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G.C.E. (Advanced Level) Examination - August 2012
10 - Combined Mathematics - II
Provisional Scheme of Marking

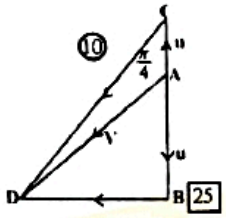
Part A

01. Let v be the true speed of the wind and let θ be the true direction of the wind.

$DB = 2u \tan\frac{\pi}{4} = 2u$ (5)

$V = \sqrt{(2u)^2 + u^2} = \sqrt{5}u$ (5)

$\tan\theta = \frac{2u}{u} = 2 \Rightarrow \theta = \tan^{-1}(2)$ (5)

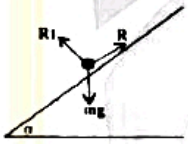


(10) 25

02. Applying F = ma for the motion of the Particle along the Slope we have.

$mg\sin\alpha - R = mf$ where f is the acceleration of the particle (5)

$f = g\sin\alpha - \frac{R}{m}$



Applying $S = ut + \frac{1}{2}at^2$ for the motion of the particle along the slope we have

$d = \frac{1}{2}\left(g\sin\alpha - \frac{R}{m}\right)^2 \Rightarrow R = m(g\sin\alpha - 2d)$ (5) (5)

15

$\therefore f = g\sin\alpha - (g\sin\alpha - 2d) = 2d$ (5)

Applying $v = u + at$ for the motion of the Particle along the slope we have

$V = 2d.1 \Rightarrow V = 2d$ (5)

10

03. Ler v be the velocity of the particle just before it strikes the plane Then $V = \sqrt{2gh}$. (5)

The velocity of the particle just after it strikes the plane is (5) ev, where e is the coefficient of restitution between the particle and the plane.

The loss of kinetic energy due to the impact

$= \frac{1}{2}mv^2 - \frac{1}{2}m(ev)^2$

$= \frac{1}{2}mv^2(1 - e^2) = \frac{mgh}{4}$

$\Rightarrow mgh(1 - e^2) = \frac{mgh}{4} \Rightarrow e = \frac{\sqrt{3}}{2}$ (5)

15

Applying $V^2 = u^2 + 2as$ for the upward motion of the Particle we have

$0 = (ev)^2 - 2gS$ (5) where S is the height to which the particle rebounds.

$S = \frac{(ev)^2}{2g} = \frac{\frac{3gh}{2}}{2g} = \frac{3}{4}h$ (5)

10

04. By the principle of Conservation of Energy we have

$\frac{1}{2}m(2g\ell) - mg\ell = \frac{1}{2}mv^2 - mg\ell\cos\frac{\pi}{3}$ (10)

$\therefore V = \sqrt{g\ell}$ (5)

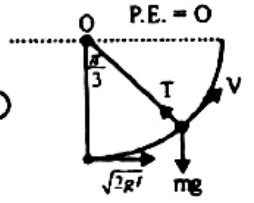
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Applying $F = ma$ for the motion of the particle along P_0 We have

$T - mg\cos\frac{\pi}{3} = \frac{mv^2}{\ell} = mg$ (5)

$T - \frac{1}{2}mg = mg \quad \because V = \sqrt{g\ell}$ (5)

$T = \frac{3}{2}mg$



10

05. $|a| = \sqrt{1^2 + (\sqrt{3})^2} = 2$ and $|b| = \sqrt{3}$ (5)

$\left(\underline{a}.\underline{b} = (\underline{i} + \sqrt{3}\underline{i})(x\underline{i} + y\underline{i}) = x + \sqrt{3}y \right.$

$\left. = |\underline{a}||\underline{b}|\cos\frac{\pi}{3} = 2\sqrt{3}.\left(\frac{1}{2}\right) = \sqrt{3}\right)$ (5)

$\Rightarrow x = \sqrt{3}(1 - y)$ (5)

$\left(x^2 + y^2 = 3(1 - y)^2 + y^2 = 3 \Rightarrow 4y^2 - 6y = 0\right.$

$\left. \Rightarrow y = 0 \text{ or } \frac{3}{2} \quad (5)\right)$

when $y = 0$ we have $x = \sqrt{3}$ ft

when $y = \frac{3}{2}$ we have $x = \sqrt{3}\left(1 - \frac{3}{2}\right) = -\frac{\sqrt{3}}{2}$

Thus $\underline{b} = -\frac{\sqrt{3}}{2}\underline{i} + \frac{3}{2}\underline{j}$ (5)

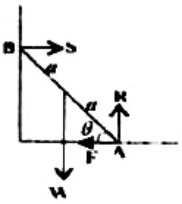
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06. Resolving forces horizontally we have $F = S$ (5)

Resolving forces vertically we have $R = W$ (5)

Taking moments about B We have



$wa\cos\theta = S.2a\sin\theta \Rightarrow S = \frac{1}{2} W\cot\theta$ (5)

$\frac{F}{R} = \frac{1}{2}\cot\theta \le \mu = \sqrt{\frac{3}{2}} \Rightarrow \tan\theta \ge \frac{1}{\sqrt{6}}$ (5)

The required angle is $\tan^{-1}\left(\frac{1}{\sqrt{6}}\right)$ (5)

25

07. $A \cup B \cup C = \Omega$ Since A, B and C are exhaustive.

