (10)

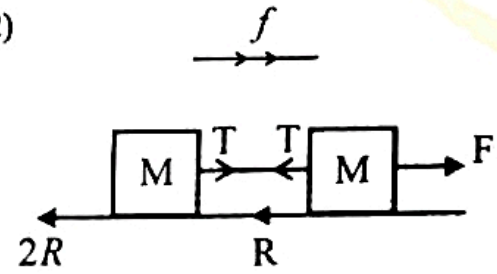
$= 2a - \frac{1}{2}g\frac{8a^2}{u^2} = 2a - \frac{4ga^2}{u_2}$ (05)

$> 2a - \frac{4ga^2}{uga}\left(u^2 > 4ga\right)$

$= a$ (05)

$\Rightarrow h > a$ and hence the particle will go over the wall. [25]

(02)



Tractive force $F = \frac{1000P}{v}N$ (05)

→ $\underline{F} = m\underline{a}$ *for the system*:

$F - 3R = 3Mf \rightarrow (1)$ (10)

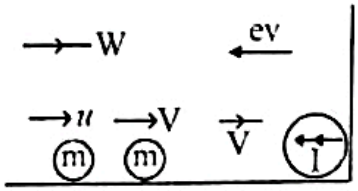
→ *For the trailer*;

$T - 2R = Mf \rightarrow (2)$ (05)

By (1) and (2) $2(T - 2R) = F - 3R$

$T = \frac{1}{2}(R + F) = \frac{1}{2}\left(R + \frac{1000P}{v}N\right)$ (05) [25]

(03)



For the first impact:

$\underline{I} = \Delta(m\underline{v})$ for the system → $mv + mw - mu = 0$ (05)

$v + w = u$

Newton's law of restitution $v - w = eu \rightarrow$ (10)

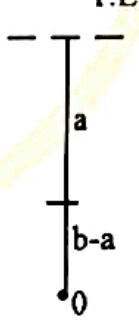
$\therefore v = \frac{1}{2}(1+e)u$

for the second impact

$\underline{I} = \Delta(mv)$, for Q → $I = mev - (-mv) = (1+e)mv$ (05)

and so the impulse on Q is $\frac{1}{2}(1+e)^2 mu$ (05) [25]

(04)



Since K.E is zero both at O and at the lowest position.
energy conservation equation.

$0 + \frac{1}{2}\frac{4mg}{a}(b-a)^2 - mgb = 0$ (15)

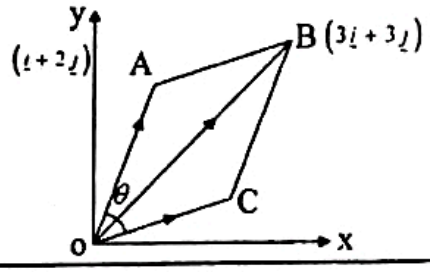
$2(b-a)^2 = ab$

$2b^2 + 2a^2 - 5ab = 0$ (05)

$(b-2a)(2b-a) = 0 \Rightarrow b = 2a\ (\because b > a)$ (05) [25]

{KE (05), P.E (05), Equation (05)}

(05)



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$\vec{AB} = \vec{AO} + \vec{OB}$

$= -(\underset{\sim}{i} + 2\underset{\sim}{j}) + (3\underset{\sim}{i} + 3\underset{\sim}{j})$

$= 2\underset{\sim}{i} + \underset{\sim}{j}$ (05)

since $\vec{OC} = \vec{AB}$ we have

$\vec{OC} = 2\underset{\sim}{i} + \underset{\sim}{j}$ (05)

scalar product $\vec{OA} \cdot \vec{OC} = (\underset{\sim}{i} + 2\underset{\sim}{j})(2\underset{\sim}{i} + \underset{\sim}{j})$

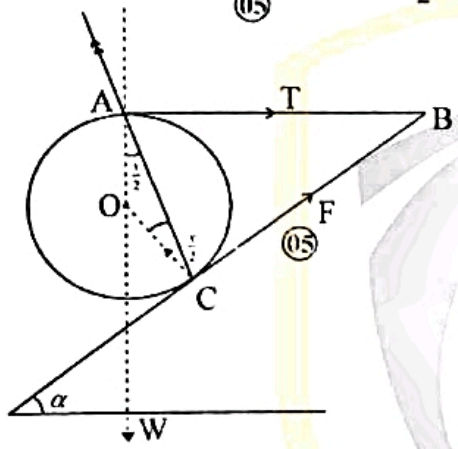
(05) $= 4$ (05)

$\vec{OA} \cdot \vec{OC} = |\vec{OA}||\vec{OC}| \cos\theta = 4$

$\cos\theta = \frac{4}{\sqrt{5}\sqrt{5}} = \frac{4}{5}$ (05)

25

(06) Angle of fiction = $O\hat{C}A$ and $O\hat{C}A = \frac{\alpha}{2}$ (05) (05)



Applying Lami's Theorem for concurrent forces at A;

$\frac{T}{Sin\left(\pi - \frac{\alpha}{2}\right)} = \frac{W}{Sin\left(\frac{\pi}{2} + \frac{\alpha}{2}\right)}$ (05)

$T = W \tan\frac{\alpha}{2}$ (05)

25

Aliter

C $W.a\,Sin\,\alpha = T(a + a\cos\alpha)$ (05)

