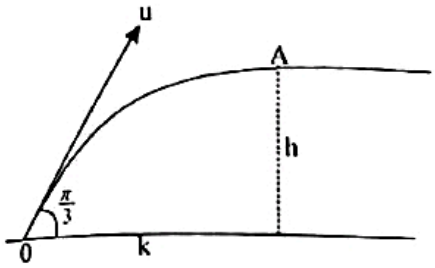
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G.C.E. (Advanced Level) Examination - August 2013
Combined Mathematics - II
Scheme of Marking

Part A

(01)



From O to A, apply $S = ut + \frac{1}{2}at^2$,

$\rightarrow k = \frac{u}{2}T \rightarrow (01)$ (05)

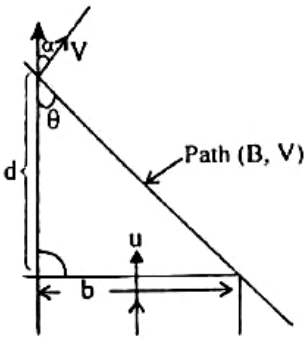
$\uparrow h = \frac{\sqrt{3}u}{2}T - \frac{1}{2}gT^2 \rightarrow (02)$ (10)

By (01) and (02) $h = \sqrt{3}k - \frac{1}{2}g\left(\frac{2k}{u}\right)^2$ (05)

$\Rightarrow \sqrt{3}k = h + \frac{1}{2}g\left(\frac{2k}{u}\right)^2$

$\Rightarrow \sqrt{3}k = h + \frac{2gk^2}{u^2}$ (05) **25**

(02)



B - Boy
V - Van

Vel (B, E) = 

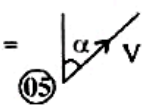
Ve (V, E) = 

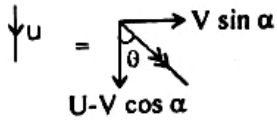
(05)

Ve (B,V) = 

(05)

Vel (B, V) = Vel (B, E) + Vel (E, V)

= 

(05) + 

$\tan\theta = \frac{V\sin\alpha}{U - V\cos\alpha}$ (05)

$\Rightarrow \frac{b}{d} = \frac{V\sin\alpha}{U - V\cos\alpha} \Rightarrow bu = (b\cos\alpha + d\sin\alpha)V$ (05)

25

(03)



For the system, applying $\underline{I} = \Delta(m\underline{v})$

$\rightarrow 0 = 5mv - (2m \times 2u - 2mu)$ (10)

$\therefore v = \frac{2u}{5}$ (05)

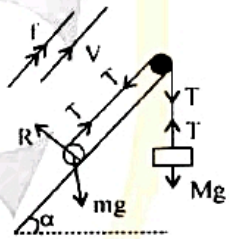
$\Delta T = \frac{1}{2}(5m)v^2 - \frac{1}{2}(2m)(2u)^2 - \frac{1}{2}(2m)u^2$ (05)

$= \frac{1}{2}(5m)\left(\frac{2u}{5}\right)^2 - 4mu^2 - mu^2$

$= \frac{2mu^2}{5} - 5mu^2 = -\frac{23}{5}mu^2$ (05)

$\therefore$ The loss of kinetic energy $= -\frac{23}{5}mu^2$ **25**

(04)





(05)

Applying $\underline{F} = m\underline{a}$

$(m) \nearrow \quad T - mg\sin\alpha = mf \rightarrow (01)$ (05)

$(M) \downarrow \quad Mg - T = Mf \rightarrow (02)$ (05)

$\therefore (01) + (02) \quad f = \frac{(M - m\sin\alpha)g}{m + M}$ (05)

Applying $v^2 = u^2 + 2as$

$(m) \nearrow \quad v^2 = 0 + 2 \times \frac{(M - m\sin\alpha)g}{M + m} \times d$

$\Rightarrow (M + m)v^2 = 2gd(M - m\sin\alpha)$ (05) **25**

Alternative Solution

For the above diagram (05)

By the Principle of conservation of energy

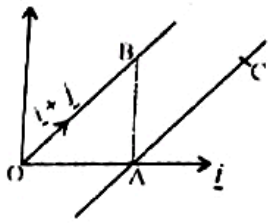
$\frac{1}{2}Mv^2 + \frac{1}{2}mv^2 = Mgd - mgd\sin\alpha$ (15)

$= (M - m\sin\alpha)gd$

$\Rightarrow (M + m)v^2 = 2gd(M - m\sin\alpha)$ (05)

25

(05)



