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G.C.E. (Advanced Level) Examination - August 2011

Combined Mathematics - II

Scheme of Marking

Part A

(01) Let T be the time required for the particle P to reach the maximum height

Let v be the required velocity of the particle Q.

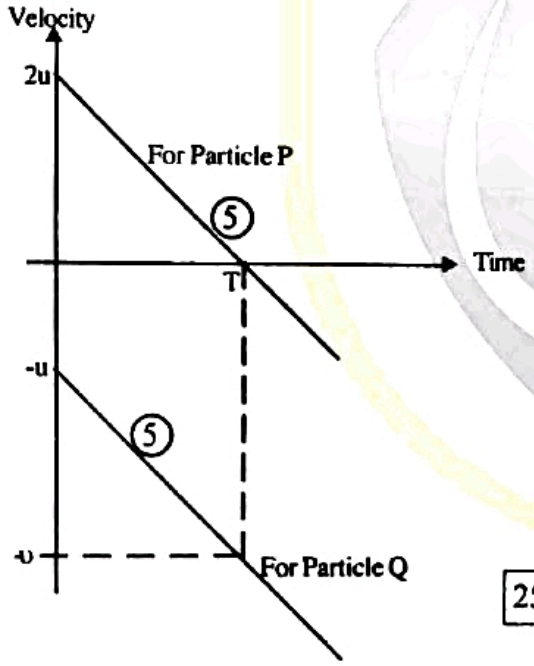
Then $\frac{2u}{T} = g \rightarrow (01)$ (5)

Also $\frac{-u-v}{T} = g \rightarrow (02)$ (5)

From (01) and (02) We get

$-u - v = 2u \Rightarrow v = -3u$ (5)

Thus, The speed of the particle Q When the particle reaches the maximum height is 3u.



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(02) Since the string is inex tensible $x + 2y$ = constant

$\therefore \ddot{x} + 2\ddot{y} = 0 \rightarrow (01)$ (5)

Applying the $F = ma$ for the motion of the particle of mass 2m vertically upwards we have

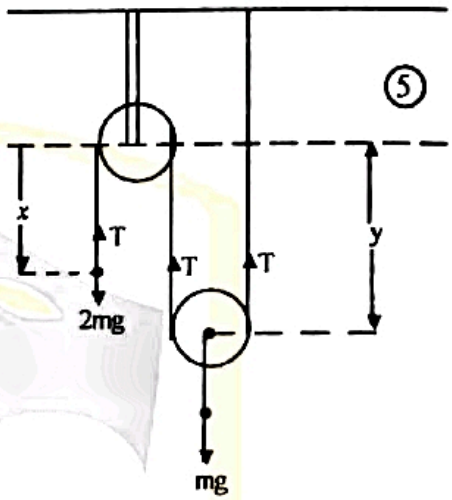
$T - 2mg = 2m(-\ddot{x}) \rightarrow (02)$ (5)

Applying $F = ma$ for the motion of the particle of mass m vertically upwards we have.

$2T - mg = m(-\ddot{y}) \rightarrow (03)$ (5)

$(02) + 4 \times (03) \Rightarrow 9T - 6mg = -2m(\ddot{x} + 2\ddot{y}) = 0$

From (01) We have $T = \frac{2}{3}mg$ (5)



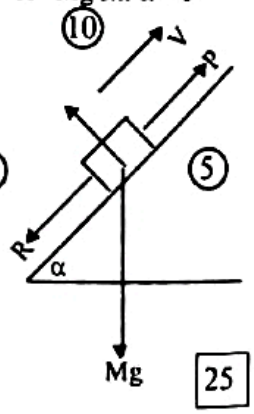
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(03) Applying F = ma for the motion of the cyclist along the road upwards we have P - R - Mg sin α = 0

Also, We have H = PV (5)

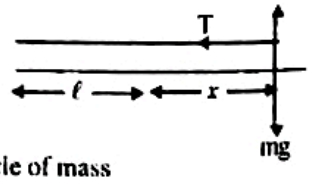
$\therefore \frac{H}{V} = R + mg \sin\alpha$

$\Rightarrow H = (R + Mg \sin\alpha)V$ (5)



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(04) $T = \lambda \frac{x}{\ell}$ (5)



Applying F = ma for the motion of the particle of mass m horizontally We have

$-T = m\ddot{x}$ (5)

$\therefore \ddot{x} = -\frac{\lambda}{m\ell}x$ (5)

Thus, the particle performs a simple harmonic motion. (5)

The periodic time.

