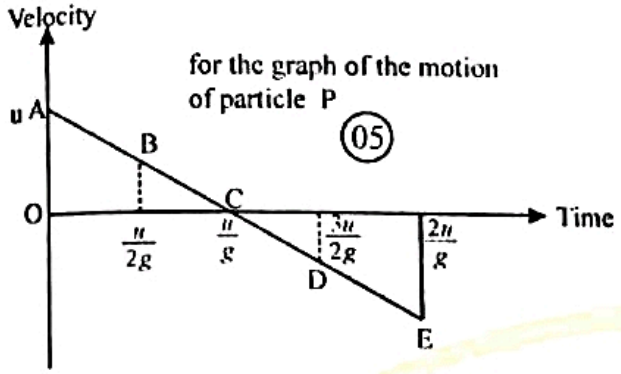
G.C.E. (Advanced Level) Examination - August 2010
Combined Mathematics II
Scheme of Marking

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(01) (a)

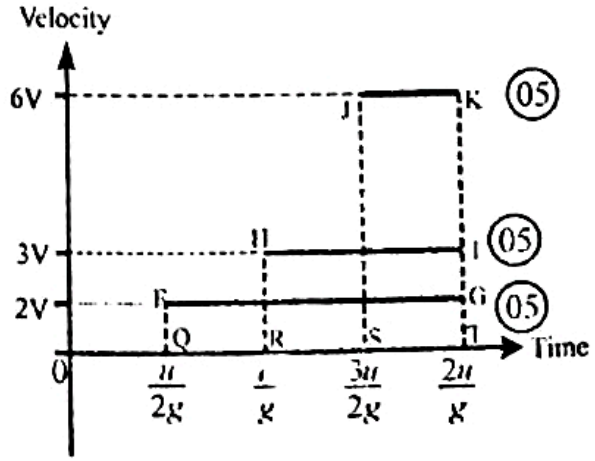


The line AE represents the velocity - time graph for the velocity of the particle p since the particles P_1, p_2 and P_3 are Projected from the particle P.

horizontally their vertical components are the same as that of the particle P at the respective times of Projection. Hence the velocity - time graphs for the vertical components of the velocities of the particles P_1, p_2 and P_3 coinside withe the portions that of the particle P.

Thus, the line BE represents the velocity - time graph for the vertical component of the velocity of the particle P_1 (05)
the line CE represents the velocity - time graph for the vertical component of the velocity of the particle P_2 (05)
and the line DE represents the velocity - time graph for the vertical component of the velocity of the particle P_3 (05)

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(i) From the first diagram it is clear that the three particles reach the ground at the same time $\frac{2u}{g}$ (05)

(ii) From the second diagram We have the horizontal distance travelled by the particle P_1

P_1 = the area of QTGF

$$=\left(\frac{2u}{g}-\frac{u}{2g}\right)2v=\frac{3uv}{g}$$ (05)

The horizontal distance travelled by the particle P_2 = the area of RTIH

$$=\left(\frac{2u}{g}-\frac{u}{g}\right)3v=\frac{3uv}{g}$$ (05)

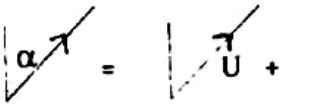
The horizontal distance travelled by the particle P_3 = the area of STKJ

$$=\left(\frac{2u}{g}-\frac{3u}{2g}\right)6v=\frac{3uv}{g}$$ (05)

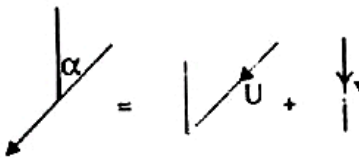
Thus the three particles P_1, P_2 and P_3 fall on the ground at the same position.

20

(b) Vel. M, B = Vel. M, W + Vel W, B where M for man, W for water and B for bank.



for upstream motion



for down stream motion

ACB is the velocity triangles of relative velocities for upstream motion and ACD is the velocity triargles of relative velocities for downstream motion.

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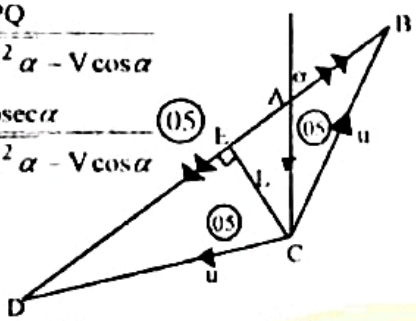
The velocity of the man relative to water along PQ is

$$\sqrt{u^2 - v^2\sin^2\alpha} - V\cos\alpha \quad (05)$$

Time taken to swim the distance PQ is

$$= \frac{PQ}{\sqrt{u^2 - v^2\sin^2\alpha} - V\cos\alpha}$$

$$= \frac{d\,\mathrm{cosec}\,\alpha}{\sqrt{u^2 - v^2\sin^2\alpha} - V\cos\alpha} \quad (05)$$



The velocity of the man relative to water along QP is

$$= \sqrt{u^2 - v^2\sin^2\alpha} + v\cos\alpha \quad (05)$$

Time taken to swim the distance QP is

$$= \frac{PQ}{\sqrt{u^2 - v^2\sin^2\alpha} + v\cos\alpha}$$

$$= \frac{d\,\mathrm{cosec}\,\alpha}{\sqrt{u^2 - v^2\sin^2\alpha} + v\cos\alpha} \quad (05)$$