$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$ and $\overrightarrow{AC} = \lambda\left(\underline{i} + \underline{j}\right)$, where $\lambda \in \mathbb{R}$ (05)

$$\therefore \overrightarrow{OC} = \underline{i} + \lambda\left(\underline{i} + \underline{j}\right)$$
$$= (1+\lambda)\underline{i} + \lambda\underline{j} \quad (05)$$

$BC \perp OB \Rightarrow \overrightarrow{BC} \cdot \overrightarrow{OB} = 0 \rightarrow (01)$ (05)

$$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$$
$$= -\underline{i} - \underline{j} + (1+\lambda)\underline{i} + \lambda\underline{j}$$
$$= \lambda\underline{i} + (\lambda - 1)\underline{j}$$

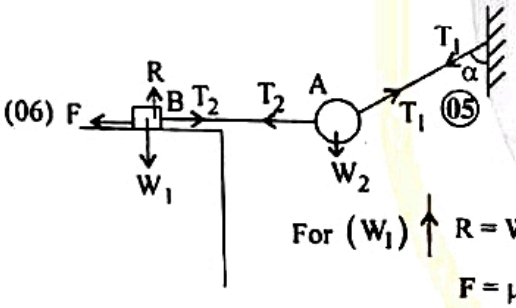
By (01), $\left|\lambda\underline{i} + (\lambda-1)\underline{j}\right| \cdot \left(\underline{i} + \underline{j}\right) = 0$ (05)

$\Rightarrow \lambda + (\lambda - 1) = 0$

$\Rightarrow \lambda = \frac{1}{2}$ (05)

25

(06)



(05)

For $(W_1) \uparrow \quad R = W_1$ (05)

$F = \mu R = \mu W_1$ (05)

for the system $\rightarrow \quad T_1 \sin\alpha = F$
for $(W_2) \uparrow \quad T_1 \cos\alpha = W_2$ } (05)

$$\Rightarrow \tan\alpha = \frac{F}{W_2}$$

$\Rightarrow W_2 \tan\alpha = \mu W_1$ (05)

25

(07) No (05)

Since A and B are mutually exclusive,

$$P(A \cup B) = P(A) + P(B)$$

$\Rightarrow P(A) + P(B) = \frac{1}{2} \rightarrow (01)$

Similarly, $P(B) + P(C) = \frac{1}{2} \rightarrow (02)$ } (05)

and $P(C) + P(A) = \frac{2}{3} \rightarrow (03)$

$(01) + (02) + (03) \Rightarrow 2\left[P(A) + P(B) + P(C)\right] = \frac{5}{3}$

$\therefore P(A) + P(B) + P(C) = \frac{5}{6} \rightarrow (04)$ (05)

Since A, B and C are exhaustive,

$$A \cup B \cup C = \Omega$$

$\therefore P(A) + P(B) + P(C) = P(\Omega) = 1 \rightarrow (05)$ (05)

By (04) and (05) it is impossible to have three such events A, B and C. (05)

25

(08) Suppose that $P(A|B) = P(A|B')$

$\Rightarrow \frac{P(A \cap B)}{P(B)}$ (05) $= \frac{P(A \cap B')}{P(B')}$ (05) $\left[P(B) \neq 0, P(B') \neq 0\right.$

$= \frac{P(A) - P(A \cap B)}{1 - P(B)}$ (05) $\left[P(B) \neq 0, \text{i.e. } 0<P(B)<1\right]$

$\Rightarrow P(A \cap B) - P(B)P(A \cap B) = P(A)P(B) - P(B)P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A)P(B)$ (05)

$\therefore$ A and B are independent (05)

25

(09) Since the mean is 4.

$2 + 3 + 6 + 2 + 1 + x + y + z = 4 \times 8$

$\therefore x + y + z = 32 - 14 = 18 \rightarrow (01)$ (05)

Since the mode is 6, at least two of the unknowns are 6. (05)

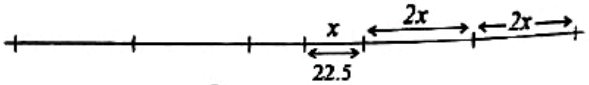
Then from (01), the remaining unknown is also is 6.

$\therefore x = y = z = 6$ (05)

$$\text{Standard Deviation} = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n}}$$
$$= \sqrt{\frac{4+1+4+4+9+3\times4}{8}} \quad (05)$$
$$= \sqrt{\frac{34}{8}}$$
$$= \frac{\sqrt{17}}{2} \quad (05)$$

25

(10) Let the width of a class interval be $2x$.



