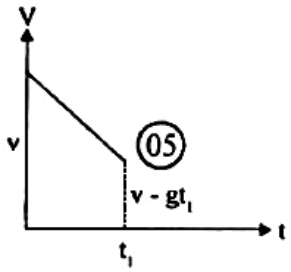
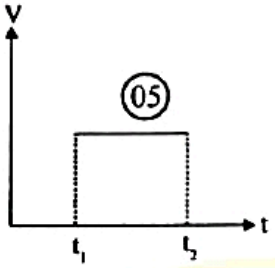
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G.C.E. (Advanced Level) Examination - August 2009
10 - Combined Mathematics - II
Marking Scheme

(01) **(a)** During the interval $O \le t \le t_1$ velocity of p, relative to B :





During the interval $t_1 \le t \le t_2$ when both particles P and Q move freely under gravity, since their relative acceleration is zero,

Velocity of Q, relative to P = constant (05)

= $V - (V - gt_1) = gt_1 \uparrow$ (05)

At time $t = t_1$, displacement of P, relative to Q

= displacement of P, relative to B

= $\frac{t_1}{2}\{V + (V - gt_1)\}$ (05)

When they meet, at time $t = t_2$

displacement of Q, relative to P is zero (05)

$$\Rightarrow gt_1(t_2 - t_1) - \frac{t_1}{2}(2v - gt_1) = 0 \quad (05)$$

and since $t_1 > 0$, $t_2 - t_1 = \frac{v}{g} - \frac{t_1}{2}$, $t_2 = \frac{v}{g} + \frac{t_1}{2}$ (05)

when they meet, at time $t = t_2$,

velocity of Q = $(U + V) - g(t_2 - t_1)$ (05)

= $U + \frac{1}{2}gt_1 \uparrow$

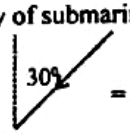
Velocity of P = velocity of P, relative to Q + Ve. of Q

= $-gt_1 + \left(U + \frac{1}{2}gt_1\right)\uparrow$

= $U - \frac{1}{2}gt_1 \uparrow$ (05) [50]

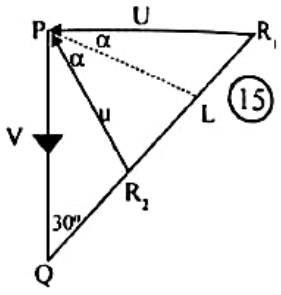
Solution : Q . 1 (b)

Velocity of submarine, relative to ship = Su, E + E, Sh



= U + V (05)

velocity Triangle : { PQR1, PQR2



The two direction in which the submarine may proceed are those of vectors $\overrightarrow{R_1P}$ and $\overrightarrow{R_2P}$ (05)

They differ by an angle $2\alpha = 2\cos^{-1}\left(\frac{V}{2u}\right)$ (05)

Time $T_1 = \frac{d}{QR_1} = \frac{d}{QL + LR_1}$, where L is the foot of the perpendicular from P to the path of sub, relative to ship. (05)

Similarly, time $T_2 = \frac{d}{QR_2} = \frac{d}{QL - LR_2}$ (05)

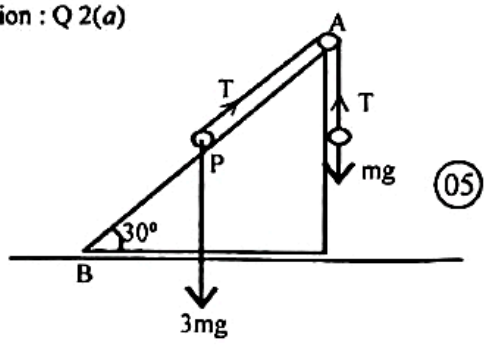
difference in times for the two possible directions is

$$T_2 - T_1 = \frac{2d(LR_1)}{QL^2 - LR_1^2} \quad (05)$$

$$= \frac{2d\sqrt{u^2 - \left(\frac{v}{2}\right)^2}}{\left(\frac{v\sqrt{3}}{2}\right)^2 - \left(u^2 - \frac{v^2}{4}\right)}$$

$$= \frac{d\sqrt{4u^2 - v^2}}{v^2 - u^2} \quad (05) \qquad [50]$$

solution : Q 2(a)