$T = \frac{W\sin\alpha}{1+\cos\alpha} = W\tan\frac{\alpha}{2}$ (05)

10

(07)

$P[(A\cup B)\cap(A\cup B')] = P[(A\cup B)\cap(A\cap B)']$ (05)

$= P(A\cup B) - P[(A\cup B)\cap A\cap B]$ (10)

$[\because P(x\cap y') = P(x) - P(x\cap y)]$

(05) $= P(A\cup B) - P(A\cap B) = P(A) + P(B) - P(A\cap B) - P(A\cap B)$

$= P(A) + P(B) - 2P(A\cap B)$ (05)

25

Aliter

$P[(A\cup B)\cap(A\cup B')]$

(05) $= P(A\cap B') + P(B\cap A')$ $\because [(A\cup B)\cap A'] \cup [(A\cup B)\cap B']$ (10)

$= (A\cap B')\cup(B\cup A')$.

$= P(A) - P(A\cap B) + P(B) - P(A\cap B)$ (05)

$= P(A) + P(B) - 2P(A\cap B)$ (05)

25

(08) R : Red, W: White

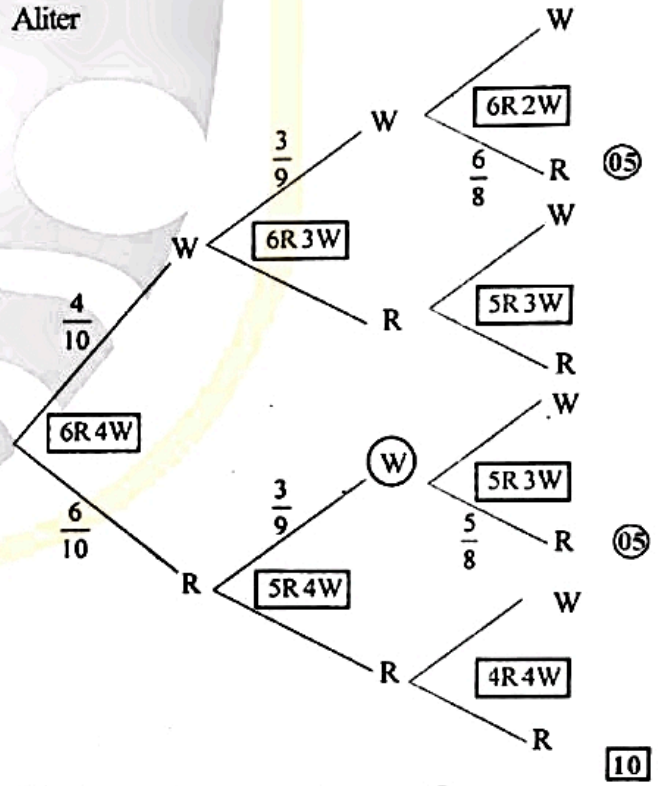
$P\left(3^{rd} R \middle| 2^{nd} W\right) = \frac{P(RWR) + P(WWR)}{P\left(2^{nd} W\right)}$ (05) (05)

$= \frac{\left(\frac{6}{10}\times\frac{4}{9}\times\frac{5}{8} + \frac{5}{10}\times\frac{3}{9}\times\frac{6}{8}\right)}{\left(\frac{6}{10}\times\frac{4}{9} + \frac{4}{10}\times\frac{3}{9}\right)}$ (05) (05)

$= \frac{2}{3}$ (05)

25

Aliter



10

(09) observations x, y, 9, 11, 11 (10)

$\frac{x+y+31}{5} = 7$ (05)

$\Rightarrow x + y = 4$

Since x and y are positive integers $(x=1, y=3)$, $(x=2, y=2)$ or $(x=3, y=1)$ Now, since the only made is 11, the observations, 1, 3, 9, 11, 11 (05)

largest observation = 11

smallest observation = 1 (05)

25

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(10)

$10\times16+20x+30\times30+40y+50\times20=31.8\times100$ (05)

$2x+4y=318-206=112$

$x+2y=56 \rightarrow (1)$

$66+x+y=100$ (05)

$x+y=34 \rightarrow (2)$

$\because x=12,\ y=22$ (05)

Median $=25+\frac{50-28}{30}\times10$ (05)

$=25+\frac{22}{3}$

$=\frac{97}{3}$

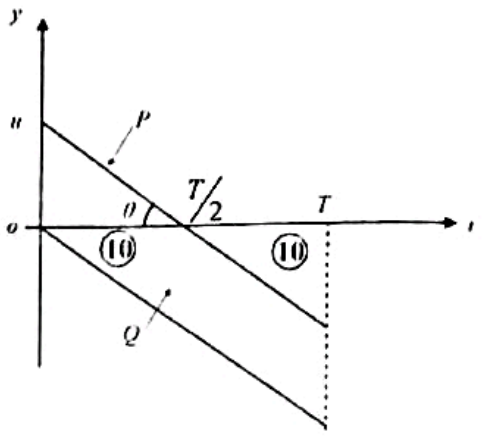
$=32.33$ (05)

25

PART B

(11)

(a)



20

$\tan\theta = g\ Sin\ \alpha = \frac{u}{T/2}$ (10)

$T=\frac{2u}{g\ Sin\ \alpha}$ (05)

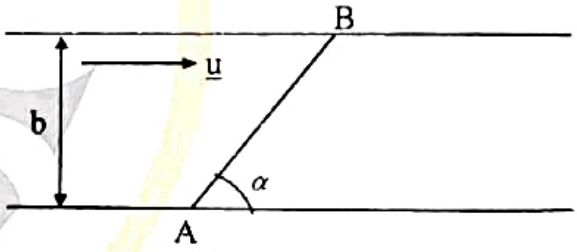
Distance of Q from O, when particle P returns to O = area of the shaded triangle.