$5x = 40 - 22.5$ (05)

$x = 3.5$ (05)

The model class is the third class interval, which is (19, 26) (05)

$\therefore$ The mode $= 19 + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)C$, (05)

Where $= (\Delta_1 = 27 - 19 = 8, \ \Delta_2 = 27 - 15 = 12, \ C = 7)$

$= 19 + \frac{8}{20} \times 7$

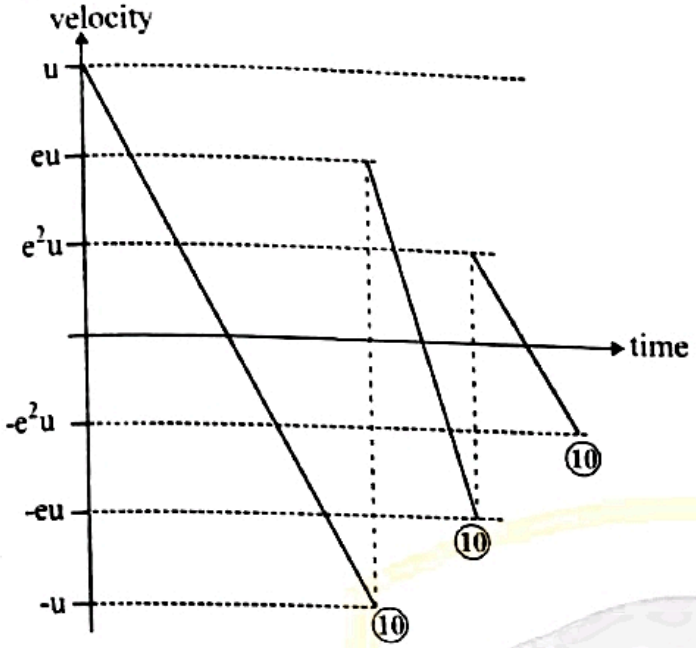
$= 21.8$ (05)

25

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(11) (i)



(10) (10) (10)

(ii) Let T_1 be the time until the first collision.

$$\frac{T_1}{2} = \frac{u}{g} \Rightarrow T_1 = \frac{2u}{g}$$ (05)

Time from 1st collision to 2nd collision

$$T_2 = \frac{2eu}{g}$$ (05)

Time from 2nd to 3rd collision

$$T_3 = \frac{2e^2u}{g}$$ (05)

Total Time till 3rd collision $= T_1 + T_2 + T_3$

$$= \frac{2u}{g}\left(1 + e + e^2\right)$$ (05)

(iii) Total time until the particle come to rest.

$= T_1 + T_2 + T_3 + \ldots\ldots$ (05)

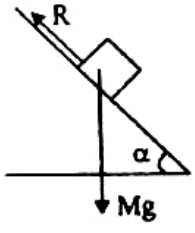
$$= \frac{2u}{g}\left(1 + e + e^2 + e^3 + \ldots\ldots\right)$$ (05)

$$= \frac{2u}{g}\sum_{r=0}^{\infty} e^r = \frac{2u}{g}\ \frac{1}{1-e}$$ (05) (05)

$$= \frac{2u}{g(1-e)}$$

70

(b) Mass of the train M = 300,000 kg (05)



$$\sin\alpha = \frac{1}{98}$$

Train moves downwards with constant velocity

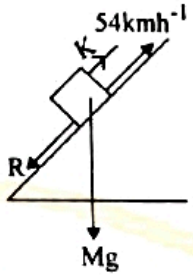
Apply $\nearrow$ $\underline{F} = m\underline{a}$

$$mg\sin\alpha - R = 0$$

$$300{,}000 \times 9.8 \times \frac{1}{98} - R = 0$$ (10)

$$R = 30{,}000N$$ (05)

For the upward movement



$$V = 54kmh^{-1} = \frac{54 \times 1000}{60 \times 60}$$

$$V = 15ms^{-1}$$ (05)

Let k be the tractive force

$\nearrow$ $\underline{F} = m\underline{a}$

$$K - R - Mg\sin\alpha = 0$$ (10)

Applying P = FV

Power P = (R +Mg sinα)V

(30,000 + 30,000) x 15 = 900,000

= 900kw (05)

40

Velocity $V = 18kmh^{-1} = \frac{18000}{60 \times 60}ms^{-1}$

$= 5ms^{-1}$ (05)

Tractive Force $= \frac{P}{V}$

$$= \frac{900{,}000}{5}N$$ (10)

$= 180{,}000$ (05)

$\rightarrow$ $\underline{F} = m\underline{a}$

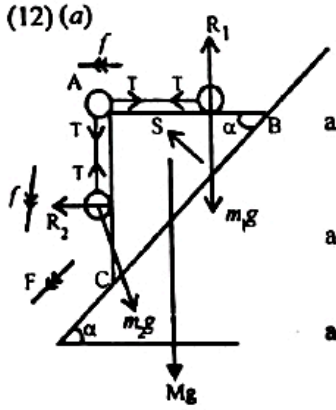
180,000 - 30,000=300,000 x a (Where a is the acceleration) (10)

$$Acceleration = \frac{150{,}000}{300{,}000}$$ (05)

$$a = \frac{1}{2}ms^{-2}$$ (05)