Thus the total time

$$= \frac{d\,\mathrm{cosec}\,\alpha}{\sqrt{u^2 - v^2\sin^2\alpha} - v\cos\alpha} + \frac{d\,\mathrm{cosec}\,\alpha}{\sqrt{u^2 - v^2\sin^2\alpha} + v\cos\alpha}$$

$$= \frac{2d\sqrt{u^2\,\mathrm{cosec}^2\alpha - v^2}}{u^2 - v^2} \quad (05)$$

35

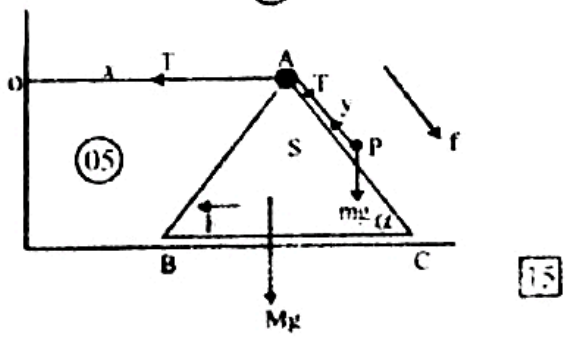
(02) At time t let OA = x and OP = y

Then $x + y = \ell$ from which have

$\dot{x} + \dot{y} = 0$ and $\ddot{x} + \ddot{y} = 0$ (05)

Since $F = -\ddot{x}$ and $f = \ddot{y}$ we have

$-F + f = 0 \Rightarrow F = f$ (05)



15

Applying p = mf for the motion of the particle p along AC we have

$$mg\sin\alpha - T = m(f - F\cos\alpha) \quad \rightarrow(01) \quad (10)$$

Applying p = mf for the motion of the system horizontally we have

$$T = MF + m(F - f\cos\alpha) \quad \rightarrow(02) \quad (05)$$

15

Adding (01) and (02) and putting $f = F$ we have

$$mg\sin\alpha = mF(1 - \cos\alpha) + MF + mF(1 - \cos\alpha)$$

(05) (05)

$$\Rightarrow F = \frac{mg\sin\alpha}{M + 2m(1 - \cos\alpha)}$$

10

Applying $S = ut + \frac{1}{2}ft^2$ (05) With u = 0 and S = d at t for the motion of the wedge we have

$$d = \frac{1}{2}\left\{\frac{mg\sin\alpha}{M + 2m(1 - \cos\alpha)}\right\}t^2 \quad (10)$$

Which gives $t = \sqrt{\dfrac{2d\{M + 2m(1 - \cos\alpha)\}}{mg\sin\alpha}}$

15

Applying $V = u + ft$ With u = 0 and $t = \sqrt{\dfrac{2d\{M + 2m(1 - \cos\alpha)\}}{mg\sin\alpha}}$ for the motion of the wedge we have (05)

$$v = \left\{\frac{mg\sin\alpha}{M + 2m(1 - \cos\alpha)}\right\}\sqrt{\frac{2d\{M + 2m(1 - \cos\alpha)\}}{mg\sin\alpha}} \quad (10)$$

$$= \sqrt{\frac{2dmg\sin\alpha}{M + 2m(1 - \cos\alpha)}}$$

15

For the motion of the particle P relative to the floor

Applying $V = u + ft$ horizontally we have $V_1 = F(1 - \cos\alpha)t$ (05)

Where V_1 is the horizontal component of the speed of the particle P relative to the floor

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Applying V = u + ft vetically we have V_2 = F sinα t, (05) where V_2 is the verticle. component of the speed of the particle P relative to the floor.

$V_0 = \sqrt{V_1^2 + V_2^2}$. (05) where V_0 is the speed of the particle relative to the floor.

$= Ft\sqrt{(1-\cos\alpha)^2 + \sin^2\alpha}$

$= Ft\sqrt{2(1-\cos\alpha)}$ (05)

$= \left\{\frac{mg\sin\alpha}{M+2m(1-\cos\alpha)}\right\}\sqrt{\frac{2d\{M+2m(1-\cos\alpha)\}}{mg\sin\alpha}}\sqrt{2(1-\cos\alpha)}$ (05)

$= 2\sqrt{\frac{dmg\sin\alpha(1-\cos\alpha)}{M+2m(1-\cos\alpha)}}$ (05) [30]

(03) Let x be the horizontal distance between the two particles P and Q initially and to be the time taken by the particle to reach the particle Q.

Applying V = u + ft vertically for the motion of the particle P we have

$$0 = u\sin\alpha - gt_0 \Rightarrow t_0 = \frac{u\sin\alpha}{g}$$ (05)

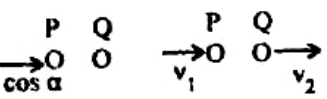
Applying $S = u + \frac{1}{2}ft^2$ horizontally for the motion of the particle P we have

$$x = u\cos\alpha\, t_0 \Rightarrow \frac{u^2\cos\alpha\sin\alpha}{g} = \frac{u^2\sin 2\alpha}{2g}$$ (05) (05)

[15]

Momentum conservation :

mu cosα = $mv_2 + mv_1$ (05)



i.e. $u\cos\alpha = v_2 + v_1 \rightarrow (01)$

Newton's law of restitution :