$=\frac{1}{2}\times T\times 2u$ (10)

$=\frac{2u^2}{g\ Sin\ \alpha}$ (05)

30

(b)



(05)

$\underline{V}(Boy,E)=\underline{V}(Boy,W)+\underline{V}(W,\Sigma)$

$=\underline{V}(W,\Sigma)+\underline{V}(Boy,W)$

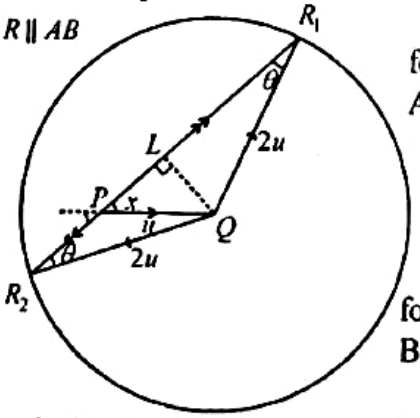
$=\overline{PQ}+\overline{QR}$, (05)

(05) $=PR$, $\nearrow t=1$ (for motion from A to B)

$t=2\swarrow$ (for motion B to A)

Velocity Triangles

$R_2R \parallel AB$



for motion from A to B (15)

for motion from B to A (15)

$QL = 2u\ Sin\ \theta = u\ Sin\ \alpha$ (05) (05)

$\Rightarrow Sin\ \theta = \frac{1}{2} Sin\ \alpha$

10

$PR_1 = 2u\cos\theta + u\cos\alpha$ (05) $PR_2 = 2u\cos\theta - u\cos\alpha$ (05)

$T_1 = \frac{AB}{2u\cos\theta + u\cos\alpha}$ (05) $T_2 = \frac{AB}{2u\cos\theta - u\cos\alpha}$ (05)

$T_2 = kT_1 \Rightarrow 2u\cos\theta + u\cos\alpha = k(2u\cos\theta - u\cos\alpha)$ (05)

$2(k-1)\cos\theta = (k+1)\cos\alpha$ (05)

$\because \cos\theta = \frac{1}{2}\left(\frac{k+1}{k-1}\right)\cos\alpha$

30

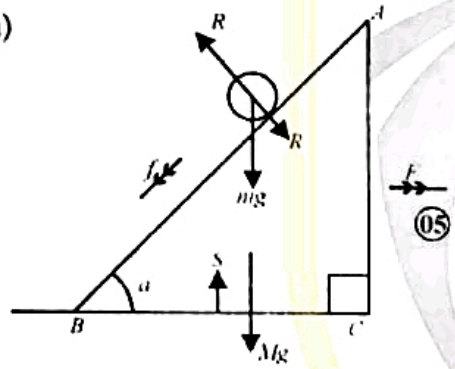
$\frac{1}{4}\sin^2\alpha + \frac{1}{4}\left(\frac{k+1}{k-1}\right)^2\cos^2\alpha = 1$ (05) $\left[\because \sin^2\theta + \cos^2\theta = \right.$

$\left[\left(\frac{k+1}{k-1}\right)^2 - 1\right]\cos^2\alpha = 3$ (05)

$\Rightarrow \cos^2\alpha = \frac{3(k-1)^2}{4k}$ $\because \cos\alpha = \frac{k-1}{2}\sqrt{\frac{3}{k}}$ (05)

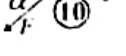
15

(12) (a)



(05)

Let $\underline{a}(M,E) = \rightarrow F$ and $\underline{a}(m,M) = f$ (05)

Then $\underline{a}(m,E) =$  (10)

Apply $\underline{F} = m\underline{a}$

for the system $\rightarrow 0 = MF + m(F - f\cos\alpha) \rightarrow (1)$

for m $\swarrow$ $mg\sin\alpha = m(f - F\cos\alpha) \rightarrow (2)$ (15)

$(1) \Rightarrow f = \frac{(m+M)F}{m\cos\alpha}$

$(2) \Rightarrow g\sin\alpha = \frac{(m+M)F}{m\cos\alpha} - F\cos\alpha$

$mg\sin\alpha\cos\alpha = (M + m - m\cos^2\alpha)F$

$F = \frac{mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}$ (10)

$f = \frac{(M+m)mg\cos\alpha\sin\alpha}{m\cos\alpha(M + m\sin^2\alpha)}$ (10)

$= \frac{(M+m)g\sin\alpha}{M + m\sin^2\alpha}$

70

$\alpha = \pi/4$ and $M = \frac{5m}{2}$ give us $F = 9/6$ and $f = \frac{7g}{6\sqrt{2}}$ (05)