$= \frac{2\pi}{\sqrt{\lambda/m\ell}} = 2\pi\sqrt{\frac{m\ell}{\lambda}}$ (5)

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(05) $\vec{AC} = p + 3q - (-2p + 5q) = 3p - 2q$ (5)

$\vec{CB} = 7p - q - (p + 3q) = 6p - 4q = 2\vec{AC}$ (5)

$\vec{AC} = \frac{1}{2}\vec{CB}$ (5)

Thus, the points A, B and C are collinear and $AC = \frac{1}{2}CB$ (5) (5)

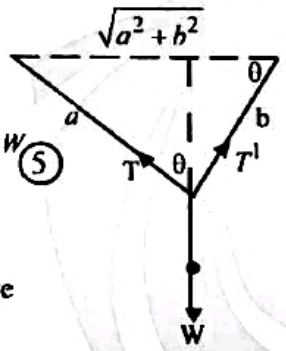
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(06) Resolving vertically we have

$T\cos\theta + T^1\sin\theta = W$ (5)

$T \cdot \frac{b}{\sqrt{a^2+b^2}} + T^1 \cdot \frac{a}{\sqrt{a^2+b^2}} = W$ (5)

$Tb + T^1 a = W\sqrt{a^2+b^2} \rightarrow (1)$



Resolving horizontally we have

$T\sin\theta - T^1\cos\theta = 0$ (5)

$T \cdot \frac{a}{\sqrt{a^2+b^2}} - T^1\frac{b}{\sqrt{a^2+b^2}} = 0$

$Ta - T^1 b = 0 \rightarrow (02)$

$(01)\times b + (02)\times a \Rightarrow T = \frac{Wa}{\sqrt{a^2+b^2}}$ (5)

$(01)\times a - (02)\times b \Rightarrow T^1 = \frac{Wb}{\sqrt{a^2+b^2}}$ (5)

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(07) (i) $p(\Omega) = p(A \cup B) = P(A) + P(B) - P(A \cap B)$

$1 = \frac{2}{5} + p(B) - \frac{1}{3} \Rightarrow p(B) = \frac{14}{15}$ (5) (5)

(ii) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{3}}{\frac{14}{15}} = \frac{05}{14}$ (5)

(iii) $P(A^1|B^1) = \frac{P(A^1 \cap B^1)}{P(B^1)} = \frac{P\{(A \cup B)^1\}}{P(B^1)} = \frac{P(\phi)}{P(B^1)} = 0$ (5) (5)

$\therefore p(\phi) = 0$

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(08) $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$. Where A and B are the events that the two friends being success in solving the problem.

(i) $P(A \cap B) = P(A)P(B) = \left(\frac{1}{3}\right)\left(\frac{1}{4}\right) = \frac{1}{12}$ (5) (5)

(ii) $P\{(A \cup B)^1\} = 1 - \{P(A) + P(B) - P(A \cap B)\}$ (5) (5)

$= 1 - \left\{\frac{1}{3} + \frac{1}{4} - \left(\frac{1}{3}\right)\left(\frac{1}{4}\right)\right\}$ (5)

$= 1 - \frac{1}{2} = \frac{1}{2}$

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(09) Since the median is 900. We have

$50 + x + \frac{500}{200} \times 100 = 500 \Rightarrow x = 200$ (5) (5)

$50 + y + \frac{500}{200} \times 100 = 500 \Rightarrow y = 200$ (5) (5)

Since the distribution is symmetrical the mean is equal to the median.

Thus, the mean is also 900. (5)

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(10) (i) Total sum = 24 x 15 = 360

Total sum of the best three months
= 35 x 3 = 105

Total sum of the lowest four months
= 11 + 14 + 16 + 22 = 63

Total of the remaining eight months
= 360 - 105 - 63 = 192 (5)

Average of the remaining eight months $= \frac{192}{8} = 24$ (5) (5)

(ii) Since there are 15 data fourth datum should be the first of the destribution. (5)

Thus, 22 is the first quartile of the distribution. (5)

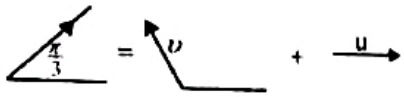
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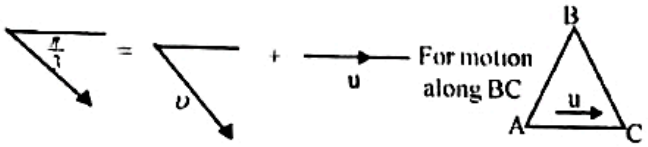
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Part B

(11) (*a*) Vel B. G = Vel B. W + Vel W. G Where B for bird W for wind and G for ground.



For motion along AB

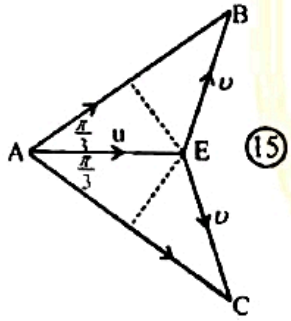


For motion along BC

AEB is the velocity triangles of relative velocities for the motion along AB and AEC is the velocity triangles of relative velocities for the motion along BC. The velocity of the bird relative to ground along AB is

$$\underbrace{\sqrt{v^2 - u^2\sin^2\frac{\pi}{3}}}_{(10)} + u\cos\frac{\pi}{3} = \sqrt{v^2 - \frac{3}{4}u^2} + \frac{1}{2}u \quad (5)$$

$$= \frac{1}{2}\left\{\sqrt{4v^2 - 3u^2} + u\right\}$$



(15)

Time taken to fly AB is $\dfrac{2a}{u + \sqrt{4v^2 - 3u^2}}$ (5)