40

(12) (a)



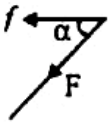
For Forces (10)

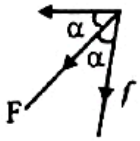
acc (W, E) = α, F

acc(P, W) = f (10)

acc (Q, W) = 2α, f

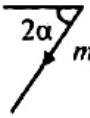
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acc P, E = 

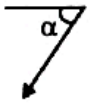
acc Q, E = 

Apply $\underline{F} = m\underline{a}$

For P ← $T = m_1(f + F\cos\alpha) \rightarrow (01)$ (10)

For Q 

$m_2 g \sin 2\alpha - T = m_2(f + F\cos\alpha) \rightarrow (02)$ (10)

For the system 

$$(M + m_1 + m_2) g \sin\alpha = \begin{pmatrix} MF + m_1(f + F\cos\alpha) \\ + m_2(f + F\cos\alpha) \end{pmatrix} \rightarrow (03) \quad (10)$$

50

$m_1 = m_2$

(01) + (02) ⇨ $m_1 g \sin 2\alpha = m_1 2(f + F\cos\alpha)$

$f + F\cos\alpha = g\sin\alpha\cos\alpha \rightarrow (04)$ (05)

(03) ⇨ $(M + 2m_1) g \sin\alpha$

$= MF + 2m_1(F + f\cos\alpha)$ (05)

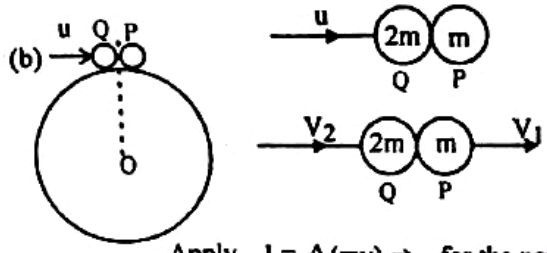
$= (M + 2m_1) F + 2m_1 f\cos\alpha$

$$g\sin\alpha = F + \frac{2m_1}{M + 2m_1} f\cos\alpha \rightarrow (05) \quad (05)$$

$$(04) + (05)\cos\alpha \Rightarrow f = \frac{2m_1}{M + 2m_1} f\cos^2\alpha \quad (05)$$

$$Mf + 2m_1 \sin^2\alpha f = 0 \Rightarrow f = 0 \quad (05)$$

Now (05) ⇨ $F = g\sin\alpha$ 25



Apply $\underline{I} = \Delta(mv) \rightarrow$ for the particles

$O = 2mv_2 + mv_1 - 2mu$ (10)

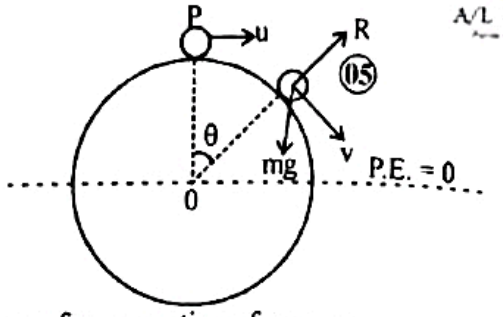
$2v_2 + v_1 = 2u \rightarrow (01)$

Newton's law of Resititution.

$v_1 - v_2 = \frac{1}{2}u$ (10)

$2v_1 - 2v_2 = u \rightarrow (02)$

(01) + (02) ⇨ $v_1 = u$ (05) 25



Using law of conservation of energy.

$$\frac{1}{2}mv^2 + mga\cos\theta = \frac{1}{2}mu^2 + mga \quad (15)$$

$v^2 = u^2 + 2ga(1 - \cos\theta)$ (05)



Apply $\underline{F} = m\underline{a}$

$$R - mg\cos\theta = \frac{-mv^2}{a} \quad (10)$$

$$R = mg\cos\theta - \frac{m}{a}\left[u^2 + 2ga(1 - \cos\theta)\right]$$

$$= \frac{m}{a}\left[ga\cos\theta - u^2 - 2ga(1 - \cos\theta)\right]$$

$$= \frac{m}{a}\left[3ga\cos\theta - 2ga - u^2\right] \quad (10)$$

$$= \frac{m}{a}\left[ga(3 - 2\cos\theta) - u^2\right]$$

$u = \sqrt{ga}$ and $\theta = 0 \Rightarrow R = 0$ (10)