For the motion (m, M), apply $\swarrow s = ut + \frac{1}{2}at^2$

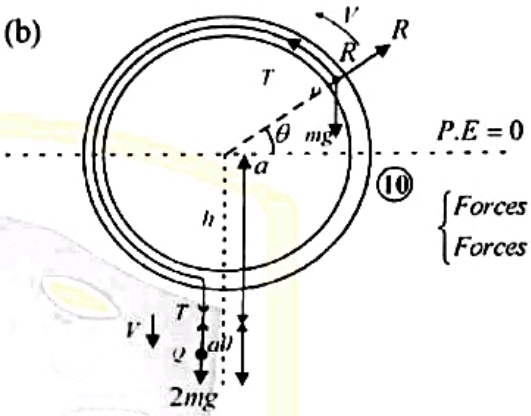
$\sqrt{2}\,a = \frac{1}{2}\cdot\frac{7g}{6\sqrt{2}}\,T^2$ (05)

$T = \sqrt{\frac{24a}{7g}}$ (05) For the motion (M,E), apply $\rightarrow v = u + at$

$v = \frac{g}{6}\sqrt{\frac{24g}{7g}} = \sqrt{\frac{2ga}{21}}$ (05)

20

(b)



(10)

{Forces at P (05)
Forces at Q (05)}

Principle of Conservation of Energy

$\frac{1}{2}mV^2 + \frac{1}{2}(2m)V^2 + mga\sin\theta - 2mg(a\theta + h) = -2mgh$ (25)

$\frac{3V^2}{2} = 2ag\theta - ga\sin\theta$

$V^2 = \frac{2}{3}ga(2\theta - \sin\theta)$ (05)

[K.E – (05)
P.E – (15)
Equation – (05)]

40

$\swarrow$ For Particle P, $\underline{F} = m\underline{a} \Rightarrow$

$mg\sin\theta - R = \frac{mv^2}{a}$ (10)

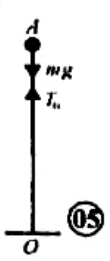
$R = mg\sin\theta - \frac{2mg}{3}(2\theta - \sin\theta)$ (05)

$= \frac{mg}{3}[3\sin\theta - 4\theta + 2\sin\theta]$ (05)

$= \frac{mg}{3}[5\sin\theta - 4\theta]$

20

(13)



(05)

For the equilibrium $\uparrow T_0 = mg$ (05)

Also $T_0 = \frac{8mg}{4a}(4a - OA)$ (05)

$\because \frac{8mg}{4a}(4a - OA) = mg$ and

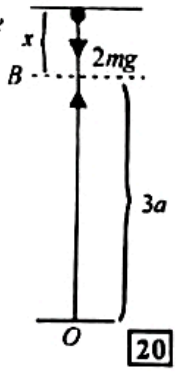
$8a - 2(OA) = a \Rightarrow OA = \frac{7a}{2}$

15

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For the motion of the composite particle

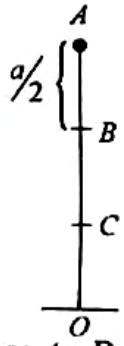


$\uparrow \underline{F} = m\underline{a}$

$T - 2mg = 2m\ddot{x}$ (10)

(05) $\frac{8mg}{4a}(a-x) - 2mg = 2m\ddot{x}$

$\Rightarrow \ddot{x} = \frac{-g}{a}x$ (05)



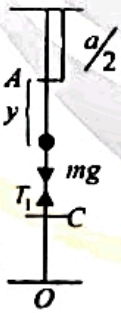
This S.H.M. has centre B, where OB=3a (05)

(05) $\because$ amplitude $= AB = a/2$ and period $= \frac{2\pi}{\sqrt{\frac{g}{a}}}$ (05)

Then $BC = a/2, OC = OB - BC = 3a - a/2 = \frac{5a}{2}$ (05)

Time taken from A to C $= \frac{1}{2} \times$ Period (05)

$= \pi\sqrt{\frac{a}{g}}$ (05) **30**



For particle P, $\downarrow \underline{F} = m\underline{a}$

$-T_1 + mg = m\ddot{y}$ (10)

(05) $\frac{-8mg}{4a}\left(y + a/2\right) + mg = m\ddot{y}$

simplification, (05)

$\ddot{y} = -\frac{2g}{a}y \rightarrow (1)$ **20**

$y = \alpha\cos\omega t + \beta \text{ Sin } \omega t \rightarrow$ (2)

$\dot{y} = -\alpha\ \omega \text{ Sin } \omega t + \beta\omega \cos\ \omega t \rightarrow$ (3) (05)

$\ddot{y} = -\omega^2(\alpha\cos\ \omega t + \beta \text{ Sin } \omega t)$ (05)

$\ddot{y} = -\omega^2 y$ (05)

comparing with (1), we have $\omega = \sqrt{\frac{2g}{a}}$ (05)

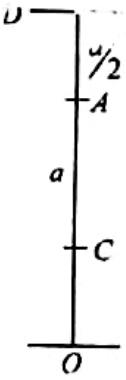
Let $t = 0$, correspond to P at C

(05) $\because v = a$, when $t = 0$, $(2) \Rightarrow \alpha = a$ (05)

(05) $\because v$ [illegible] when $t = 0$, $(3) \Rightarrow \beta = 0$ (05)

$\because y = a$ [illegible] ωt

40



Let t_1 be the time taken from C to D

$y = -a/2$ when $t = t_1$, we have $-a/2 = a\cos$ (05)

$\omega t_1 = 2\pi/3$ (05)

$t_1 = 2\pi/3\sqrt{\frac{a}{2g}} = \pi/3\sqrt{\frac{2a}{g}}$ (05)