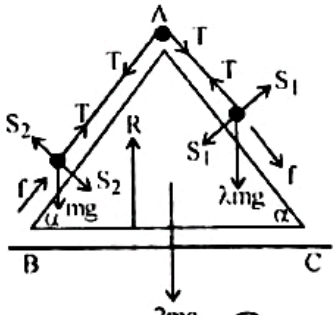
By symmetry the time taken to fly BC is $\dfrac{2a}{u + \sqrt{4v^2 - 3u^2}}$ (10)

Thus the total time taken to fly AB and BC is

$$\frac{4a}{u + \sqrt{4v^2 - 3u^2}} \quad (5)$$

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(*b*)



(10) for force

(10) for acceleration

Let the acceleration of the particle P relative to the wedge be f along BA.

Then the acceleration of the particle Q relative to the wedge is f along AC.

Let the acceleration of the wedge be F along CA.

Applying F = ma for the motion of the particle P along BA we have

$-mg\sin\alpha + T = m(f - F\cos\alpha) \rightarrow (01)$ (15) 35

Applying F = ma for the motion of the particle Q along AC We have

$\lambda mg\sin\alpha - T = \lambda m(f - F\cos\alpha) \rightarrow (01)$ (15) 15

Applying F = ma for the motion of the system horizontally along CA We have

$0 = 2mF + m(F - f\cos\alpha) + \lambda m(F - f\cos\alpha) \rightarrow (03)$ (15)

15

$$\therefore F = \frac{1+\lambda}{3+\lambda} f\cos\alpha \quad (5)$$

$$(1)+(2) \Rightarrow -g(1-\lambda)\sin\alpha$$

$$= (1+\lambda)f - (1+\lambda)F\cos\alpha \quad (5)$$

$$= (1+\lambda)f\left\{1 - \frac{(1+\lambda)}{3+\lambda}\cos^2\alpha\right\} \quad (5)$$

$$= \frac{(1+\lambda)}{(3+\lambda)}\left\{(3+\lambda) - (1+\lambda)\cos^2\alpha\right\} f$$

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Thus We have $f = \dfrac{(\lambda-1)(3+\lambda)g\sin\alpha}{(1+\lambda)\{(3+\lambda)-(1+\lambda)\cos^2\alpha\}}$ (5)

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i.e. the magnitude of the aceleration of the particle P or Q relative to the wedge.

is $\dfrac{(-\lambda+1)(3+\lambda)g\sin\alpha}{(1+\lambda)\{(3+\lambda)-(1+\lambda)\cos^2\alpha\}}$ (10)

The magnitude of the acceleration of the particle P relative to the wedge just after the string is broken can be found by setting $\lambda = 0$ in

$$f = \frac{(-\lambda+1)(3+\lambda)g\sin\alpha}{(1+\lambda)\{(3+\lambda)-(1+\lambda)\cos^2\alpha\}}$$ (5)

Thus we have the magnitude of the acceleration of the particle p relative to the wedge just after the string is broken as

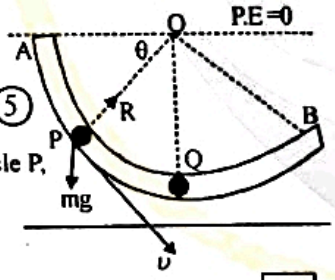
$$f_1 = \frac{3g\sin\alpha}{3-\cos^2\alpha}$$

15

(12) The reaction R is perpendicular to the direction of mition and so it does no work.

Thus, by the conservation of energy for the system we have

$$\frac{1}{2}mV^2 - mga\sin\theta = \frac{1}{2}m(2ga) - mga$$ (15)



i.e. $V^2 = 2ga(1+\sin\theta)$

$\therefore V = \sqrt{2ga(1+\sin\theta)}$ (5)

i.e. the speed of the particle P, When OP makes an angle θ with OA, is $\sqrt{2ga(1+\sin\theta)}$

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Applying F = ma for the motion of the particle.

P along PO We have,

$$R - mg\sin\theta = m\frac{v^2}{a} = m\cdot 2ga(1+\sin\theta)$$ (15)

$\therefore R = mg(2+3\sin\theta)$ (5)

i.e. the magnitude of the reaction on the particle P from the tube is mg(2 + 3 sinθ)

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Let V_1 be Velocity of the particle P when it reaches the point C.