$P(A \cup B \cup C) = P(\Omega) = 1$ (5)

But $P(A \cup B \cup C) = P(A) + P(B) + P(C)$ since A, B and C are mutually exclusive (5)

$= 2P + P^2 + 4P - 1 = 1$ (5)

i.e $P^2 + 6P - 2 = 0 \Rightarrow P = -3 \pm \sqrt{11}$ (5)

$P = -3 + \sqrt{11}$ since P cannot be negative (5)

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08. $P\{A \cap (B \cup C)\} = P\{(A \cap B) \cup (A \cap C)\}$ (5)

$= P(A \cap B) + P(A \cap C) - P\{(A \cap B) \cap (A \cap C)\}$ (5)

$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

$= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C)$

Since A, B, and C are independent events. (5)

$= P(A)\{P(B) + P(C) - P(B)P(C)\}$ (5)

$= P(A)\{P(B) + P(C) - P(B \cap C)\}$

$= P(A)\{P(B \cup C)\}$ (5)

Thus A and $(B \cup C)$ are independent events. 25

09. The correct total sum $= 100 \times 30 - 40 + 30 = 2990$ (5)

the correct mean $= \frac{2990}{100} = 29.9$ (5) 10

The correct total Sum of Squares

$= 100(4.1^2 + 30^2) - 40^2 + 30^2$

$= 100(16.81 + 900) - 700$

$= 91681 - 700 = 90981$

∴ the correct variance $= \frac{90981}{100} - 29.9^2$

$= 909.81 - 894.01$

$= 15.8$ (5)

The correct standard deviation $= \sqrt{15.8}$

$= 3.975$ or (5)

$= 3.97$ (5) 15

10. Then We have $63 = 85a + b \rightarrow (01)$ (5)

$\bar{y} = a\bar{x} + b \Rightarrow 31 = 45a + b \rightarrow (02)$ (5)

$(01) - (02) \Rightarrow 40a = 32 \Rightarrow a = 08$

From (01) We have $31 = 45 \times 0.8 + b \Rightarrow b = -5$

Thus, the linear transformation is $y = 0.8x - 5$ (5) 15

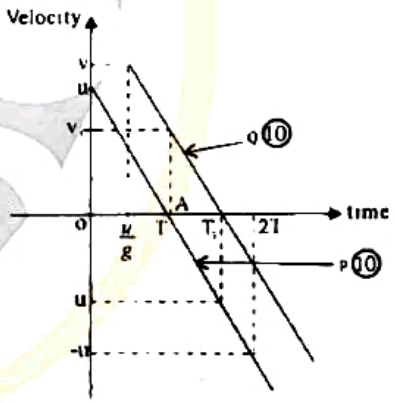
S.d of $y = a$ S.d of $x \Rightarrow 5 = \frac{4}{5} \times x$ (5)

S.d of $x = 6.25$ (5) 10

Combined Mathematics - II
Part B

11. (a) (i) Let T be the time taken by the Particle P to reach the point A

Then $T = \frac{u}{g}$ (5) from the graph Also from the graph we have $OA = \frac{1}{2} uT = \frac{u^2}{2g}$ (5) 10



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(ii) Let v_1 be the velocity of the particle Q at the point A. Then from the graph for Q we have.

$\frac{v - v_1}{T - \frac{u}{2g}} = g \Rightarrow v_1 = v - g\left(T - \frac{u}{2g}\right) = v - \frac{u}{2} \rightarrow (01)$ (5) (5)

Also from the graphs we have

$\left(\frac{V + V_1}{2}\right)\left(T - \frac{u}{2g}\right) = \frac{u^2}{2g} \Rightarrow V + V_1 = 2u \rightarrow (02)$ (5) (5)

From (1) and (2) We have $V = \frac{5u}{4}$ (5) and $V_1 = \frac{3u}{4}$ (5).

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(i) Let T_1 be the time taken by the Particle Q to reach its highest Point and let u_1 be the velocity of the Particle P at this time instant. Then from the graph for Q we have.

$$T_1 - \frac{u}{2g} = \frac{v}{g} \Rightarrow T_1 = \frac{5u}{4g} + \frac{u}{2g} = \frac{7u}{4g}$$ (5) and

$$\frac{u_1}{T_1 - T} = -g \Rightarrow u_1 = \frac{-3u}{4}$$ (5)

Let h be the height of the particle P from the Point O, When the Particle Q is at its highest point.

Then from the graph we have.

$$h = \frac{1}{2}\left(u + \frac{3u}{4}\right)\left(2T - T_1\right) = \left(\frac{7u}{8}\right)\left(\frac{2u}{g} - \frac{7u}{4g}\right) = \frac{7u^2}{32g}$$
(5) (5)

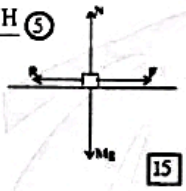
[20]

(b) At the maximum speed there is no acceleration, So the forces acting on the car are in eqilibrium.

Thus we have $F - R = 0$

$F\upsilon = P \Rightarrow Fv = 10^3 H \Rightarrow F = \frac{10^3 H}{V}$ (5)

Thus $R = \frac{10^3 H}{V}$ (5)



[15]

(i) Let a be the acceleration of the car when it is moving directly up the road.