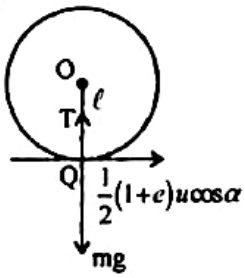
$\cdot (V_1 - V_2) = eu\cos\alpha \rightarrow (02)$ (05)

(01) + (02) $\Rightarrow v_2 = \frac{(1+e)u\cos\alpha}{2}$ *and* (05)

(01) - (02) $\Rightarrow v_1 = \frac{(1-e)u\cos\alpha}{2}$ (05) [20]

Applying P = mf for the motion of the particle Q along OQ we have

$T - mg\cos\theta = m\frac{v^2}{\ell} \rightarrow (03)$ (05)



In the initial position

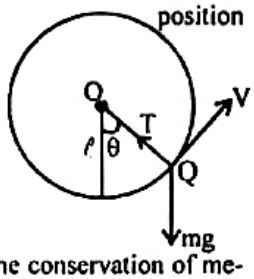
P.E. = 0 and

K.E. $= \frac{1}{2}m\left\{\frac{1}{2}(1+e)u\cos\alpha\right\}^2$

Total mechanical energy $= \frac{1}{8}m(1+e)^2u^2\cos^2\alpha$ (05)

In an intermediate position

P.E. = mg ℓ (1 - cosθ) and K.E. = $\frac{1}{2}mv^2$ (05) Total Mechanical energy $= mg\ell(1-\cos\theta) + \frac{1}{2}mv^2$



During the intermediate motion no work is done by the tension becouse it is always perpendicular to the direction of motion so we can use the conservation of mechanical energy, i.e. initial energy = final energy Hence we have

$\frac{1}{8}m(1+e)^2u^2\cos^2\alpha = mg\ell(1-\cos\theta) + \frac{1}{2}mv^2$ (05) [20]

which gives

$v^2 = \frac{1}{4}(1+e)^2u^2\cos^2\alpha - 2g\ell(1-\cos\theta)$

Substituting this value of V in (03) we have

$T = \frac{m}{\ell}\left\{\frac{1}{4}(1+e)^2u^2\cos^2\alpha - 2g\ell(1-\cos\theta)\right\} + mg\cos\theta$ (05)

To complete the circular motion the string must be tant in the highest position.

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i.e. $T \geq 0$ *when* $\theta = \pi$ (05)

$$T = \frac{m}{\ell}\left\{\frac{1}{4}(1+e)^2 u^2 \cos^2\alpha - 4g\ell\right\} - mg$$

$$= \frac{m}{\ell}\left\{\frac{1}{4}(1+e)^2 u^2 \cos^2\alpha - 5g\ell\right\} \geq 0$$

i.e. $(1+e)^2 u^2 \cos^2\alpha - 20g\ell \geq 0$ (05)

i.e the particle Q completes circular motion if

$$u\cos\alpha \geq \frac{2\sqrt{5g\ell}}{1+e}$$ 15

Applying $2fs = V^2 - u^2$ vertically for the motion of the particle P before the collision

We have $-2gh = -\left(\frac{u\sin\alpha}{g}\right)^2$ (05) where h is the height that the particle attanined to reach the particle Q

$$h = \frac{u^2\sin^2\alpha}{2g}$$

Let x, be the horizontal distance that the particles P travels after the collision and t_1 be the time taken to travel this distance.

Applying $S = ut + \frac{1}{2}ft^2$ vertically for the mition of the particle P after the collision we have.

$$\frac{u^2\sin^2\alpha}{2g}\ (05) = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \frac{u\sin\alpha}{g}$$

Applying $S = ut + \frac{1}{2}ft^2$ horizontally for the motion of the rarticle P after the collision we have

$$x_1 = \left(\frac{(1-e)u\cos\alpha}{2}\right)t_1 \Rightarrow x_1 = \frac{u^2(1-e)\cos\alpha\sin\alpha}{2g}\ (05)$$

$$= \frac{u^2(1-e)\sin 2\alpha}{4g}$$

The total horizontal distance travelled by the particle P

$$= x + x_1 = \frac{u^2\sin 2\alpha}{2g} + \frac{u^2(1-e)\sin 2\alpha}{4g}\ (05)$$

$$= \frac{u^2(3-e)\sin 2\alpha}{4g}\ (05)$$ 25

It e = 3 the total horizontal distance travelled by the particle P is zero so the particle P comes back to the point of projection. (05) 05

(04) Let T_E be the tension in the string in equilibrium position.

Then $T_E = \frac{\lambda a}{\ell}$ (05) by Hooke's law

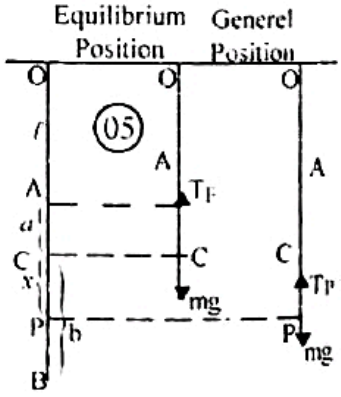
But $T_E = mg$ (05)

Thus we have $a = \frac{mg\ell}{\lambda}$ (05) 15

When the particle is in a general position extention in the string is $a + x$ and the acceleration of the particle is $\ddot{x}$ in the direction CP.

Again by Hooke's law we have

$$T_P = \frac{\lambda(a+x)}{\ell} = \frac{mg}{a}(a+x)\ (05)$$



The maximum distance of the particle from C is the displacement b which is greater than a.