Then taking $\theta = \dfrac{\pi}{2}$ in $v = \sqrt{2ga(1+\sin\theta)}$ we have

$$v_1 = \sqrt{2ga\left(1+\sin\frac{\pi}{2}\right)} = 2\sqrt{ga}$$ (10)

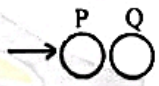
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Momentum Conservation :

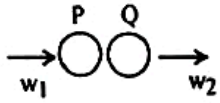
$$m\cdot 2\sqrt{ga} = mW_2 + mW_1$$ (10)

i.e. $W_2 + W_1 = 2\sqrt{ga} \rightarrow (01)$

Before collision



After Collision



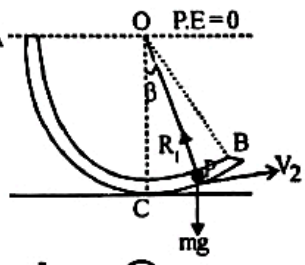
Newton's law of restitution :

$$-(W_1 - W_2) = eV_1 = \frac{1}{2}\cdot 2\sqrt{ga} = \sqrt{ga} \rightarrow (02)$$ (10)

$(01)+(02) \Rightarrow W_2 = \dfrac{3\sqrt{ga}}{2}$ (5) and $(01)-(02) \Rightarrow W_1 = \dfrac{\sqrt{ga}}{2}$ (5)

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The reaction R_1 is perpendicular to the direction of motion and so it does no work thus, by the conservation of energy for the particle P we have,



$$\frac{1}{2}mV_2^2 - mga\cos\beta = \frac{1}{2}mW_1^2 - mga$$ (10)

$$\frac{1}{2}mV_2^2 - mga\cos\beta = \frac{1}{2}m\left(\frac{ga}{4}\right) - mga$$

i.e. $V_2^2 = 2ga\left(\cos\beta - \dfrac{7}{8}\right)$ (5)

$V_2 = 0 \Leftrightarrow \cos\beta = \dfrac{7}{8} > \cos\dfrac{\pi}{3} \Rightarrow \beta < \dfrac{\pi}{3}$ (5)

$\therefore O \le \beta < \dfrac{\pi}{2}$

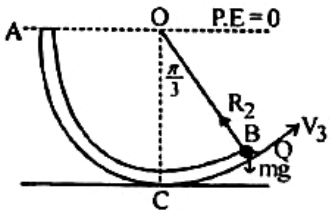
Let this angle be β_0 (5)

Thus the particle P oscillates between $-\beta_0$ and β_0 Hence, the particle P never leaves the tube. (5)

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The reaction R_2 is perpendicular to the direction of motion and so it does no work thus, by the conservation of energy for the particle Q When it reaches the point B, We have,



$$\frac{1}{2}mV_3^2 - mga\cos\frac{\pi}{3} = \frac{1}{2}mW_2^2 - mga \quad (15)$$

$$\frac{1}{2}mV_3^2 - \frac{1}{2}mga = \frac{1}{2}m\left(\frac{9ga}{4}\right) - mga$$

i.e. $V_3^2 = \frac{5ga}{4} \Rightarrow V_3 = \frac{1}{2}\sqrt{5ga}$ (5)

Thus the particle Q reaches the point B with speed

$\frac{1}{2}\sqrt{5ga}$ 20

Applying $v^2 = u^2 + 2fs$ vertically form B to the maximum point it reaches for the motion of the particle Q We have,

(10) $0 = V_3^2\sin^2\frac{\pi}{3} - 2gs = \left(\frac{5ga}{4}\right)\left(\frac{3}{4}\right) - 2gs$

Where S is the maximum height the particle Q reaches in its vertical motion.

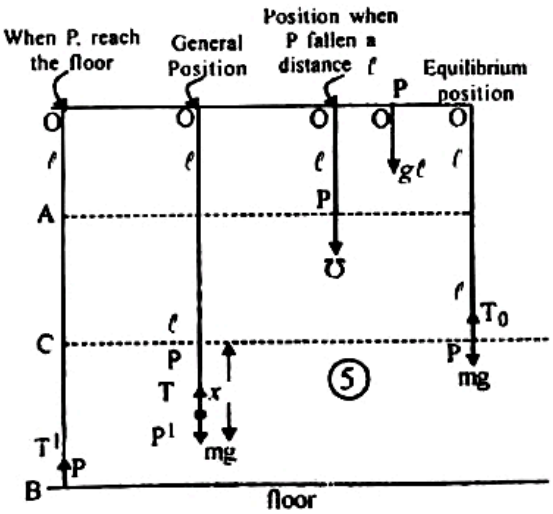
$$S = \frac{15}{32}a$$

$$S = \frac{15}{32}a + \frac{a}{2}$$

$$S = \frac{31a}{32} \quad (5)$$

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(13)



Let T_0 be the tension in the string in equilibrium Position.

Then $T_0 = \frac{\lambda\ell}{\ell}$ by Hooke's law. (10)

But $T_0 = mg$ (10)

Thus we have $\lambda = mg$ (5) 30

Applying $V^2 = u^2 + 2as$.

Vertically downwards to the particle P We have,

$V^2 = g\ell + 2g(\ell) = 3g\ell$ (10)

Where V is the velocity of P

Where it has fallen a distance ℓ

Thus We have $v = \sqrt{3g\ell}$ (5) 15

Again by Hooke's law we have [for the figure] (5)

$T = \frac{mg(\ell + x)}{\ell}$ (10)

Applying Newton's Vertically downwards for the particle P we have.