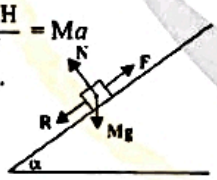
$$F = \frac{10^3 H}{\frac{v}{3}} = \frac{3 \times 10^3 H}{v}$$ (5)

Then we have

$F = Mg\sin\alpha - R = Ma$ (5)

$$\frac{3 \times 10^3 H}{V} - Mg\sin\alpha - \frac{10^3 H}{V} = Ma$$

$$a = \frac{2 \times 10^3 H}{MV} - g\sin\alpha$$ (5)



[15]

(ii) Let a' be the acceleration of the car when it is moving directly down the road.

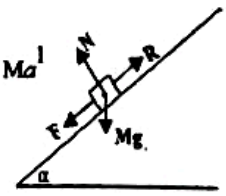
$$F = \frac{10^3 H}{\frac{v}{2}} = \frac{2 \times 10^3}{V} H$$ (5)

Then we have

$F + Mg\sin\alpha - R = Ma'$ (5)

$$\frac{2 \times 10^3 H}{V} + Mg\sin\alpha - \frac{10^3 H}{V} = Ma^1$$

$$\Rightarrow a^1 = \frac{10^3 H}{MV} + g\sin\alpha$$ (5)

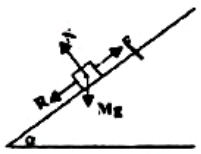


$$a' = 2a \Rightarrow \frac{10^3 H}{MV} + g\sin\alpha = 2\left[\frac{2 \times 10^3 H}{MV} - g\sin\alpha\right]$$
(5)

$$\Rightarrow \text{Sin}\,\alpha = \frac{10^3 H}{MgV}$$ (5) [10]

Let V_1 be the maximum speed of the car when it is moving directly up the road.

$$F_1 = \frac{10^3 H}{V_1}$$ (5)



At the maximum speed there is no acceleration so the forces acting on the car are in equilibrium.

thus, we have

$F_1 - Mg\sin\alpha - R = 0$ (5)

$$\frac{10^3 H}{V_1} - Mg\left(\frac{10^3 H}{Mgv}\right) - \frac{10^3 H}{V} = 0 \Rightarrow V_1 = \frac{v}{2}$$ (5) [15]

12. (a) Applying $S = ut + \frac{1}{2}at^2$ vertically we have

$$y - k = u\sin\theta - \frac{1}{2}gt^2 \rightarrow (01)$$ (10)

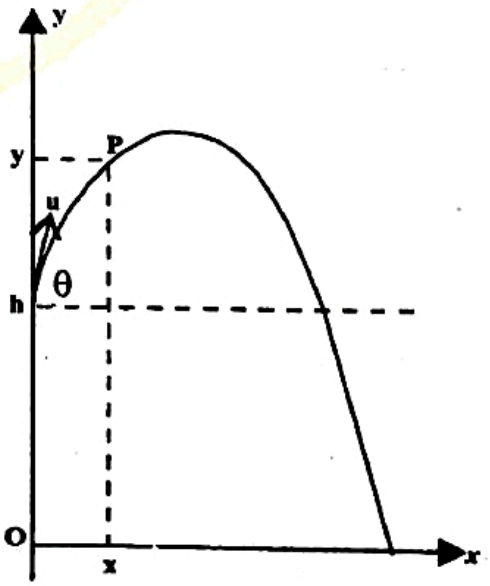
Applying $S = ut + \frac{1}{2}at^2$ horizontally we have

$$x = u t\cos\theta \Rightarrow t = \frac{x}{u\cos\theta}$$ (5)

Form (01) we have

$$y - k = x\tan\theta - \frac{gx^2 Sec^2\theta}{2u^2}$$ (5)

$$y = k + x\tan\theta - \frac{gx^2\sec^2\theta}{2u^2}$$



[20]

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Applying $S = ut + \frac{1}{2}at^2$ horizontally for the particle P we have ⑤

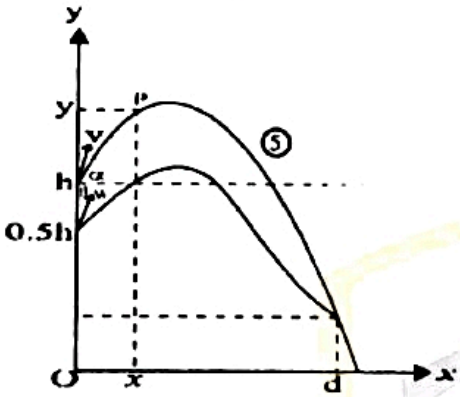
$d = vt_0 \cos\alpha \rightarrow (02)$ Where t_0 is the time at which the two particles meet.

Applying $S = ut + \frac{1}{2}at^2$ horizontally for the particle Q We have

$d = Wt_0 \cos\beta \rightarrow (03)$ ⑤

From (02) and (03) we have

$v \cos\alpha = W \cos\beta$ ⑤



⑤ 20

Using the result obtained in the first part for the particle P we have.