The particle therefore rises above the level of A and the string goes slac!. when the particle just passes back A.

For all positions of P between A and B applying Newton's law vertically downwards at P we have

$$mg - T_P = m\ddot{x} \Rightarrow mg - \frac{mg}{a}(a+x) = m\ddot{x}$$ (05)

$$\Rightarrow \ddot{x} + \frac{g}{a}x = 0\ (05)$$ 20

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The solution of the above equation is of the form $x = A\cos\sqrt{\frac{g}{a}}\,t + B\sin\sqrt{\frac{g}{a}}\,t$

When t = 0, x = b we have A = b and when x = 0, $t = \frac{\pi}{2}\sqrt{\frac{a}{g}}$ we have B = 0 (05) (05) [10]

$x = b\cos\sqrt{\frac{g}{a}}t$

When x = -a the particle P completes the simple harmonic motion. (05)

When x = -a using $x = b\cos\sqrt{\frac{g}{a}}\,t$

we have $t = \sqrt{\frac{a}{g}}\cos^{-1}\left(\frac{-a}{b}\right)$ (05)

$\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha = \frac{-a}{b} \quad \therefore \sin\alpha = \frac{a}{b}$ (05)

Thus the particle P performs simple harmonic motion for a time, $\sqrt{\frac{a}{g}}\left(\frac{\pi}{2}+\alpha\right)$, where $\alpha = \sin^{-1}\left(\frac{a}{b}\right)$ [15]

Differentiating $x = b\cos\sqrt{\frac{g}{a}}\,t$ with respect to t we have

$\dot{x} = -b\sqrt{\frac{g}{a}}\sin\sqrt{\frac{g}{a}}\,t$ (05)

Where $t = \sqrt{\frac{a}{g}}\left(\frac{\pi}{2}+\alpha\right)$ we have

$\dot{x} = -b\sqrt{\frac{g}{a}}\sin\left(\frac{\pi}{2}+\alpha\right) = b\sqrt{\frac{g}{a}}\cos\alpha$ (05)

$= \sqrt{\frac{g}{a}\left(b^2-a^2\right)}$ (05)

Thus the velocity of the particle P at time when it leaves simple harmonic motion is $\sqrt{\frac{g}{a}\left(b^2-a^2\right)}$ upwards. [15]

The particle will move vertically up under gravity with initial velocity $\sqrt{\frac{g}{a}\left(b^2-a^2\right)}$ at A.

Applying $2fs = v^2 - u^2$ for the vertical motion of the particle under gravity with s = l,

f = -g and $u = \sqrt{\frac{g}{a}\left(b^2-a^2\right)}$ (05) we have the velocity at O as.

$$\left[\begin{array}{l} -2gl = v^2 - \frac{g}{a}\left(b^2-a^2\right) \quad (05) \\ \Rightarrow -\frac{2\lambda a}{m} = v^2 - \frac{g}{a}\left(b^2-a^2\right) \end{array}\right] \quad \therefore gl = \frac{\lambda a}{m}\ (05)$$

$$v^2 = \frac{g}{a}\left(b^2-a^2\right) - \frac{2\lambda a}{m} = \frac{m}{2a}\left\{\frac{mg\left(b^2-a^2\right)}{2a^2} - \lambda\right\} \quad (05)$$

If $\lambda < \frac{\left(b^2-a^2\right)mg}{2a^2}$ then the velocity at O is non zero and hence the particle will strike the ceiling. (05) [25]

(05) (*a*) Resolving forces horizontally we have

→ X = - 3p + 7p cos 60^0 - p cos 60^0 = 0 (05)

Resolving forces vertically We have

↑ Y = 6P sin 60^0 - 2P sin 60^0 = $2\sqrt{3}$P (05)

R = $\sqrt{x^2+y^2}$ Where R is the resultant force

= Y = $2\sqrt{3}$ P (05)

The resultant force acts along AC Taking moment about A we have (05)

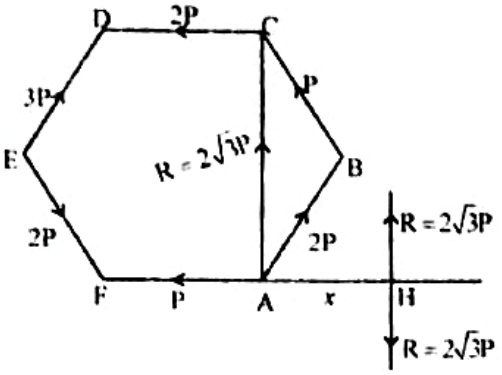
G = P 2 a sin 60^0 + 2P 4a cos 30^0 - 3P . 4a cos 30^0 + 2P 2a sin 60^0 = $\sqrt{3}$ Pa (05) (05) [30]

Introduce a force $2\sqrt{3}$ P along CA and a force $2\sqrt{3}$ P in the direction parallel to AC at the point H on FA produced at a distance x meters from A. (05)

Then the system is reduced to a single resultant force $2\sqrt{3}$ P in the direction parallel to AC at the point H if x satisfies (05)

The equation $2\sqrt{3}$ P x = $\sqrt{3}$ P a (05)

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i.e. the system is equivalent to a single resultant force $2\sqrt{3}$ P in the direction parallel to AC at the point H on FA produced at a distance $\frac{a}{2}$ meters from A. [15]

Introduction a force $2\sqrt{3}P$ at H in the direction parallel to CA.