$mg - T = m\ddot{x}$ (10)

i.e. $mg - \frac{mg(\ell + x)}{\ell} = m\ddot{x} \Rightarrow \ddot{x} + \frac{g}{\ell}x = 0$ (5) (5) 35

$\dot{x} = v = \sqrt{3g\ell}$ When $x = -\ell$ (10)

Thus from $\dot{x}^2 = \frac{g}{\ell}\left(c^2 - x^2\right)$ We have

$3g\ell = \frac{g}{\ell}\left(c^2 - \ell^2\right) \Rightarrow c = 2\ell$ (5) (5) 20

$\therefore \dot{x}^2 = \frac{g}{\ell}\left(4\ell^2 - x^2\right)$ (5)

$\dot{x} > 0$ for $-\ell \le x < 2\ell$ and $\dot{x} = 0$ for $x = 2\ell$

(5) Thus the particle P comes to instantaneous rest When it reaches the floor. (5) 15

Let t_1 be time taken for the particle P to fall under gravity from O to A.

Then from $v = u + at$ We have

$$\sqrt{3g\ell} = \sqrt{g\ell} + gt_1 \Rightarrow t_1 = \left(\sqrt{3} - 1\right)\sqrt{\frac{\ell}{g}}$$

(10) (5)

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Let t_2 be time taken for the particle P to move in simple harmonic motion from A to B.

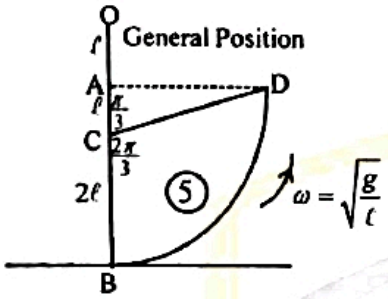
Then from the figure just above we have,

$$\sqrt{\frac{g}{\ell}}\,t_2 = \frac{2\pi}{3} \Rightarrow t_2 = \frac{2\pi}{3}\sqrt{\frac{\ell}{g}} \quad (10)\ (5)$$

$$t_1 + t_2 = \left(\sqrt{3}-1\right)\sqrt{\frac{\ell}{g}} + \frac{2\pi}{3}\sqrt{\frac{\ell}{g}} = \frac{1}{3}\left(3\sqrt{3}-3+2\pi\right)\sqrt{\frac{\ell}{g}} \quad (5)$$

Thus the time taken from O to reach the floor is

$$\frac{1}{3}\left(3\sqrt{3}-3+2\pi\right)\sqrt{\frac{\ell}{g}}$$



(5) 40

(14) $\underline{a}\cdot\underline{b} = |\underline{a}||\underline{b}|\cos\theta$ Where θ is the angle between the two vectors a and b. (10) 10

Taking $\underline{c} = \underline{a}$ and $\underline{d} = \underline{b}$ in

$$(\underline{a}+\underline{b})\cdot(\underline{c}+\underline{d}) = \underline{a}\cdot\underline{c} + \underline{b}\cdot\underline{c} + \underline{a}\cdot\underline{d} + \underline{b}\cdot\underline{d}$$

We have

$$(\underline{a}+\underline{b})\cdot(\underline{a}+\underline{b}) = \underline{a}\cdot\underline{a} + \underline{b}\cdot\underline{a} + \underline{a}\cdot\underline{b} + \underline{b}\cdot\underline{b} \quad (10)$$

But

$$(\underline{a}+\underline{b})\cdot(\underline{a}+\underline{b}) = |\underline{a}+\underline{b}||\underline{a}+\underline{b}|, \quad \underline{a}\cdot\underline{a} = |\underline{a}||\underline{a}|, \quad (5)\ (5)$$

$$\underline{b}\cdot\underline{b} = |\underline{b}||\underline{b}| \text{ and } \underline{a}\cdot\underline{b} = \underline{b}\cdot\underline{a} \quad (5)$$

$$|\underline{a}+\underline{b}|^2 = |\underline{a}|^2 + 2(\underline{a}\cdot\underline{b}) + |\underline{b}|^2 \quad (5)$$

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$$|\underline{a}-\underline{b}|^2 = |\underline{a}|^2 + 2(\underline{a}\cdot\underline{b}) + |\underline{b}|^2 \quad (10)$$

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$$|\underline{a}+\underline{b}|^2 - |\underline{a}-\underline{b}|^2 = 4\underline{a}\cdot\underline{b} \quad (5)$$

$$|\underline{a}+\underline{b}|^2 = |\underline{a}-\underline{b}|^2 \qquad \underline{a}\cdot\underline{b} = 0 \quad (5)$$

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Let $\underline{a}$, $\underline{b}$ and $\underline{c}$ be the position vectors A, B and C of a paralleogram OACB with respect to the point O. (5)

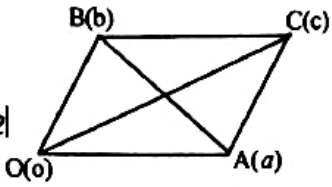
$OC = |\underline{C}|$ and $AB = |\underline{b}-\underline{a}|$

But $\underline{c} = \underline{a}+\underline{b}$

Thus we have

$OC = |\underline{a}+\underline{b}|$

$OC = AB \Rightarrow |\underline{a}+\underline{b}| = |\underline{a}-\underline{b}|$ (5)



From the above result proved we have $\underline{a}\cdot\underline{b} = 0$ (5)

This means that OA is perpendicular to OB. (5)

Thus OABC is a rectangle.