$Y_0 = h + d\tan\alpha - \frac{gd^2\sec^2\alpha}{2V^2}$ ⑤ $\rightarrow (04)$ where y_0 is the height at which the two particles meet. Using the result obtained in the first part for the particle Q we have

$$y_0 = \frac{h}{2} + d\tan\beta - \frac{gd^2\sec^2\beta}{2W^2} \rightarrow (05)$$ ⑤

From (04) and (05) We have

$$h + d\tan\alpha = \frac{h}{2} + d\tan\beta \Rightarrow h = 2d(\tan\beta - \tan\alpha)$$ ⑤ 15

$$t = \frac{h}{2(\tan\beta - \tan\alpha)} = \frac{h\cos\alpha\cos\beta}{2(\sin\beta\cos\alpha - \sin\alpha\cos\beta)} = Wt_0\cos\beta$$ ⑤

i.e. $$t_0 = \frac{h}{2w\left[\sin\beta - \sin\alpha\left(\frac{\cos\beta}{\cos\alpha}\right)\right]} = \frac{h}{2w\left[\sin\beta - \sin\alpha\left(\frac{v}{w}\right)\right]}$$ ⑤

$$= \frac{h}{2(w\sin\beta - V\sin\alpha)}$$ 15

(b) Let a be the acceleration of the Particle Q

Applying $F = ma$ for the motion of the Particle Q vertically down we have

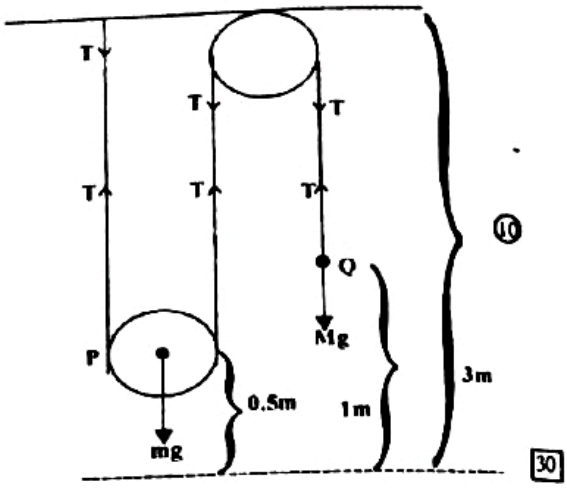
$Mg - T = Ma \rightarrow (01)$ ⑤

Applying $F = ma$ for the motion of the pulley P vertically up we have

$2T - mg = m\frac{a}{2} \rightarrow (02)$ ⑩

① × ② + 1. We have $2Mg - mg = \left(2M + \frac{m}{2}\right)a$

i.e $a = 2\left(\frac{2M - m}{4M + m}\right)g$ ⑤



⑩ 30

From (01) we have $T = Mg - Ma$

$$= Mg\left[1 - 2\left(\frac{2M - m}{4M + m}\right)\right] = \frac{3mMg}{4M + m}$$ ⑤ 10

Applying $S = ut + \frac{1}{2}at^2$ for the motion of the Particle Q Vertically downward we have

$$1 = 0 + \frac{1}{2}\left(\frac{2(2M - m)}{4M + m}\right)gt_0^2$$ ⑩

Where t_0 is the time required to reach the floor.

$t_0 = \sqrt{\frac{4M + m}{(2M - m)g}}$ Seconds 10

Let h_0 be the height that the pulley P rises during the time t_0

Applying $S = ut + \frac{1}{2}at^2$ for the motion of the Pulley P Vertically upward we have.

$$h_0 = 0 + \frac{1}{2}\left(\frac{2M - m}{4M + m}\right)\left(\frac{4M + m}{2M - m}\right) = \frac{1}{2} \text{ meters.}$$ ⑤

Let V be the Velocity of the Pulley P at time t_0 Applying $V = u + at$ for the motion of the Pulley P vertically upward we have.

$$V = 0 + \left(\frac{2M - m}{4M + m}\right)g\sqrt{\frac{4M + m}{2(M - m)g}} = \sqrt{\left(\frac{2M - m}{4M + m}\right)g}$$ ⑤

Let h_1 be the height that the Pulley P rises safter the Particle Q reach the floor.

Applying $V^2 = u^2 + 2aS$ for the motion of the Pulley P vertically upward after the particle Q reaches the floor we have.

⑤ $0 = \left(\frac{2M - m}{4M + m}\right)g - 2gh_1 \Rightarrow h_1 = \frac{1}{2}\left(\frac{2M - m}{4M + m}\right)$ ⑤

Total height

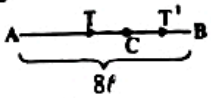
$$= \frac{1}{2} + h_0 + h_1 = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}\left(\frac{2M - m}{4M + m}\right)$$

$$= \frac{1}{2} + \frac{3M}{4M + m} \text{ metres}$$ ⑤ 30

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13. Let T be the tention of the String AP When the Particle P is at C.

Let T' be the tention of the String PB When the Particle P is at C.



Then by Hooke's law we have ⑤

$T = \left(\frac{AC - 3\ell}{3\ell}\right) 4\lambda$ and $T^1 = \left(\frac{8\ell - AC - 2\ell}{2\ell}\right)\lambda$ ⑩

⑩

$= \left(\frac{6\ell - AC}{2\ell}\right)\lambda$

Since the particle P is rest in equilibrium at the Point C we have.