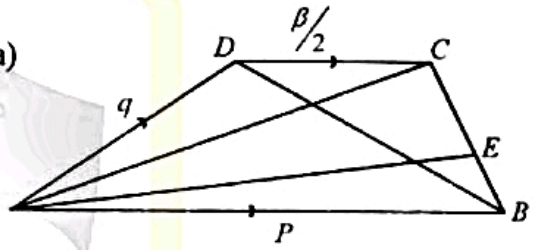
When $t = t_1$ (at point D) $\dot{y} = -a\ \omega \text{ Sin } \omega t_1 = -a\sqrt{\frac{2g}{a}}\cdot\sqrt{}$

$= -\sqrt{\frac{3ga}{2}}$ (05)

Required speed $= \sqrt{\frac{3ga}{2}}$ (05)

10

(14) (a)



$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$ (05)

$= \underline{q} + \frac{\underline{p}}{2}$ (05)

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$ (05)

$= -\underline{p} + \underline{q} + \frac{\underline{p}}{2}$ (05)

$= \underline{q} - \frac{\underline{p}}{2}$ (05)

$\because \overrightarrow{BE} = \frac{1}{3}\overrightarrow{BC} = \frac{1}{3}\left[\underline{q} - \underline{p}/2\right]$

$= \frac{\underline{q}}{3} - \frac{\underline{p}}{6}$ (05)

$\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE}$ (05)

$= \underline{p} + \frac{\underline{q}}{3} - \frac{\underline{p}}{6}$

$= \frac{5\underline{p}}{6} + \frac{\underline{q}}{3}$ **40**

$\overrightarrow{AF} = \overrightarrow{AB} + \overrightarrow{BF}$

$= \underline{p} + \lambda\overrightarrow{BD}$ (05)

$= \underline{p} + \lambda(\underline{q} - \underline{p}) = (1-\lambda)\underline{p} + \lambda\underline{q}$ (05) **10**

$\overrightarrow{AF} = k\overrightarrow{A\Sigma}$ (05)

$(1-\lambda)\underline{p} + \lambda\underline{q} = \frac{k}{6}(5\underline{p} + 2\underline{q})$ (05)

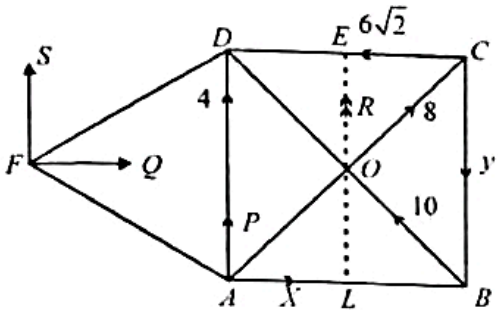
(05) $1 - \lambda = \frac{5k}{6}$

(05) $\lambda = \frac{2k}{6} \Rightarrow$ $\lambda = \frac{2}{7}$ (05)

25

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(b)



(10)

Resolving → $X - 6\sqrt{2} - \frac{10}{\sqrt{2}} + \frac{8}{\sqrt{2}} = 0$ (05)

$\Rightarrow X = 6\sqrt{2} + 5\sqrt{2} - 4\sqrt{2} = 7\sqrt{2}\ N$ (05)

Taking moments about O

$X \times a/2 - y \times a/2 + 6\sqrt{2} \times a/2 - 4 \times a/2 = 0$ (10)

$y = 7\sqrt{2} + 6\sqrt{2} - 4 = 13\sqrt{2} - 4\ N$ (05)

Resultant R $= 4 - y + 4\sqrt{2} + 5\sqrt{2}$ (05)

$= 4 - 13\sqrt{2} + 4 + 9\sqrt{2} = 4\left(2 - \sqrt{2}\right)$

$= 4k\ N \uparrow$, where $k = 2 - \sqrt{2}$ (05)

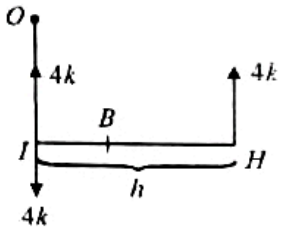
$\rightarrow Q = 0 \uparrow P + S = 4k$ (05)

$\overset{\curvearrowright}{F}$ $4k \times \sqrt{2}a = P \times \frac{\sqrt{2}\,a}{2}$ (05)

$P = 8k\ N$ and $S = -4k$ (05)

(05) *∵ The force at F = 4k N ↓ and the force along AD = 8kN ↑*

45



$\overset{\curvearrowright}{O}$ $4kh = 6ka$

$h = \frac{3a}{2}\ m$ (05)

20

Line of action of the resultant of new system: parallel to BC and through H, where H is on AB extended such that BH = a (05)

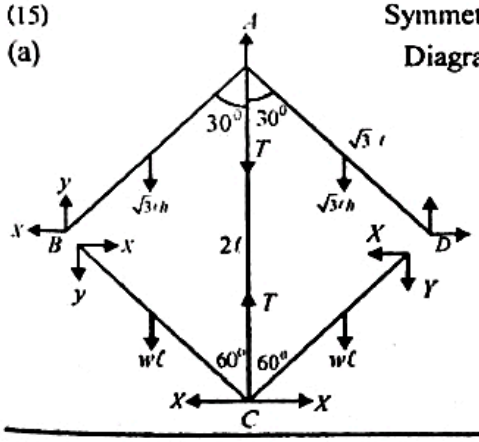
10

(15)

(a)

Symmetry (05)

Diagram (05)