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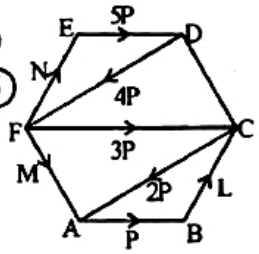
Resolving forces horizontally along → We have

(b) →

$$\left(\begin{array}{l} P - 2P\cos\frac{\pi}{6} + L\cos\frac{\pi}{3} + 3P - 4P\cos\frac{\pi}{6} \\ +M\cos\frac{\pi}{3} + N\cos\frac{\pi}{3} + 5P \end{array}\right) = 0 \quad (10)$$

$$9p - 3\sqrt{3}p + \frac{1}{2}L + \frac{1}{2}M + \frac{1}{2}N = 0$$

$$L + M + N = \left(\sqrt{3}-3\right)6P \rightarrow (1) \quad (5)$$



Resolving forces vertically upwards we have,

$$\uparrow\ -2P\cos\frac{\pi}{3} - 4P\cos\frac{\pi}{3} - M\sin\frac{\pi}{3} + N\sin\frac{\pi}{3} + L\sin\frac{\pi}{3} = 0 \quad (10)$$

$$-3P - \frac{\sqrt{3}}{2}(M - N - L) = 0$$

$$L - M + N = 2\sqrt{3}P \rightarrow (02) \quad (5)$$

F) Taking moment about F in the anti clock wise We have

$$-2P\cdot 2a + P\cdot 2a\sin\frac{\pi}{3} - 5P\cdot 2a\sin\frac{\pi}{3} + L\cdot 2\cdot 2a\sin\frac{\pi}{3} = 0 \quad (10)$$

$$-2P - 2\sqrt{3}P + \sqrt{3}L = 0$$

$$L = \frac{2}{\sqrt{3}}\left(1+\sqrt{3}\right)P \quad (10)$$

$$(01) - (02) \Rightarrow 2M = \left(\sqrt{3}-3\right)6P - 2\sqrt{3}P$$

$$= \left(4\sqrt{3}-18\right)P$$

$$\Rightarrow M = \left(2\sqrt{3}-9\right)P \quad (10)$$

$$(01) + (02) \Rightarrow 2N = \left(\sqrt{3}-3\right)6P + 2\sqrt{3}P - 2\ell$$

$$= \left(8\sqrt{3}-18\right)P - \frac{4}{\sqrt{3}}\left(1+\sqrt{3}\right)P$$

$$= \left(\frac{20\sqrt{3}}{3} - 22\right)P$$

$$N = \left(\frac{10}{\sqrt{3}} - 11\right)P \quad (10)$$

70

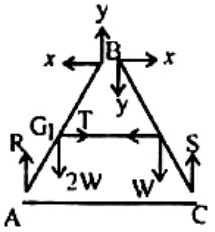
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(15) (a) Let AB = BC = 2a

Taking moment about C in the anticlookwise sense for the system We have

$$W \cdot a \cdot \sin\theta + 2W \cdot 3a\sin\theta - R \cdot 4a\sin\theta = 0 \quad (15)$$

$$R = \frac{7}{4}W \quad (5)$$



Taking moment about B in the anticlookwise sense for the rod AB We have

$$T \cdot a\cos\theta + 2W \cdot a\sin\theta - R \cdot 2a\sin\theta = 0 \quad (15)$$

$$T = -2W\tan\theta + 2R\tan\theta = \left(-2W + \frac{7}{2}W\right)\tan\theta \quad (5)$$

$$= \frac{3}{2}W\tan\theta$$

40

Resolving horizontally for the rod AB We have

$$x = T = \frac{3}{2}W\tan\theta \quad (5)$$

Resolving vertically for the rod AB We have

$$Y + R - 2W = 0 \quad (5)$$

$$y = -R + 2W = -\frac{7}{4}W + 2W = \frac{1}{4}W \quad (5)$$

Thus, the reaction at the join B.

$$\sqrt{x^2 + y^2} = \sqrt{\left(\frac{3}{2}W\tan\theta\right)^2 + \left(\frac{1}{4}W\right)^2} = \frac{W}{4}\sqrt{1 + 36\tan^2\theta} \quad (5)$$

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The angle that the reaction makes with the horizontal is

$$\tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{W/4}{\frac{3}{2}W\tan\theta}\right) = \tan^{-1}\left(\frac{1}{6}\cot\theta\right) \quad (10)$$

10

(b) (i) Taking moments about D We have

$$P \cdot AD - 10\sqrt{3} \cdot AB = 0 \quad (5)$$

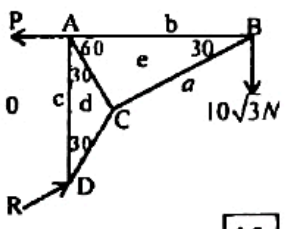
But $AD = 2AC \cdot \cos 30^0$

$$= 2AB\cos 60^0 \cos 30^0$$

$$= \frac{\sqrt{3}}{2}AB \quad (5)$$

$$P \cdot \frac{\sqrt{3}}{2}AB - 10\sqrt{3} \cdot AB = 0$$

$$P = 20N \quad (5)$$



15

Let R be the reaction at E and θ be the angle that R makes with the horizontal.