∴ P leaves the surface just after the collision. 50

(13)



Let λ be the coefficient of elasticity.

$$T_0 = \frac{\lambda\left(\frac{\ell}{3}\right)}{\ell} \quad (05)$$

$T_0 = mg$

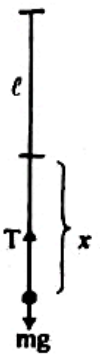
$\frac{\lambda\ell}{3\ell} = mg$

• $\lambda = 3mg$ (05) 10

$v^2 = u^2 + 2fs$

$v^2 = 2g\left(\frac{\ell}{2}\right)$

$v = \sqrt{g\ell}$ (05) 05



$$T = \frac{\lambda x}{\ell} = \frac{3mgx}{\ell} \quad (05)$$

$\underline{F} = m\underline{a}$

$mg - T = m\ddot{x}$ (10)

$$mg - \frac{3mgx}{\ell} = m\ddot{x}$$

$$\ddot{x} + \frac{3g\left(x - \frac{\ell}{3}\right)}{\ell} = 0; \qquad x \geq 0 \quad (05) \quad 20$$

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$x = \frac{l}{3} + \alpha\cos\omega t + \beta\sin\omega t$

$x = 0$ *when* $t = 0$. (05)

$0 = \frac{\ell}{3} + \alpha$ *gives us* $\alpha = \frac{-\ell}{3}$ (05)

$\dot{x} = -\alpha\omega\ \sin\omega t + \beta\omega\ \cos\omega t$ (05)

$\dot{x} = \sqrt{g\ell}$ *when* $t = 0$;

$\sqrt{g\ell} = \beta\omega$ (05)

$\ddot{x} = -\alpha\omega^2\cos\omega\ t - \beta\omega^2\sin\omega t$ (05)

$$\frac{-3g\left(x-\frac{\ell}{3}\right)}{\ell} = -\alpha\omega^2\cos\omega t - \beta\omega^2\sin\omega t$$ (05)

$$\frac{-3g}{\ell}(\alpha\cos\omega t + \beta\sin\omega t) = -\omega^2(\alpha\cos\omega t + \beta\sin\omega t)$$ (05)

$\therefore \omega^2 = \frac{3g}{\ell}$

So $\omega = \sqrt{\frac{3g}{\ell}}$ (05) *and* $\beta = \sqrt{g\ell}\sqrt{\frac{\ell}{3g}} = \frac{\ell}{\sqrt{3}}$ (05) **45**

Now $x = \frac{\ell}{3} + \alpha\cos\omega t + \beta\sin\omega t$ *gives us*

$x - \frac{\ell}{3} = \alpha\cos\omega t + \beta\sin\omega t$ (05)

Centre of the SHM is given by $x - \frac{\ell}{3} = 0$ (05)

$\therefore x = \frac{\ell}{3}$

Thus the centre C is a depth of $\frac{\ell}{3}$ below A. **10**

Amplitude = BC

$\dot{x} = 0$ When $t = t_1$ gives $-a\omega\sin\omega t_1 + \beta\omega\cos\omega t_1$ (05)

(05) $\Rightarrow \frac{\ell}{3}\sin\omega t_1 = -\frac{1}{\sqrt{3}}\cos\omega t_1$

$\tan\omega t_1 = -\sqrt{3}$

$\omega t_1 = \frac{2\pi}{3}$ (05)

Find x when $t = t_1$

$x = \frac{\ell}{3} - \frac{\ell}{3}\cos\omega t_1 + \frac{1}{\sqrt{3}}\sin\omega t_1$

$= \frac{\ell}{3} - \frac{\ell}{3}\cos\frac{2\pi}{3} + \frac{1}{\sqrt{3}}\sin\frac{2\pi}{3}$ (05)

$= \frac{\ell}{3} + \frac{\ell}{6} + \frac{\ell}{2}$

$= \ell$ (05)

$\therefore BC = \ell - \frac{\ell}{3} = \frac{2\ell}{3}$ (05) **30**

Let t_0 be the time taken to reach A for the first time

Applying $S = ut + \frac{1}{2}ft^2$ ↓

$u = 0,\ a = g,\ s = \frac{\ell}{2},\ t = t_0$

$\frac{\ell}{2} = \frac{1}{2}gt_0^2$ (05)

$t_0 = \sqrt{\frac{\ell}{g}}$ (05)

Total time to reach B $= \sqrt{\frac{\ell}{g}} + \frac{2\pi}{3\omega}$ (05)