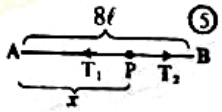
$T = T^1 \Rightarrow \left(\frac{AC - 3\ell}{3\ell}\right) 4\lambda = \left(\frac{6\ell - AC}{2\ell}\right)\lambda \Rightarrow 11AC = 42\ell$

⑤ ⑤

$\Rightarrow AC = \frac{42}{11}\ell$ ⑤ 40

Let T_1 be the tention of the string AP when the Particle P is at a distance x form the Pcint A

Let T_2 be the tention of the string BP when the Particle P is at a distance x from the point A. Then by Hooke's law we have.



$T_1 = \left(\frac{x - 3\ell}{3\ell}\right) 4\lambda$ ⑤

$T_2 = \left(\frac{8\ell - x - 2\ell}{2\ell}\right)\lambda = \left(\frac{6\ell - x}{2\ell}\right)\lambda$ ⑤

Applying Newton's law along AB for the Particle P We have

$T_2 - T_1 = m\ddot{x}$

i.e $\left(\frac{6\ell - x}{2\ell}\right)\lambda - \left(\frac{x - 3\ell}{3\ell}\right)4\lambda = m\ddot{x}$ ⑤

i.e $m\ddot{x} + \frac{\lambda}{\ell}\left(\frac{4}{3} + \frac{1}{2}\right)x - 7\lambda = 0$ ⑤

i.e $\ddot{x} + \frac{11\lambda}{6m\ell}x - \frac{7\lambda}{m} = 0$

i.e $\ddot{x} + \frac{11\lambda}{6m\ell}\left(x - \frac{42}{11}\ell\right) = 0$ ⑤ 40

Let $y = x - \frac{42}{11}\ell$

Then $\dot{y} = \dot{x}$ and $\ddot{y} = \ddot{x}$ ⑤

Thus We have $\ddot{y} + \frac{11\lambda}{6m\ell}y = 0 \rightarrow (01)$ ⑤ 10

$y = A\cos\omega t + B\sin\omega t \rightarrow (02)$

$\dot{y} = A\omega\sin\omega t + B\omega\cos\omega t \rightarrow (03)$

$x = 4\ell$ and $\dot{x} = 0$ at $t = 0 \Rightarrow y = \frac{2}{11}\ell$ and $\dot{y} = 0$ ⑤

at $t = 0$

Thus (02) $\Rightarrow A = \frac{2}{11}\ell$ and (03) $\Rightarrow B = 0$ ⑤ ⑤

$\because y = \frac{2}{11}\ell\cos\omega t \Rightarrow \ddot{y} = -\frac{2}{11}\ell w^2\cos\omega t$

$= -\omega^2 y$

Comparing this with (01) we have

$\omega^2 = \frac{11\lambda}{6m\ell} \Rightarrow \omega = \sqrt{\frac{11\lambda}{6m\ell}}$ ⑤ 20

$y = \frac{2}{11}\ell\cos\left[\sqrt{\frac{11\lambda}{6m\ell}}\right]$ for $\frac{-2}{11}\ell \le y \le \frac{2}{11}\ell$ ⑤

$x = \frac{42}{11}\ell + \frac{2}{11}\ell\cos\left[\sqrt{\frac{11\lambda}{6m\ell}}\right]t$ for $\frac{40}{11}\ell \le x \le 4\ell$ ⑤

Let t_0 be the time for the Particle P to reach the Point for $x = \frac{41}{11}\ell$

Then we have

$\frac{2}{11}\cos\left[\sqrt{\frac{11\lambda}{6m\ell}}\right]t_0 = \frac{41}{11} - \frac{42}{11} \Rightarrow \cos\left[\sqrt{\frac{11\lambda}{6m\ell}}\right]t_0 = -\frac{1}{2}$

⑤ ⑤

$\cos\left[\sqrt{\frac{11\lambda}{6m\ell}}\right]t_0 = -\frac{1}{2} \Rightarrow \sqrt{\frac{11\lambda}{6m\ell}}\,t_0 = \frac{2\pi}{3}$ ⑤

$\dot{x} = -\frac{2}{11}\ell\left[\sqrt{\frac{11\lambda}{6m\ell}}\right]\sin\left[\sqrt{\frac{11\lambda}{6m\ell}}\right]t$, for $\frac{40}{11}\ell \le x \le 4\ell$

⑤

The Velocity of the Particle P at this Point is

$\dot{x} = -\frac{2}{11}\ell\left[\sqrt{\frac{11\lambda}{6m\ell}}\right]\sin\left[\sqrt{\frac{11\lambda}{6m\ell}}\right]t_0$

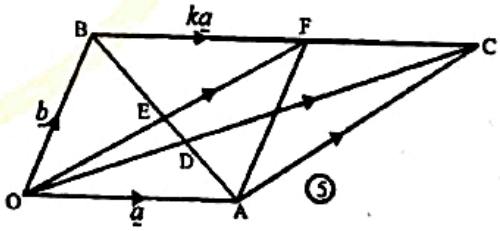
$= -\frac{2}{11}\ell\left[\sqrt{\frac{11\lambda}{6m\ell}}\right]\left[-\frac{\sqrt{3}}{2}\right] = \sqrt{\frac{\lambda\ell}{22m}}$ ⑤