Resolving forces vertically We have $R\sin\theta = 10\sqrt{3}$ (5)

Resolving forces horizontally We have

$$R = \sqrt{\left(10\sqrt{3}\right)^2 + 20^2} = 10\sqrt{7}N \quad (5)$$

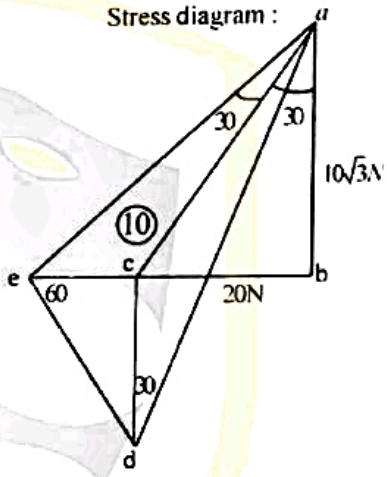
$$\tan\theta = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right) \quad (5)$$

Since the system is in equilibrium under three forces the reaction R should also passes through B.

15

Stress diagram :



10

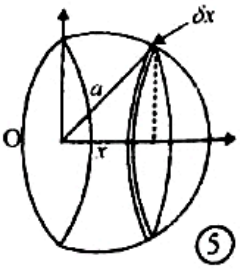
Rod	Stress	Magnitude
AB	Tention	30N
BC	Thrust	$20\sqrt{3}N$
AC	Thrust	20N
DC	Thrust	40N
AD	Tention	$10\sqrt{3}N$

(20) (20)

40

(16) By symmetry the centre of mass of the hemisphere lies on the axis of symmetry. (5)

Let $\bar{x}$, be the distance to the centre of mass of the hemisphere from O the centre of the base of the hemisphere.



(5)

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Let ρ be the density of the hemi spere.

$\frac{1}{2}\cdot\frac{4}{3}\pi a^3\rho\bar{x}$ (10)

$= \int_0^a \pi\left(a^2 - x^2\right)x\rho\,dx$ (10)

i.e. $\frac{2}{3}a^3\bar{x} = \int_0^a\left(a^2x - x^3dx\right) = \left[a^2\frac{x^2}{2} - \frac{x^4}{4}\right]_0^a$ (5)

$= \left(\frac{1}{2} - \frac{1}{4}\right)a^4 = \frac{1}{4}a^4$ (5)

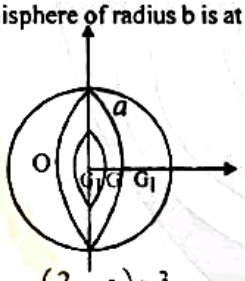
$\Rightarrow \bar{x} = \frac{3}{8}a$ (5)

Thus the centre of mass of a uniform solid hemisphere of radius a is at a distance $\frac{3}{8}a$ from the base of the hemisphere. 45

The centre of mass of the solid hemisphere of radius a is at a distance $\frac{3}{8}a$ from O. (5)

The centre of mass of the solid hemisphere of radius b is at a distance $\frac{3}{8}b$ from O. (5)

Let $\bar{x}$ be the distance to the centre of mass of the shell from O.



$\left(\frac{2}{3}\pi b^3 - \frac{2}{3}\pi a^3\right)\rho\,\bar{x} = \left(\frac{2}{3}\pi b^3\right)\rho\,\frac{3}{8}b - \left(\frac{2}{3}\pi a^3\right)\rho\,\frac{3}{8}a$

(10) (10) (10)

i.e. $\bar{x} = \frac{\frac{3}{8}\left(b^4 - a^4\right)}{b^3 - a^3} = \frac{3}{8}\frac{(a+b)\left(a^2+b^2\right)}{a^2+ab+b^2} \rightarrow (01)$ (5)

45

Resolving horizontally We have $S = \mu R \rightarrow (01)$ (5)

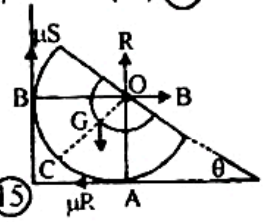
Resolving Vertically We have $R + \mu S = W \rightarrow (02)$ (5)

From (01) and (02) We have

$R = \frac{W}{1+\mu^2}$ and $S = \frac{\mu W}{1+\mu^2}$ (5) (5)

Taking moment about O We have

$W.OG\sin\theta = \mu R.OA + \mu S.OB$ (15)



i.e. $W\cdot\frac{3}{8}\frac{(a+b)\left(a^2+b^2\right)}{a^2+ab+b^2}\sin\theta = \frac{\mu W}{1+\mu^2}b + \frac{\mu^2 W}{1+\mu^2}b$ (10)

i.e. $\frac{3}{8}\frac{(a+b)\left(a^2+b^2\right)}{a^2+ab+b^2}\sin\theta = \frac{\mu(1+\mu)}{1+\mu^2}h$ (5)

i.e. $\sin\theta = \frac{8}{3}\frac{\mu b(1+\mu)\left(a^2+ab+b^2\right)}{\left(1+\mu^2\right)(a+b)\left(a^2+b^2\right)}$ (5)

$\theta = \sin^{-1}\left\{\frac{8}{3}\frac{\mu b(1+\mu)\left(a^2+ab+b^2\right)}{\left(1+\mu^2\right)(a+b)\left(a^2+b^2\right)}\right\}$ (5)

60

(17) (a) Let N - for Nimal getting a tial,
S - for Sunil getting a tail and
P - for Piyal getting a tail.