For AB and BC $\overset{\curvearrowright}{A}$ $w\ell\left(1+\sqrt{3}\right)\frac{\sqrt{3}\ell}{4} - x \times 2\ell = 0$ (10)

$\Rightarrow x = \frac{\left(1+\sqrt{3}\right)\sqrt{3}}{8}w\ell$ (05)

For CD $\overset{\curvearrowright}{C}$ $x \times \frac{\ell}{2} - y \times \frac{\sqrt{3}\ell}{2} - w\ell \times \frac{\sqrt{3}\ell}{4} = 0$ (10)

$y = \frac{1}{2\sqrt{3}}\left[2x - w\ell\sqrt{3}\right] = \frac{w\ell}{2\sqrt{3}}\left[\frac{3+\sqrt{3}}{4} - \sqrt{3}\right]$ (05)

$= \frac{w\ell}{8\sqrt{3}}\left(3 - 3\sqrt{3}\right) = \frac{w\ell}{8}\left(\sqrt{3} - 3\right)$

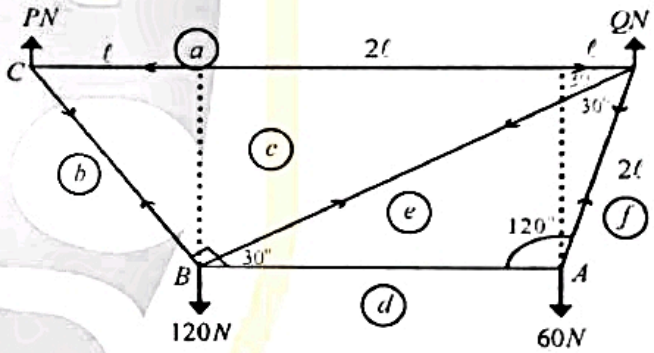
For BC and CD ↑ $T - 2w\ell - 2y = 0$ (10)

(05) $T = 2w\ell + \frac{w\ell}{4}\left(\sqrt{3} - 3\right) = \frac{w\ell}{4}\left[8 + \sqrt{3} - 3\right]$

$= \frac{w\ell}{4}\left(5 + \sqrt{3}\right)$ (05)

60

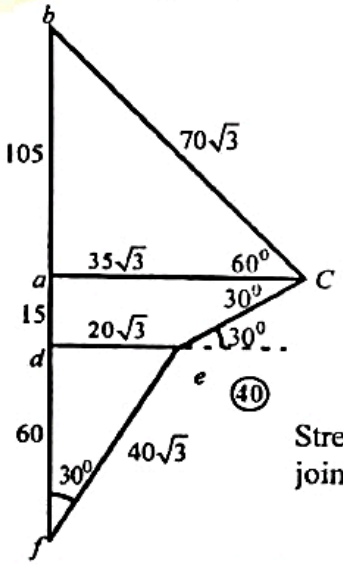
(b)



$\overset{\curvearrowright}{D}$ $P \times 4\ell = 120 \times 3\ell + 60 \times \ell$

$P = 105N$

Note: For finding Por Q without the stress diagram, allocate (10)



(40)

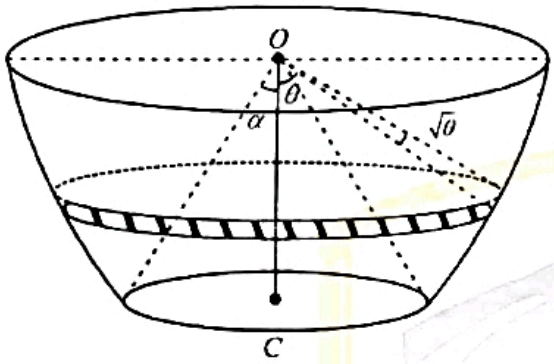
Stress diagram for joints C, A, B

40

Rod	Magnitude	Tension Thrust	
$CD(ca)$	$35\sqrt{3}$	Thrust	(10)
$BC(bc)$	$70\sqrt{3}$	Tension	(10)
$BD(ec)$	30	Tension	(10)
$AB(ed)$	$20\sqrt{3}$	Tension	(10)
$AD(fe)$	$40\sqrt{3}$	Tension	(10)

50

(16)



By symmetry the centre of gravity lies on OC (05)
Let $\bar{x}$ be the distance of the center of gravity from O.
The weight of the in finitesimal ring shown in the diagram

$$= (2\pi a \sin\theta)\, a\, \delta\theta\, \sigma g = 2\pi a^2 \sigma g\, Sin\theta\, \delta\theta$$

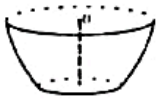
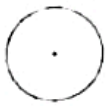
$$Then\ \bar{x} = \frac{\int_a^{\pi/2} 2\pi a^2 \sigma g\, Sin\theta . a\cos\theta\, d\theta}{\int_a^{\pi/2} 2\pi a^2 \sigma g\, Sin\theta\, d\theta} \quad (10)\ (10)$$

$$\bar{x} = a\frac{\int_a^{\pi/2} 2\, Sin\,\theta\cos\theta\, d\theta}{\int_a^{\pi/2} 2\, Sin\,\theta\, d\theta} = \frac{\left[-\frac{\cos 2\theta}{2}\right]_a^{\pi/2}}{\left[-2\cos\theta\right]_a^{\pi/2}} a = \frac{1}{4}\frac{(1+\cos 2\alpha)}{\cos\alpha} a \quad (05)\ (05)$$

$$= \frac{1}{2}\frac{\cos^2\alpha}{\cos\alpha} a = \frac{a\cos\alpha}{2} \quad (05)$$