⑤ 40

14. $\overline{OD} = \overline{OA} + \overline{AD}$ ⑤

$= \underline{a} + \frac{1}{3}(\underline{b} - \underline{a})$ ⑤

$= \frac{1}{3}(2\underline{a} + \underline{b})$



⑤

$\overline{OC} = \underline{b} + k\underline{a}$ ⑤

Since O, D and C are Collinear we can write

$\overline{OD} = \lambda\overline{OC}$ where λ is a Parameter.

i.e. $\frac{1}{3}(2\underline{a} + \underline{b}) = \lambda(\underline{b} + k\underline{a})$ ⑤

$(3\lambda k - 2)\underline{a} + (3\lambda - 1)\underline{b} = \underline{0}$

Since $\underline{a}$ and $\underline{b}$ are two non - Parallel vectors we have

⑤ $3\lambda k - 2 = 0$ and $3\lambda - 1 = 0$

$\Rightarrow \lambda = \frac{1}{3}$ and $k = 2$ ⑤ 20

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$\overline{OD} = \lambda\overline{OC} \Rightarrow OD = \frac{1}{3}OC \Rightarrow OD = \frac{1}{3}(OD + DC)$ ⑤

$\Rightarrow 2OD = DC \Rightarrow OD : DC = 1 : 2$ ⑤ 20

$\therefore \overline{OC} = \underline{b} + 2\underline{a}$

$\overline{AC} = \overline{AO} + \overline{OC} = -\underline{a} + (\underline{b} + 2\underline{a})$

$= \underline{a} + \underline{b}$ ⑤

Extend the line OE to meet BC at a Point F.
Then OACF is a Parallelogram since OE is Parallel to AC

$\overline{OF} = \overline{AC} = \underline{a} + \underline{b}$ ⑤

Thus OAFB is a Parallelogram Therefore, E is the mid Point of AB

$\overline{DE} = \overline{DA} + \overline{AE} = -\frac{1}{3}(\underline{b} - \underline{a}) + \frac{1}{2}(\underline{b} - \underline{a}) = \frac{1}{6}(\underline{b} - \underline{a})$ ⑤

$6\overline{DE} = \overline{AB} \Rightarrow 6DE = AB$ ⑤ 20

(b) Resolving forces along ox we have

$X = 6P + 4P\cos\frac{\pi}{3} + 2P\cos\frac{\pi}{3} = 9P$ ⑩

Resolving forces along on we have

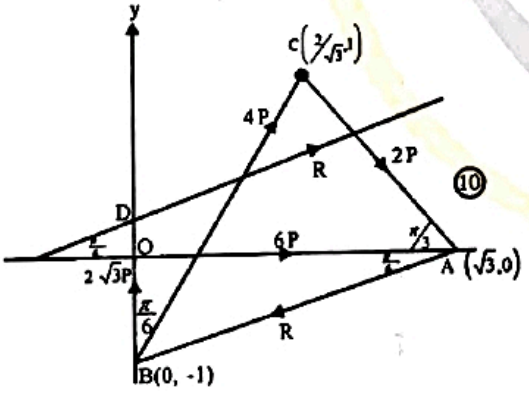
$Y = 2\sqrt{3}P + 4P\sin\frac{\pi}{3} - 2P\sin\frac{\pi}{3} = 3\sqrt{3}P$ ⑩

$R = \sqrt{X^2 + Y^2} = \sqrt{81+27}\,P$

$= 6\sqrt{3}P$ ⑤

$\tan\theta = \frac{Y}{X} = \frac{3\sqrt{3}P}{9P} = \frac{1}{\sqrt{3}}$ ⑤

$\Rightarrow \theta = \frac{\pi}{6}$ ⑤



⑩ 45

Suppose that the resultant passes through the point D such that OD = d
Taking moments about the point D in the anticlokwise we have.

$\left[4P\cos\frac{\pi}{3} \times (d+1) + (6P \times d) + 2P\cos\frac{\pi}{3} \times d - 2P\sin\frac{\pi}{3} \times \sqrt{3} = 0\right]$ ⑩

$\left[2 \times (d+1) + 6 \times d + 1 \times d - \sqrt{3} \times \sqrt{3} = 0 \Rightarrow d = \frac{1}{9}\right]$ ⑤ 15

Let (x,y) be the any [illegible] resultant with respect to the cartesian system of coordinates through the point O.

Then we have

$\frac{y-d}{x} = \tan\frac{\pi}{6} \Rightarrow y - \frac{1}{9} = \frac{1}{\sqrt{3}}x \Rightarrow 9y - 3\sqrt{3}x - 1 = 0$ ⑤ 05

The resultant of the system and the force introduced along AB are equal and opposite.
Thus, the system reduces to a couple of magnitude.

$6\sqrt{3}P \times \left(1 + \frac{1}{9}\right)\sin\frac{\pi}{3} = 10P$ newten metre ⑤ 10

15. (a) Let 2a be the length of each of the rods.
Resolving forces vertically we have

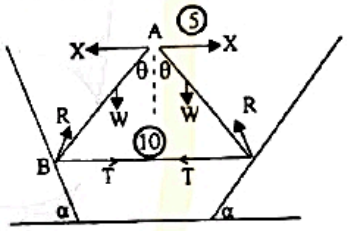
$2R\cos\alpha = 2W \Rightarrow R = W\sec\alpha$ ⑤ ⑤ 10

Taking moment about A in the anti - clockwise sense for the equilibrium of the rod AB wehave.