$= \sqrt{\frac{\ell}{g}} + \frac{2\pi}{3}\sqrt{\frac{\ell}{3g}}$

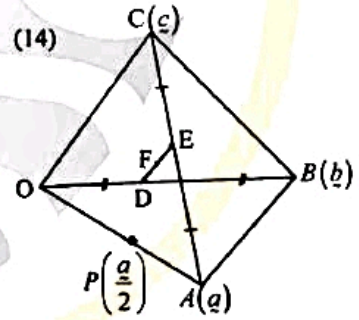
$= \sqrt{\frac{\ell}{g}}\left(1 + \frac{2\pi}{3\sqrt{3}}\right)$ (05) **2**

The tension when the particle is at B $= \frac{3mg}{\ell}(AB)$ (05)

$= \frac{3mg}{\ell}(\ell)$

$= 3mg$ (05) **10**

(14)



$\overrightarrow{OD} = \frac{1}{2}\overrightarrow{OB} = \frac{1}{2}\underline{b}$ (05)

$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$ (05)

$= \underline{a} + \frac{1}{2}(\overrightarrow{AC})$

$= \underline{a} + \frac{1}{2}(\underline{c} - \underline{a})$

$= \frac{\underline{a} + \underline{c}}{2}$ (05)

$\overrightarrow{OF} = \overrightarrow{OD} + \overrightarrow{DF}$ (05)

$= \frac{\underline{b}}{2} + \frac{1}{2}\overrightarrow{DE}$

$= \frac{\underline{b}}{2} + \frac{1}{2}(\underline{e} - \underline{d})$

$= \frac{\underline{b}}{2} + \frac{1}{2}\left[\frac{\underline{a} + \underline{c}}{2} - \frac{\underline{b}}{2}\right]$

$= \frac{\underline{a} + \underline{b} + \underline{c}}{4}$ (05) **25**

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$\vec{OQ} = \vec{OB} + \frac{1}{2}\vec{BC} = \underline{b} + \frac{1}{2}(\underline{c} - \underline{b}) = \frac{1}{2}(\underline{b} + \underline{c})$ (05)

(05)

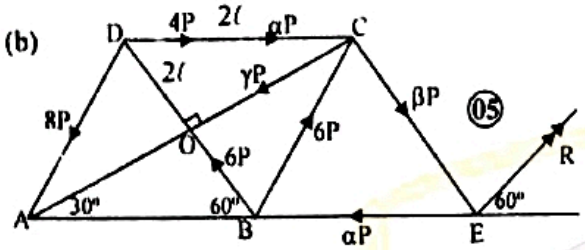
$\vec{PF} = \vec{PO} + \vec{OF} = \frac{-\underline{a}}{2} + \frac{\underline{a} + \underline{b} + \underline{c}}{4} = \frac{-\underline{a} + \underline{b} + \underline{c}}{4}$ (05)

$\vec{FQ} = \vec{FO} + \vec{OQ} = \frac{-\underline{a} + \underline{b} + \underline{c}}{4} + \frac{1}{2}(\underline{b} + \underline{c}) = \frac{-\underline{a} + \underline{b} + \underline{c}}{4}$ (05)

$\vec{PF} = \vec{FQ}$ (05)

$\Rightarrow$ P, F and Q are colinear (05)
Also PF : FQ = 1 : 1 (05) **35**

(b)



(05)

$\vec{OC}$ $X = 2P\cos 30 + 6P\cos 30 + 4P\cos 30 - 8P\cos 30$ (05)

$= 2P \cdot \frac{\sqrt{3}}{2}(1 - 3 + 2 + 4)$

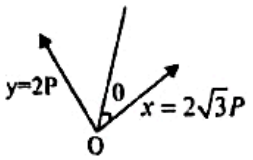
$= 2\sqrt{3}P$ (05)

$\vec{OD}$ $Y = 6P - 2P\cos 60 + 6P\cos 60 - 4P\cos 60 - 8P\cos 60$ (05)

$= 6P - 2P - \frac{1}{2}(1 - 3 + 2 + 4)$

$= 6P - 4P$

$= 2P$ (05)



$\tan\theta = \frac{y}{x} = \frac{2P}{2\sqrt{3}P} = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30^0$ (05)

$\therefore$ The resultant is Parellet to BC (05) **35**

O Taking moments

$M_0 = 2P \cdot \ell\cos 30 + 6P \cdot \ell\cos 30 - 4P\ell\cos 30 + 8P\ell\cos 30$

$= 2P\ell\frac{\sqrt{3}}{2}(1 + 3 - 2 + 4)$

$= 6\sqrt{3}P\ell$ (10) **10**

$R^2 = x^2 + y^2 = \left(2\sqrt{3}P\right)^2 + (2P)^2$

$= 12P^2 + 4P^2$

$= 16P^2$

$R = 4P$ (05)

O Taking moments.