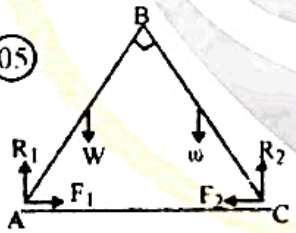
Then the system is in equilisbrium. (05) [05]

(b) Let AB = BC = 2a

Taking moment about B for the rod AB we have,

$F_1 \cdot 2a \sin 45^0 + W \cdot a \sin 45^0 - R_1\ 2a \cos 45^0 = 0$

i.e. $R_1 - F_1 = \frac{W}{2} \rightarrow (01)$ (05)



Taking moment about B for the rod BC we have

$F_2\ 2a \sin 45^0 + \omega \cdot a \sin 45^0 - R_2\ 2a \cos 45^0 = 0$

i.e. $R_2 - F_2 = \frac{1}{2}\omega \rightarrow (02)$ (05)

Resolving forces vertically we have

$R_1 + R_2 = W + \omega \rightarrow (03)$ (05)

Resolving forces horizontally we have

$F_1 - F_2 = 0 \rightarrow (04)$ (05)

From (01), (02) and (04) we have

$$R_1 - R_2 = \frac{1}{2}(W - \omega) \rightarrow (05)$$

From (03) and (05) we have

$$R_1 = \frac{1}{4}(3W + \omega) \quad \text{and} \quad R_2 = \frac{1}{4}(W + 3\omega)$$

From (01) and (04) we have

$$F_1 = F_2 = \frac{1}{4}(W + \omega)$$

$$\frac{F_1}{R_1} = \frac{\frac{1}{4}(W + \omega)}{\frac{1}{4}(3W + \omega)} = \frac{W + \omega}{3W + \omega} \quad \text{and} \quad (05)$$

$$\frac{F_2}{R_2} = \frac{\frac{1}{4}(W + \omega)}{\frac{1}{4}(W + 3\omega)} = \frac{W + \omega}{W + 3\omega} \quad (05)$$

In order to preserve the equilibrium we should have

$$\frac{F_1}{R_1} \le \mu \quad \text{and} \quad \frac{F_2}{R_2} \le \mu \quad (05)$$

i. e. We should have

$$\frac{W + \omega}{3W + \omega} \le \mu \quad \text{and} \quad \frac{W + \omega}{W + 3\omega} \le \mu$$

i. e. We should have $\frac{W + \omega}{W + 3\omega} \le \mu$ (05) [40]

If $\mu = \frac{W + \omega}{W + 3\omega}$ then $F_2 = \mu R_2$ and friction is limiting at C but still $F_1 < \mu R_1$ and (05)

friction is not limiting at A (05)

Thus the slipping is about to occur at C but not at A. [10]

(06) (a) By symmetry the reaction at C is horizontal. (05)

Consider the equilibrium of the rod BC.

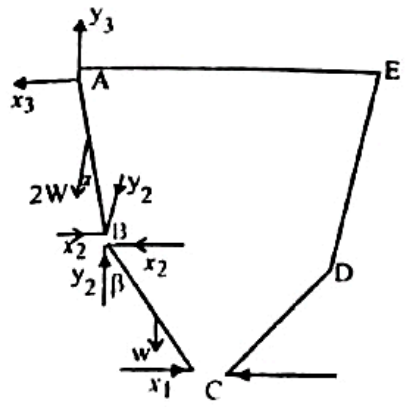
Resolving verically we have $y_2 = W$

Resolving horizontally we have $x_1 = x_2$ (05)

Taking moments about B for BC we have

$x_1\ 2a \cos\beta - W\ a \sin\beta = 0$

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$\Rightarrow x_1 = \frac{l}{2} W \tan\beta$ (05)

Now consider the equilibrium of the rod AB

Taking moments about A for AB we have

$x_2\ 2a\cos\alpha - W\ 2a\sin\alpha - y_2\ 2a\sin\alpha = 0$ (05)

$\Rightarrow x_2 = 2W\tan\alpha \qquad \therefore y_2 = W$ (05)

$x_1 = x_2 \Rightarrow \frac{W}{2}\tan\beta = 2W\tan\alpha \Rightarrow \tan\beta = 4\tan\alpha$ (05)

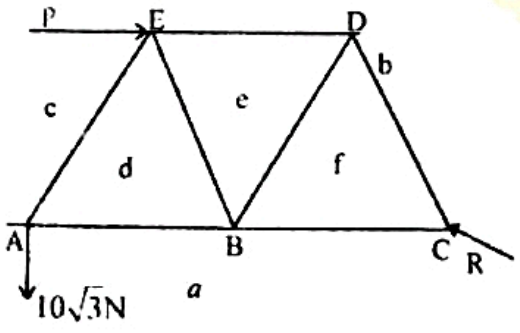
30

(i) Taking moments about C

we have $P \cdot a\cos 30^0 - 10\sqrt{3}\ 2a = 0$ (05)

Where a is the length of a rod

P = 40 N (05)



10

Let R be the reaction at C and θ be the angle that R makes with the horizontal.