$T.\,2a\cos\theta + R\sin\alpha\,.\,2a\cos\theta + Wa\sin\theta - R\cos\alpha.\,2a\sin\theta = 0$ ⑩

$2T + 2W\tan\alpha + W\tan\theta - 2W\tan\theta = 0$ ⑩

$T = \frac{W}{2}(\tan\theta - 2\tan\alpha)$ ⑤



⑤ 40

Taking moment about B in the anti - clockwise sense for the equilibrium of the rod AB we have

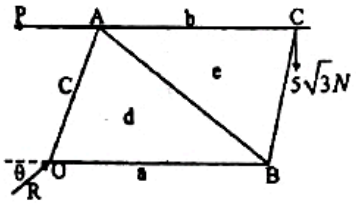
$X.\,2a\cos\theta - W.\,a\sin\theta = 0$ ⑤

i.e. $X = \frac{W}{2}\tan\theta$ ⑤ 10

(b) Let 2a be the length of any rod.
Taking moments about O we have.

$P.2a\sin\frac{\pi}{3} - 5\sqrt{3}\left\{2a + 2a\cos\frac{\pi}{3}\right\} = 0$ ⑤

$P = 15N$ ⑤



10

Let R be the reaction at O and θ be the angle that R makes with the Horizontal.
Resolving forces vertically we have.

$R\sin\theta = 5\sqrt{3}$ ⑤

Resolving forces horizontally we have.

$R\cos\theta = P = 15$ ⑤

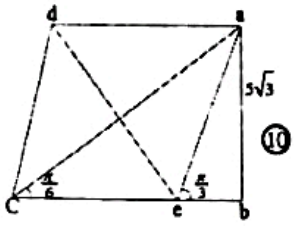
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$R = \sqrt{75+225} = \sqrt{300} = 10\sqrt{3}N$ (5)

$\tan\theta = \frac{5\sqrt{3}}{15} = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$ (5) [20]

Thus the reaction at O makes an angle $\frac{\pi}{6}$ with the horizontal

Stress diagram:



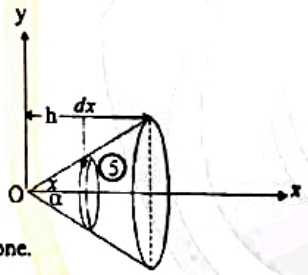
[10]

Rod	Stress	Magnitude
OA	thrust	10N
OB	thrust	10N
AC	tension	5N
AB	tension	10N
BC	thrust	10N

(25) (25) [50]

16. By symmetry the centre of gravity of the cone lies on the axis of summetry. (5)

Let $\bar{x}$ be the distance to the centre of gravity of the cone from its vertex O.



Let τ be the density of the cone.

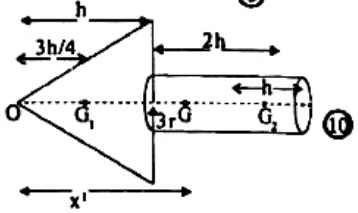
$\frac{1}{3}\pi(h\tan\alpha)^2 \, h\tau\,\bar{x} = \int_0^h \pi (x\tan\alpha)^2 \, x\,\tau\,dx$ (05) (15)

i.e. $\frac{1}{3}h^3\bar{x} = \int_0^h x^3\,dx = \frac{h^4}{4} \Rightarrow \bar{x} = \frac{3}{4}h$ (5)

Thus the centre of gravity of the cone is on its axis of symmetry at a distance $\frac{1}{4}h$ from the base of the cone. (5)

[40]

By symmetry the centre of mass of the solid right circular cylindrical block lies on the axis of symmetry at a distance 2h from the vertex O of the cone. (5)



By first Part the centre of mass of the cone is on the axis of symmetry at a distance $\frac{3}{4}h$ from the vertex O of the cone. (5)

Let x' be the distance to the centre of mass of the composite body along the axis of symmetry from the vertex O of the cone.

$\left[\pi r^2(2h) + \frac{1}{3}\pi(3r^2)h\right]\tau x^1 = \left(\pi r^2(2h)\right)2h\tau + \left[\frac{1}{3}\pi(3r)^2 h\right]\tau\left(\frac{3h}{4}\right)$

(10) (5) (5)

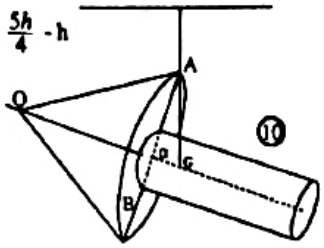
$5x^1 = 4h + \frac{9}{4}h \Rightarrow x^1 = \frac{5}{4}h$ (5)

For the correct equation (5) [30]

$AB = 3r$ and $BG = OG - OB = \frac{5h}{4} - h$

$= \frac{h}{4}$ (5)

$\tan\alpha = \frac{AB}{BG} = \frac{12r}{h}$ (5)



[20]