C. G. lies at the mid point of O C

40

Object	Weight	↓ Distance from O
	$\sigma g\ 2\pi a^2 \cos\alpha$ (05)	$\frac{a\cos\alpha}{2}$ (05)
	$\sigma g\ \pi a^2\ Sin^2\alpha$ (05)	$a\cos\alpha$ (05)
	$\sigma g\ \pi a^2 \left(2\cos\alpha + Sin^2\alpha\right)$ (05)	$\bar{y}$

By Symmetry C.G. lies on OC, at a distance $\bar{y}$ from O where (05)

$$\sigma\, g\pi a^2 \left(2\cos\alpha + Sin\,\alpha\right)\bar{y} = \sigma g\ 2\pi a^2 \cos\alpha \times \frac{a\cos\alpha}{2} + \sigma g\ \pi a^2\ Sin^2\alpha \times a\cos\alpha \quad (10)$$

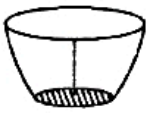
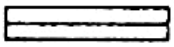
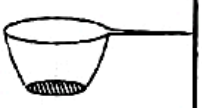
$$\bar{y} = \frac{a\cos\alpha\left(\cos\alpha + Sin^2\alpha\right)}{2\cos\alpha + Sin^2\alpha} \quad (05)$$

$$= \left(\frac{1+\cos\alpha-\cos^2\alpha}{1+2\cos\alpha-\cos^2\alpha}\right) a\cos\alpha$$

45

$$when\ \alpha = \pi/3,\ \bar{y} = \left(\frac{1+1/2-1/4}{1+1-\frac{1}{4}}\right) a/2$$

$$\bar{y} = \frac{5a}{14} \quad (05)$$

Object	Weight	Distance from AB	Distance from the O C
	W	$\frac{5a}{14}$	O (05)
	$W/4$	O (05)	$a+b/2$ (05)
	$\frac{5W}{4}$ (05)	$\bar{y}$	$\bar{x}$

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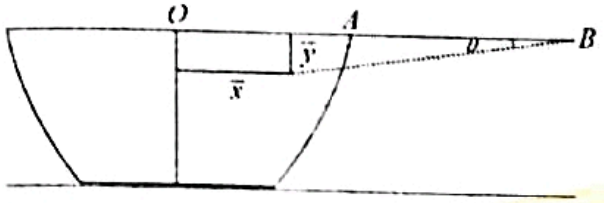
$\overrightarrow{AB}$ $\frac{5W}{4}\bar{y} = W\frac{5a}{14}$

$\bar{y} = \frac{2a}{7}$ (10)

$(\overrightarrow{OX})$ $\frac{5W}{4}\bar{x} = \frac{W}{4}\left(a+\frac{b}{2}\right)$

$\bar{x} = \frac{2a+b}{10}$ (10)

45



$\tan\theta = \frac{\bar{y}}{(a+b-\bar{x})}$ (10)

$\frac{1}{7} = \frac{\frac{2a}{7}}{a+b-\left(\frac{2a+b}{10}\right)} = \frac{20a}{7(8a+9b)}$ (05)

$8a+9b = 20a$

$4a = 3b$ (05)

20

(17) (a) $P(A|B) = \frac{P(A\cap B)}{P(B)}$ (05)

$P(A) = P\left[(A\cap B)\cup\left(A\cap B'\right)\right]$ (05) $\because [(A\cap B)\cup(A\cap B')]$

$= A\cap\left(B\cup B'\right) = A\cap\Omega = A]$

$= P(A\cap B) + P\left(A\cap B'\right)$ (05) $\because A\cap B$ & $A\cap B'$ are mutually exclusive

$= \frac{P(B)P(A\cap B)}{P(B)} + \frac{P(B')P(A\cap B')}{P(B')}$ (05)

$= P(B)P(A\setminus B) + P(B')P(A\setminus B')$

15

M - Male F-Female

OL - Highest qualification is O/L

A/L - Highest qualification is A/L

G - Graduates

P(M) = 0.8 P(F) = 0.2

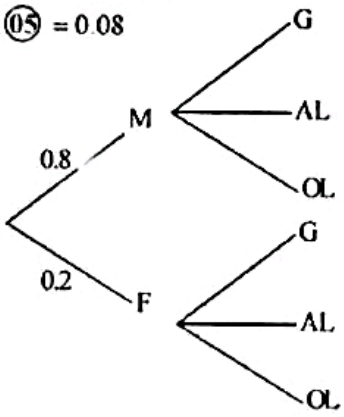
P (OL) = 0.57 P(AL) = 0.32 P(G) = 0.11

P(OL/F) = 0.4 P(AL/F) = 0.45

(i) $P(F\cap OL) = P(F)P(OL\setminus F)$ (05)

$= 0.2\times 0.4$

(05) $= 0.08$



10

(ii) $P(OL) = P(OL\cap M) + P(OL\cap F)$ (05)

$P(M\cap OL) = 0.57 - 0.08 = 0.49$ (05)

10

(iii) $P(G\setminus M) = ?$

$P(G) = P(M)P(G\setminus M) + P(F)P(G\setminus F)$ (05)