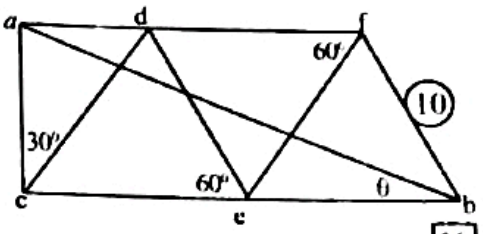
Resolving forces vertically we have $R\sin\theta = 10\sqrt{3}$

Re solving forces horizontally we have $R\cos\theta = P$

$R = \sqrt{(10\sqrt{3})^2 + 40^2} = 10\sqrt{19}$ N (05)

$\tan\theta = \frac{10\sqrt{3}}{40} \Rightarrow \theta = \tan^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (05) 10

Stress diagram :

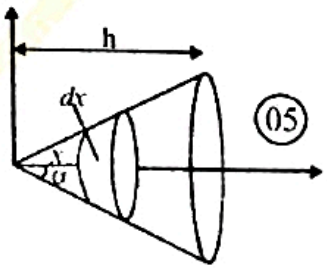


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Rod	Stress	Magnitude
AB	Thrust	10N
BC	Thrust	30N
CD	Thrust	20N
DE	Thrust	20N
EA	Tention	20N
EB	Thrust	20N
DB	Tention	20N

(20) (20) 40

(07) By symmetry the centre of gravity of the cone lies on the axis of symmetry. (05)



(05)

Let $\bar{x}$ be the distance to the centre of gravity of the cone.

from its verte x O.

Let p be the density of the cone.

$\frac{1}{3}\pi(h\tan\alpha)^2\, hp\bar{x} = \int_0^h \pi(x\tan\alpha)^2\, xp\,dx$ (10)

(05)

i.e. $\frac{1}{3}h^3\bar{x} = \int_0^h x^3 dx = \frac{h^4}{4} \Rightarrow \bar{x} = \frac{3}{4}h$ (05)

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Thus the centre of gravity of the cone is on its axis of symmetry at a distance $\frac{1}{4}h$ from the base of the cone. [30]

By symmetry the centre of gravity of the solid right circular cylindrical block lies on the axis of symmetry at a distance $\frac{1}{2}H$ from a base. (05)

By symmetry the centre of gravity of mould also line on the axis of symmetry. (05)

Let x be the distance to the centre of gravity of the mould from the diameter PQ.

$$\left(\pi R^2 H - \frac{1}{3}\pi r^2 h\right)p\bar{x} = \left(\pi R^2 H\right)p\frac{H}{2} - \left(\frac{1}{3}\pi r^2 h\right)p\frac{h}{4}$$

(05) (05) (05)

$$i.e.\quad \bar{x} = \frac{6R^2H^2 - r^2h^2}{4\left(3R^2H - r^2h\right)} \rightarrow (01)\ (05)$$ [30]

Since the centre of gravity of the mould is at the vertex of the conical hollow we have $\bar{x} = h$. It is also given that R = 2r

Writing $\bar{x} = h$ and R = 2r in (01) we have

$$h = \frac{6(2r)^2H^2 - r^2h^2}{4\left(3(2r)^2H - r^2h\right)} = \frac{24H^2 - h^2}{4(12H - h)}$$ (05)

$3h^2 - 48Hh + 24H^2 = 0$

i.e. $h^2 - 16Hh + 8H^2 = 0$ (05)

$$h = \frac{16H \pm \sqrt{(16H)^2 - 32H^2}}{2} = \left(8 \pm \sqrt{56}\right)H = 2\left(4 \pm \sqrt{14}\right)H$$ (05)

$H > h \Rightarrow h = 2(4 + \sqrt{14})H$ is not admissible. (05)

Thus, we have $h = 2\left(4 - \sqrt{14}\right)H$ [20]

Since the centre of gravity of the mould is at the vertex of the conical hollow PG is vertical. (05)

Let θ be the inclination of PQ with PG, the downward vertical.

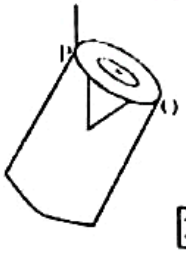
PO = R = 2r

$OG = 2\left(4 - \sqrt{14}\right)H = 6\left(4 - \sqrt{14}\right)r \quad \therefore H = 3r$ (05)

$$\tan\theta = \frac{OG}{OC} = \frac{6\left(4 - \sqrt{14}\right)r}{2r}$$ (05)

$$= 3\left(4 - \sqrt{14}\right)$$

$$\theta = \tan^{-1}\left\{3\left(4 - \sqrt{14}\right)\right\}$$ (05)



[20]

(08) $A = (A \cap B) \cup (A \cap B')$

We have, $P(A) = P\{(A \cap B) \cup (A \cap B')\}$ (05)

$= P(A \cap B) + P((A \cap B')$

$\therefore ((A \cap B) \cap (A \cap B') = \phi$ (05)

i.e. $P((A \cap B') = P(A) - P((A \cap B)$ [10]

$A \cup B = B \cup \left(A \cap B'\right)$

$P(A \cup B) = P\left\{B \cup \left(A \cap B'\right)\right\}$ (05)

$= P(B) + P\left(A \cap B'\right) \quad \because A \cap \left(A \cap B'\right) = \phi$ (05)

$= P(A) + P(B) - P(A \cap B)$ *from first part* [10]

(i) $P(A \cap B') = P(A) - P(A \cap B)$

$= P(A) - P(A)\,P(B)$

$\therefore$ A and B are independent events. (05)

$= P(A)\{1 - P(B)\} = P(A)\,P(B')$ (05)

By the definition of independence A and B' are independent. [10]

(ii) $P\left(A' \cap B'\right) = P\left\{(A \cup B)'\right\}$ by de Morgan's Law (05)

$= 1 - P(A \cup B) = 1 - P(A) - P(B) + P(P \cap B)$

$= 1 - P(A) - P(B) + P(A)\,P(B)$

$= \{1 - P(A)\}\{1 - P(B)\}$ (05)

$= P\left(A'\right)P\left(B'\right)$

By the definition of independence A' and B' are independent. [10]

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Let C be the event that a regular batsman sustains an injury prior to a series and D be the event that a regular bowler sustains an injury prior to a series.