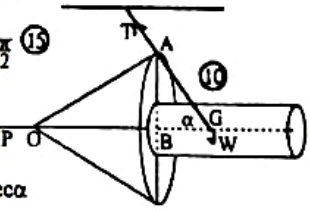
Applying Lami's Theorem we have

$\frac{W}{\sin(\pi-\alpha)} = \frac{P}{\sin\left(\frac{\pi}{2}+\alpha\right)} = \frac{T}{\sin\frac{\pi}{2}}$ (15)

$\frac{P}{\cos\alpha} = T = \frac{W}{\sin\alpha}$ (05)

$P = W\cot\alpha$, and $T = W\operatorname{cosec}\alpha$

(05) (05)



[40]

17. (a) (i) $P(BBB) = P(B)\ P(B/B)\ P(B/BB)$ (05)

$= \frac{3}{15} \times \frac{2}{14} \times \frac{1}{13} = \frac{1}{455}$ (10) (05) [20]

or $P(BBB) = \frac{{}^3C_3}{{}^{15}C_3} = \frac{3!}{15\times14\times13} = \frac{1}{455}$ (10) (05) (05) [20]

(ii) $P(W^1\,W^1\,W^1) = P(W^1)\,P(W^1/W^1)\,P(W^1/W^1W^1)$ (05)

$= \frac{10}{15} \times \frac{9}{14} \times \frac{8}{13} = \frac{24}{91}$ (10) (05) [20]

or $P(W^1\,W^1\,W^1) = \frac{{}^{10}C_3}{{}^{15}C_3} = \frac{10\times9\times8}{15\times14\times13} = \frac{24}{91}$ (10) (05) (05) [20]

(iii) P (at least one ball is white) = 1-P (none of the three balls are white) (05)

$= 1 - \frac{24}{91} = \frac{67}{91}$ (05) (05) [15]

(iv) P (balls are of different colour)

$\frac{{}^5C_1 \times {}^3C_1 \times {}^7C_1}{{}^{15}C_3} = \left(\frac{5\times3\times7}{15\times14\times13}\right)3! = \frac{3}{13}$ (05) (05) (05) [15]

(v) $P(BRW) = P(B)\,P(R/B)\,P(W/BR)$

$= \frac{3}{15} \times \frac{7}{14} \times \frac{5}{13} = \frac{1}{26}$ (05) (05) [10]

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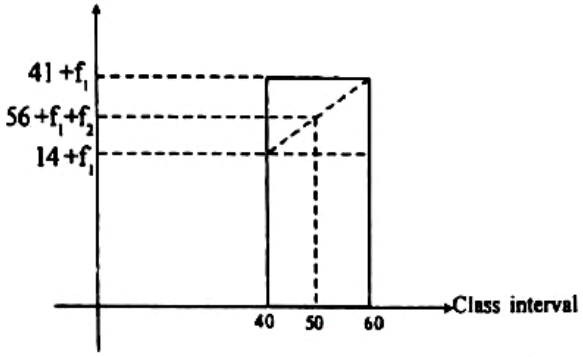
(b) Median = 50 So the median class is 40 - 60

$$14 + f_1 + \frac{27}{20} \times 10 = \frac{56 + f_1 + f_2}{2}$$ (10)

$$28 + 2f_1 + 27 = 56 + f_1 + f_2$$

i.e. $f_1 - f_2 = 1 \rightarrow (01)$

Comulative Frequency



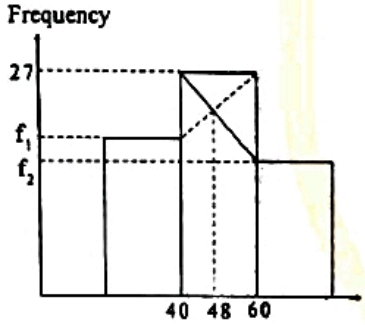
Mode = 48 Sothe modal class is 40 - 60

$$\frac{27 - f_1}{27 - f_2} = \frac{48 - 40}{60 - 48} = \frac{8}{12} = \frac{2}{3}$$ (10)

i. e. $3f_1 - 2f_2 = 27 \rightarrow (02)$

From (01) and (02) we have

$f_1 = 25$ and $f_2 = 24$ (5)



30

The total number of students who sat for the Statistics Paper.

$= 56 + f_1 + f_2 = 56 + 25 + 24 = 105$ (5) 05

Class interval	Mid value (x)	Frequency (f)	$d = \frac{x - 50}{20}$	fd	fd^2
00 - 20	10	14	-2	-28	56
20 - 40	30	25	-1	-25	25
40 - 60	50	27	0	0	0
60 - 80	70	24	1	24	24
80 - 100	90	15	2	30	60
	Total	105		1	165

(5) (5)

10

Mean $= 50 + 20d = 50 + 20 \times \frac{1}{105} = 50 + \frac{4}{21} = 50.19$ (5)

$$\text{Variance} = 20^2 \left\{ \frac{1}{105} \sum_{i=1}^{5} f_i d_i^2 - \left(\frac{\sum_{i=1}^{5} f_i d_i}{105} \right)^2 \right\}$$ (5)

$$= \left(\frac{20}{105}\right)^2 (165 \times 105 - 1) = 17324 \left(\frac{4}{21}\right)^2$$ (5)

Standard deviation $= \frac{4}{21}\sqrt{17324} = \frac{4 \times 131.62}{21} = 25.07$ (5)

25

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