$6\sqrt{3}P\ell = 4P(\ell\cos 30 + x\cos 30)$ where $x = BE$ (05)

$6\sqrt{3}P\ell = 4P\frac{\sqrt{3}}{2}(\ell + x)$

$3\ell = \ell + x$

$x = 2\ell$ (05) **15**

For the equilibrium,

Re solving along $\overrightarrow{OC}$

$2\sqrt{3}P - \gamma P = 0$

$\gamma = 2\sqrt{3}$ (05)

Re solving along $\overrightarrow{OD}$

$2P - \beta P = 0$

$\beta = 2$ (05)

E Taking moments

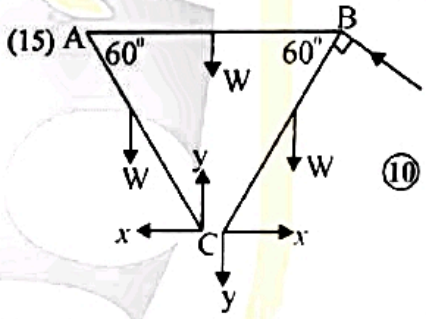
$\alpha P \cdot 2\ell\cos 30 - \gamma P \cdot 2\ell = 0$

$\alpha \cdot \sqrt{3} = \gamma \cdot 2$ (10)

$\alpha = 2 \cdot \frac{2.2\sqrt{3}}{\sqrt{3}} = 4$

For α, β, γ (10) **30**

(15)



(10)

For the system, taking moments about A

$W\left[a\cos 60 + a + (2a - a\cos 60)\right] = P \cdot 2a\cos 60$ (15)

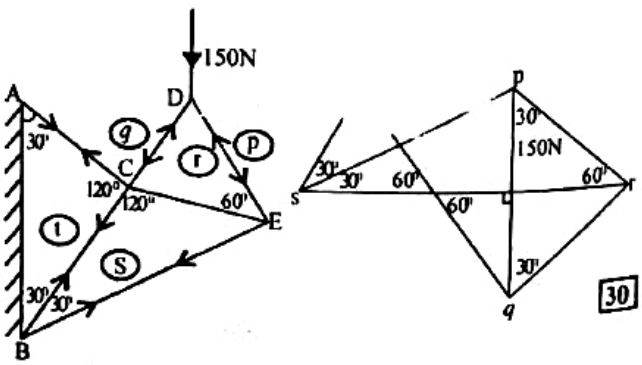
$W\left(\frac{a}{2} + a + 2a - \frac{a}{2}\right) = 2a \cdot \frac{1}{2}P$ (10)

$P = 3W$ (05) **30**

A $ya - xa\sqrt{3} = \frac{W \times a}{2}$ (10)

B $ya + x \cdot a\sqrt{3} = -\frac{Wa}{2}$ (10)

$\Rightarrow$ $y = 0$ (05) $\therefore x = \frac{-W}{2\sqrt{3}}$ (05) **30**



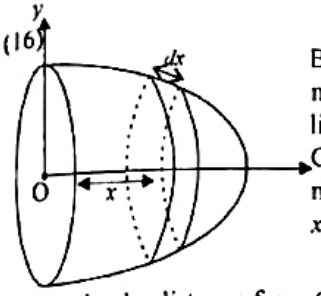
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Rod	Thurst	Tension	Magnitude
AC	-	✓ (05)	$100\sqrt{3}$ N (05)
CD	✓ (05)	-	$50\sqrt{3}$ N (05)
DE	✓ (05)	-	$50\sqrt{3}$ N (05)
CE	-	✓ (05)	$100\sqrt{3}$ N (05)
BC	-	✓ (05)	$50\sqrt{3}$ N (05)
BE	✓ (05)	-	150 N (05)

60

(16)



By sysmmetry the centre of mass of the solid hemisphere lies on the axis of symmetry. Consider a disc of infinitesimal thickness δx at a distance x from the centre O. (05)

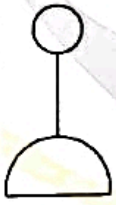
Let $\bar{x}$ be the distance from O to the centre of mass.
γ be the density of the material.
Mass of the infinitesimal disc with thickness
$\delta x \approx \pi(a^2-x^2)\delta x\gamma$ (05)

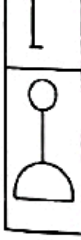
$$\bar{x} = \frac{\int_0^a \pi\left(a^2-x^2\right)x\gamma\,dx}{\int_0^a \pi\left(a^2-x^2\right)\gamma\,dx} \text{ (10)} = \frac{\left[\frac{a^2x^2}{2} - \frac{x^4}{4}\right]_0^a}{\frac{2}{3}a^3} \text{ (05)}$$

$$= \frac{3}{8}a \text{ (05)}$$

40

By symmetry, the centre of mass of the composite object lies on the axis of symmetry. (05)