$0.11 = 0.8\times P(G\setminus M) + 0.2\times\underbrace{(1-0.4-0.45)}_{0.15}$ (05)

$P(G\setminus M) = \frac{0.11-0.03}{0.8} = \frac{0.08}{0.8} = \frac{1}{10} = 0.1$ (05)

15

(iv) $P\left(F\setminus G'\right) = \frac{P\left(F\cap G'\right)}{P\left(G'\right)} = \frac{P(F)P\left(G'\setminus F\right)}{1-P(G)}$ (05) (05)

$= \frac{0.2\times(0.40+0.45)}{1-0.11} = \frac{17}{89}$ (05)

20

(17) (b) $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_1 \quad \sigma_x^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$

(i) $\sigma_X^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_1^2 - 2x_1\bar{x} + \bar{x}^2\right)$ (05)

$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2\bar{x}\frac{1}{n}\sum_{i=1}^{n} x_1 + \frac{1}{n}\bar{x}^2\sum_{i=1}^{n}1$

$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2\bar{x}^2 + \frac{1}{n}\bar{x}2\times n$

$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2$ (05)

10

(ii) $\sum_{i=1}^{n}(\alpha x_i+\beta)^2 = \sum_{i=1}^{n}\left(\alpha^2x_i^2 + 2\alpha\beta x_i + \beta^2\right)$

$= \alpha^2\sum_{i=1}^{n} x_i^2 + 2\alpha\beta\sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\beta^2$

$= \alpha^2\left\{\underset{(05)}{n\sigma_x^2 + n\bar{x}^2}\right\} + \underset{(05)}{2\alpha\beta n\bar{x}} + n\beta^2$ a S n $\sigma_x^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$

$= n\alpha^2\sigma_x^2 + n\left\{\alpha^2\bar{x}^2 + 2\alpha\beta\bar{x} + \beta^2\right\}$

$= n\alpha^2\sigma_x^2 + n(\alpha\bar{x}+\beta)^2$ (05)

15

Aliter

$$\sum_{i=1}^{n}(\alpha x_i+\beta)^2=\sum_{i=1}^{n}\left[\alpha(x_i-\bar{x})+\alpha\bar{x}+\beta\right]^2$$

$$=\sum_{i=1}^{n}\left\{\alpha^2(x_i-\bar{x})^2+2(\alpha\bar{x}+\beta)\alpha(x_i-\bar{x})+(\alpha\bar{x}+\beta)^2\right\}$$ (05)

$$=\alpha^2\sum_{i=1}^{n}(x_i-\bar{x})^2+2\alpha(\alpha\bar{x}+\beta)\sum_{i=1}^{n}(x_i-\bar{x})+\sum_{i=1}^{n}(\alpha\bar{x}+\beta)^2$$

$$=\alpha^2 n\,\sigma_x^2+2\alpha(\alpha\bar{x}+\beta)\left(\sum_{i=1}^{n}x_i-\sum_{i=1}^{n}\bar{x}\right)+(\alpha\bar{x}+\beta)^2 n$$ (05)

$$=n\,\alpha^2\sigma_x^2+2\alpha(\alpha\bar{x}+\beta)(n\bar{x}-n\bar{x})+n(\alpha\bar{x}+\beta)^2$$

$$=n\alpha^2\sigma_x^2+n(\alpha\bar{x}+\beta)^2$$ (05)

15

$$\bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i=\frac{1}{n}\sum_{i=1}^{n}(\alpha x_i+\beta)=\alpha\frac{1}{n}\sum_{i=1}^{n}x_i+\frac{1}{n}\sum_{i=1}^{n}\beta$$ (05)

$$=\alpha\bar{x}+\beta\frac{1}{n}n$$ (05)

$$=\alpha\bar{x}+\beta$$

10

$\because y_i=\alpha x_i+\beta$, by (ii)

$$\sum_{i=1}^{n}y_i^2=n\,\alpha^2\,\sigma_x^2+n\bar{y}^2$$ (05)

$$\therefore\frac{1}{n}\sum_{i=1}^{n}y_i^2-\bar{y}^2=\alpha^2\sigma_x^2$$

$\therefore$ by (i), $\sigma_y^2=\alpha^2\sigma_x^2$ (05)

10

Let $\{x_1,x_2.....x_n\}$ be the set of marks.

given : $\bar{x}=45$

Let $y_i=\alpha x_i+\beta$ be the scaled marks.

Then $\bar{y}=50$ and $\sigma_y^2=15$

$\bar{y}=\alpha\bar{x}+\beta\Rightarrow 50=45\alpha+\beta\rightarrow(1)$ (05)

also, $x_i=60$ when $y_i=68$

$\Rightarrow 68=60\alpha+\beta\rightarrow(2)$ (05)

(i) and (ii) $\Rightarrow 15\alpha=18$

$\alpha=6/5$ (05)

$\beta=50-45\times 6/5=-4$

$\sigma_y^2=\alpha^2\,\sigma_x^2\Rightarrow 15=\frac{6}{5}\,\sigma_x^2$

$\because \sigma_x^2=\frac{15\times 5}{6}=12.5$ (05)

20

$x_i=m\Rightarrow y_i\geq m$

$\Rightarrow\frac{6}{5}x_i-4\geq m$ (05)

$\Rightarrow\frac{6}{5}m-4\geq m$

$\Rightarrow\frac{m}{5}\geq 4$

$\Rightarrow m\geq 20$ (05)

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