Then P(C) = 0.2 and P(D) = 0.1

P(N)= $P(C' \cap D') = P(C')\,P(D')$ (05)

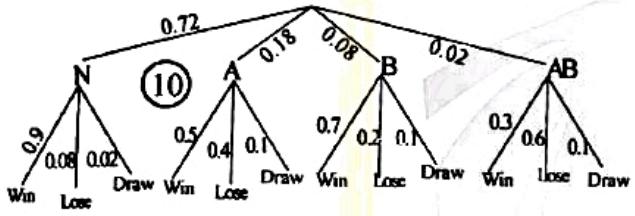
$= \{1 - P(C)\}\,\{1 - P(D)\} = 0.8 \times 0.9 = 0.72$ (05)

10

$P(A) = P(C \cap D') = P(C)P(D') = 0.2 \times 0.9 = 0.18$ (05) 05

$P(B) = P(C' \cap D) = P(C')P(D) = 0.8 \times 0.1 = 0.08$ (05) 05

$P(AB) = P(C \cap D) = P(C)P(D) = 0.2 \times 0.1 = 0.02$ (05) 05



From the tree diagram we have

P(Win) = 0.72 x 0.9 + 0.18 x 0.5 + 0.08 x 0.7 + 0.02 x 0.3 (05)

= 0.648 + 0.09 + 0.056 + 0.006

= 0.8 (05) 20

Aliter.

P(Win) = P(N) P(Win/N) + P(A) P(Win/A) + P(B) P(Win/B) + P (AB) P(Win/AB) (10)

= 0.72 x 0.9 + 0.18 x 0.5 + 0.08 x 0.7 + 0.02 x 0.3 (05)

= 0.648 + 0.09 + 0.056 + 0.006

= 0.8 (05) 20

From the tree diagram we have

P(Lose) = 0.72 x 0.08 + 0.18 x 0.4 + 0.08 x 0.2 + 0.02 x 0.6

= 0.0576 + 0.072 + 0.016 + 0.012

= 0.1576 (05)

$P\{(Lose) \cap B\} = 0.08 \times 0.2 = 0.016$ (05)

$P\{B/(Lose)\} = \dfrac{P\{(Lose) \cap B\}}{P\{Lose\}} = \dfrac{0.016}{0.1576} = 0.1$ (05)

15

Aliter.

P(Lose) = P(N) P(Lose/N) + P(A) P(Lose/A) + P(B) P(Lose/B) + P (AB) P(Lose/AB)

= 0.72 x 0.08 + 0.18 x 0.4 + 0.08 x 0.2 + 0.02 x 0.6

= 0.0576 + 0.072 + 0.016 + 0.012

= 0.1576 (05)

$P\{(Lose) \cap B\} = P(B)\,P\{(Lose)/B\}$

$= 0.08 \times 0.2 = 0.016$ (05)

$P\{B/(Lose)\} = \dfrac{P\{(Lose) \cap B\}}{P\{Lose\}} = \dfrac{0.016}{0.1576} = 0.1$ (05)

15

(09) (a) Mean $= \bar{x} = \dfrac{1}{n}\sum_{i=1}^{n} x_i$ (05)

and variance $= \dfrac{1}{n}\sum_{i=1}^{n} (x_i - \bar{x})^2$ (05) 10

Let $\{x_1, x_2, \ldots\ldots x_{38}\}$ be the values entered correctly

(i) Then

$\bar{x} = 58 = \dfrac{1}{40}\left\{\sum_{i=1}^{38} x_i + 53 + 65\right\}$ (05)

$= \dfrac{1}{40}\left\{\sum_{i=1}^{38} x_i + 55 + 63\right\} =$ mean $\left(\begin{matrix}\text{after the}\\ \text{correction}\end{matrix}\right)$ (05)

The mean is not affected due to the error.

10

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Aliter :

(i) Sun of the two observations = 53 + 65 = 118
(before correction)
Sun of two observations = 55 + 63 = 118 (05)
(After the correction)
Since the two sums are the same the mean is not affected due to the error. (05) [10]

(ii) Variance

$$= 3.2 = \frac{1}{40}\left\{\sum_{i=1}^{38}(x_i - 58)^2 + (53-58)^2 + (65-58)^2\right\} \quad (05)$$

$$= \frac{1}{40}\left\{\sum_{i=1}^{38}(x_i - 58)^2 + 25 + 49\right\}$$

(before correction)

Let S^2 be the variance after correction.