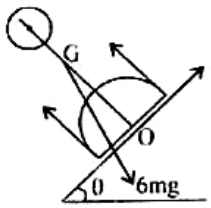
Object	mass	distance to CM from
	$m = \frac{4}{3}\pi r^3\gamma$ (05)	$3r+2\ell$ (05)
	$\frac{2}{3}\pi(2r)^3\gamma = \frac{2}{3}\pi 8r^3\gamma$ (05) $= 4m$	$\frac{3}{8}2r = \frac{3}{4}r$ (05)
	m (05)	$2r+\ell$ (05)
	$6m$ (05)	$\bar{y}$

$$6m\bar{y} = m(3r+2\ell) + 4m\left(\frac{3r}{4}\right) + m(2r+\ell) \text{ (10)}$$

$$6\bar{y} = 8r+3\ell$$

$$\bar{y} = \frac{1}{6}(8r+3\ell) \text{ (05)}$$

55



$$OG = \frac{1}{6}(8r+3\ell)$$

Composite body will not topple, if OH < 2r (10)
i.e. OG tan θ < 2r (05)

$$\tan\theta < \frac{2r\times 6}{8r+3\ell} = \frac{12r}{8r+3\ell} \text{ (05)}$$

20

$$\ell = \frac{4r}{3}, \quad \theta = \frac{\pi}{6}$$

$$\tan\theta = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} \text{ (05)}$$

$$\frac{12r}{8r+3\ell} = \frac{12r}{8r+3\cdot\frac{4r}{3}} = \frac{12r}{12r} = 1 \text{ (05)}$$

$$\tan\theta = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} < 1 \text{ (05)}$$

$$\tan\theta < \frac{12r}{8r+3\ell}$$

$\therefore$ composite body will not topple. (05)

20

Resolving $\quad R - 6mg\cos 30 = 0$ (10)

$\therefore \quad R = 3\sqrt{3}mg$ (05)

15

(17) (a) Let S, M, N and X be defined as follows: (10)
S - Participated in sports activities
M - Paricipated in musical activities
N - Not participated in Sports or Musical activities
X - Passed the examination

Then $P(S) = \frac{50}{100}, P(M) = \frac{30}{100}, P(N) = \frac{20}{100}, P(X) = \frac{48}{100}$ (05)

$P(X|S) = 0.60, P(X|N) = 0.3$ (05) (05)

(i) From the theroem of total probabilities. (10)

$$P(X) = P(S)P(X|S) + P(M)P(X|M) + P(N)P(X|N)$$

$$\frac{48}{100} = \frac{50}{100}\times 0.6 + \frac{30}{100}\times P(X|M) + \frac{20}{100}\times 0.3 \text{ (05)}$$

$$P(X|M) = \frac{48-30-6}{30} = \frac{12}{30} = \frac{2}{5} \text{ (10)}$$

(ii) By Bayes Theroem

$$P(S|X) = \frac{P(S)P(X|S)}{P(X)} \text{ (10)} = \frac{50\times 0.6}{48} = \frac{30}{48} = \frac{5}{8} \text{ (05)}$$

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(b) The first quartile = Value at the $= \frac{50^{th}}{4}$ observation

Value at the 12.5^{th} observation.

The class interval where the first quartile lies is (0.82 - 0.83) (10)

$\therefore$ The first quartile $= 0.82 + \frac{12.5 - 4}{9} \times 0.01$

$= 0.82 + 0.009$ (10)

$= 0.829$ (05)

25

Mean of the diameters of 50 balls = 0.835
Mean of the diameters of 100 balls = 0.835
Mean of the diameters of 150 balls = 0.835 } (10)

10

Variance of the diameters of 50 balls $S_1^2 = 0.01^2 = 0.0001$

Variance of the diameters of 100 balls,

$$S_2^2 = 0.015^2 = 0.000225$$

Variance of the diameters of the combined set of 150 balls.

$$S^2 = \frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2}$$ (10)

$$= \frac{50 \times 0.0001 + 100 \times 0.000225}{150}$$

$$= \frac{0.0050 + 0.0225}{150}$$ (05)

$= 0.00018$

15

Let y be the true value of the diameter of 100 balls. Then (05)

$y = x + 0.015$ where x's are the original values. (05)

(05)

$\therefore \bar{y} = \bar{x} + 0.015$ and true s.d = original s.d.

The true mean = 0.835 + 0.015 = 0.85 (05)

and true standard deviaton = 0.015 (05)

25