Then $P(N) = P(S) = P(P) = 1 - P = q$ (say) (5)

P(Nimal wins the second turn)

= $P(N^1S^1P^1N)$ (5)

= $P(N^1)P(S^1)P(P^1)P(N)$ (5)

= $p\cdot p\cdot p\cdot q$ (5)

= $P^3(1-P)$

20

P(Nimal wins the third turn)

= $P(N^1S^1P^1N^1S^1P^1N)$ (5)

= $P(N^1)P(S^1)P(P^1)P(N^1)P(S^1)P(P^1)P(N)$ (5)

= $p^6q = (1-P)p^6$ (5)

15

P(Nimal wins the game event eventually.)

= $P(N\cup N^1S^1P^1N\cup N^1S^1P^1N^1S^1P^1N\cup\ldots\ldots)$ (5)

= $P(N) + P(N^1S^1P^1N) + P(N^1S^1P^1N^1S^1P^1N) + \ldots$

= $P(N) + P(N^1)P(S^1)P(P^1)P(N) + P(N^1)P(S^1)$ (5)

$P(P^1)P(N^1)P(S^1)P(P^1)P(N) + \ldots\ldots$ (5)

= $q + p^3q + p^6q + \ldots\ldots$ (5)

= $q(1 + p^3 + p^6 + \ldots\ldots) = q\frac{1}{1-p^3} = \frac{1}{1+p+p^2}$ (5)

25

If the coin is such that it is more likely to lands tails than heads, then $p < \frac{1}{2}$

Thus We have $1 + p + p^2 < 1 + \frac{1}{2} + \frac{1}{4} = \frac{7}{4}$ (5)

$\therefore$ P(Nimal wins the game)

(5) $= \frac{1}{1+p+p^2} > \frac{4}{7} = 0.571 > 0.5$ (5)

Nimal has more than 50% chance of winings the game

15

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(b) $\sum_{i=1}^{n} y_i = \sum_{i=1}^{n}(a+bx_i) = na + \sum_{i=1}^{n} x_i$ (5)

$$\frac{1}{n}\sum_{i=1}^{n} y_i = a + \frac{1}{n}\sum_{i=1}^{n} x_i$$

i.e. $\bar{y} = a + b\bar{x}$ (5) 10

$$S_y^2 = \sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n}\{(a+bx_i) - (a+b\bar{x})\}^2$$ (5)

$$= b^2\sum_{i=1}^{n}(x_i - \bar{x})^2 = b^2 S_x^2$$ (5) 10

$$\bar{x} = \frac{1+2+3+4+5+6+7}{7} = \frac{28}{7} = 4$$ (5) 5

$$S_x = \sqrt{\frac{1}{7}\left\{\begin{matrix}(1-4)^2+(2-4)^2+(3-4)^2+(4-4)^2 \\ +(5-4)^2+(6-4)^2+(7-4)^2\end{matrix}\right\}}$$

$$= \sqrt{\frac{1}{7}\{9+4+1+1+4+9\}} = \sqrt{\frac{28}{7}} = 2$$ (5) 5

(i) $y = 1 + 1.01x$ will transforms the set $\{1, 2, 3, 4, 5, 6, 7\}$ in to the set $\{2.01, 3.02, 4.03, 5.04, 6.05, 7.06, 8.07\}$ (5)

$\bar{y} = 1 + 1.01 \times 4 = 5.04$ (5)

$S_y = bS_x = 1.01 \times 2 = 2.02$ (5) 15

(ii) consider the transformation $y = a + bx$, Where a and b are constant.

Then We have,

$\bar{y} = a + b\bar{x}$ and $S_y = bS_x$

Thus We get

$5 = a + 4b$ and $6 = 2b \Rightarrow b = 3$ and $a = -7$ (5) (5)

$y = -7 + 3x$

The seven numbers are -4, -1, 2, 5, 8, 11, 14 (5) 15

(c) Consider the transformation $y = -25 + x$ (5)

Then ~~from~~ to $\bar{y} = -25 + \bar{x}$ We have (5)

$$\frac{27.2}{80} = -25 + \bar{x} \Rightarrow \bar{x} = 25 + 0.34$$

$$\Rightarrow \bar{x} = 25.34$$ (5) 15

From $S_y^2 = S_x^2$

We have $S_x = \sqrt{\frac{85.1}{80}} = \sqrt{1.06375}$ (5)

$= 1.031$ (5) 10