Then

$$S^2 = \frac{1}{40}\left\{\sum_{i=1}^{38}(x_i - 58)^2 + (55-58)^2 + (63-58)^2\right\} \quad (05)$$

$$= \frac{1}{40}\left\{\sum_{i=1}^{38}(x_i - 58)^2 + 9 + 25\right\}$$

(after correction)

$$\therefore S^2 - 3.2 = \frac{9-49}{40} = -1 \Rightarrow S^2 = 2.2 \quad (05)$$

The variance is reduced due to the correction. [15]

Aliter :

(ii) Sun of squares of the deviations of the two observations from mean.
$= (53 - 58)^2 + (65 - 58)^2 = 25 + 49 = 74$
(before correction) (05)

Sun of squares of the deviations of the two observations from mean.
$= (55 - 58)^2 + (63 - 58)^2 = 9 + 25 = 34$
(after the correction) (05)

Since the two sum after the correction is reduced the variance is reduced due to the correction. (05) [15]

(b) (i) Mean

Long method

Mid Point (x)	Frequency (f)	fx
5	10	50
15	27	405
25	33	825
35	35	1225
45	38	1710
55	30	1650
65	19	1235
75	8	600
Total	200	7700
	Mean	38.5

$$Mean = \frac{7700}{200} = 38.5$$

(10)

Coding Method

Mid Point (x)	Frequency (f)	$d = \frac{x-45}{10}$	df
5	10	-4	-40
15	27	-3	-81
25	33	-2	-66
35	35	-1	-35
45	38	0	0
55	30	1	30
65	19	2	38
75	8	3	24
Total	200		-130

$$Mean = 45 + \left(-\frac{130}{200}\right) \times 10$$

$$= 38.5$$

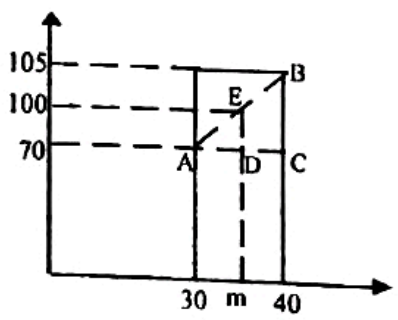
[10]

Median : Median class is 30 - 40

Let m is the median of the distribution.

$$\frac{m-30}{40-30} = \frac{100-70}{105-70} = \frac{30}{35} \quad (05)$$

$\Rightarrow$ m = 30 + 8.57 = 38.57 (05)

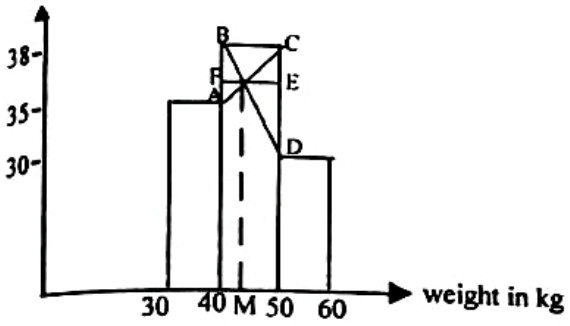


[10]

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Mode : Modal class is 40 - 50

Let M is the mode of the distribution.



$$\frac{M-40}{50-M} = \frac{38-35}{38-30}$$ (05)

$$\frac{M-40}{50-M} = \frac{3}{8}$$

$$\Rightarrow 11M = 150 + 320$$

$$\Rightarrow M = \frac{470}{11} \Rightarrow M = 42.73$$ (05) [10]

The maximum number of passengers when mean is used

$$= \frac{1500}{38.5} = 38.96 \ \underline{\Omega}\ 39$$ (05)

The maximum number of passengers when mode is used

$$= \frac{1500}{42.73} = 35.1 \ \underline{\Omega}\ 35$$

The maximum number of passengers who can be transported safely at a time is 35. (05) [10]

Mid Point (x)	Frequency (f)	$d = \frac{x-45}{10}$	fd	fd^2
5	10	-4	-40	160
15	27	-3	-81	243
25	33	-2	-66	132
35	35	-1	-35	35
45	38	0	0	0
55	30	1	30	30
65	19	2	38	76
75	8	3	24	72
Total	200		-130	748

(05)

$$d = \frac{-130}{200} = -6.5 \quad S_d^2 = \frac{748}{200} - (-0.65)^2$$ (05)

$$= 3.74 - 0.4225$$

$$= 3.3175$$

$$S_d = 1.82$$

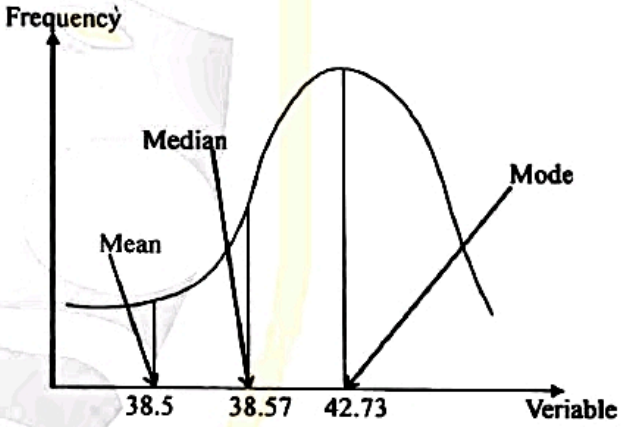
$$S_x = 10 \times 1.82 = 18.2$$ (05) [15]

Coefficient of skewness

$$= \frac{3(mean - median)}{standard\ deviation}$$

$$= \frac{3(38.5 - 38.57)}{18.21} = -0.115$$ (05)

Shape of the distribution is negative skewed. (05)



[10]