(05)

Let the acceleration of P relative to wedge = f

Then, the acceleration of Q, relative to wedge is of the same magnitute f, but directed vertically upwards = $f\uparrow$ (05)

Let the acceleration of wedge = $\rightarrow F$

Then, the acceleration of P = $\underrightarrow{F} + f\swarrow$ (05)

and acceleration of Q = $\underrightarrow{F} + f\uparrow$ (05)

Applying $F = m\underline{a}$

(i) For the system, horizontally,

$\rightarrow \quad O = 2m\,F + mF + 3m\left(F - \frac{\sqrt{3}}{2}f\right) \rightarrow (01)$ (05)

(ii) For P, along the inclined face,

$\swarrow \quad \frac{1}{2}(3mg) - T = 3m\left(f - \frac{\sqrt{3}}{2}F\right) \rightarrow (02)$ (05)

(iii) For Q, vertically upwards,

$\uparrow \quad T - mg = mf \rightarrow (03)$ (05)

From (01), $F = \frac{\sqrt{3}}{4}f$ (05)

Also (02) + (03) $\Rightarrow \frac{1}{2}mg = 4mf - \frac{3\sqrt{3}}{2}mf$,

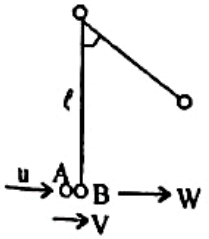
Eliminating f from the last two equations, and multiplying by $\frac{2\sqrt{3}}{m}$

$g\sqrt{3} = F(32-9) \Rightarrow F = \frac{\sqrt{3}\,g}{23}$ (05)

From (03), $T = m(g + f)$

$= mg\left(1 + \frac{4}{23}\right) = \frac{27mg}{23}$ (05) [50]

Solution : Q 2(b)



For direct impact of the two spheres:
Conservation of Linear Momentum gives,

$\rightarrow mV + mW = mu$

$\Rightarrow V + W = u$ (05)

Newton's law of Restitution gives,

$\rightarrow W - V = eu$ (05)

Solving $V = (1-e)\frac{u}{2}$ and $W = (1-e)\frac{u}{2} \rightarrow (1)$ (05) (05)

For motion of projectile A :

$\rightarrow \frac{\ell}{2} = vt, \quad \downarrow 2\ell = \frac{1}{2}gt^2$ (05)

Eliminating between the last two equations,

$\left(\frac{\ell}{2v}\right)^2 = \frac{4\ell}{g}$

$\Rightarrow v^2 = \frac{g\ell}{16} \rightarrow (2)$ (05)

For motion of the bob B (after impact), until it comes to instantaneous rest :

Principle of conservation of Energy gives,

$\frac{1}{2}mW^2 = mg\ell(1-\cos\alpha)$ (05)

$W^2 = 4g\ell\sin^2\left(\frac{\alpha}{2}\right) \rightarrow (3)$ (05)

From (1), (2) and (3) $\frac{1+e}{1-e} = \frac{W}{v} = 8\sin\left(\frac{\alpha}{2}\right)$ (05)

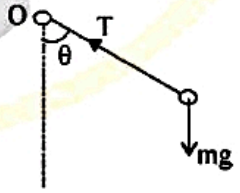
$\Rightarrow e = \frac{8\sin\left(\frac{\alpha}{2}\right)-1}{8\sin\left(\frac{\alpha}{2}\right)+1}$ (05) [50]

Solution : Q 3

Applying the principle of conservation of Energy, for the motion of Particle P :

$\frac{1}{2}mv^2 - mg\ell\cos\theta = \frac{1}{2}mu^2 - mg\ell$ (10)

$v^2 = u^2 - 2g\ell(1-\cos\theta)$ (05)



Applying $\underrightarrow{F} = m\underline{a}$, for particle P along $\overrightarrow{PO}$:

$T - mg\cos\theta = m\left(\frac{v^2}{\ell}\right)$ (10)

The tension : $T = mg\cos\theta + \frac{m}{\ell}\left[u^2 - 2g\ell(1-\cos\theta)\right]$ (05)

$T = m\left(3g\cos\theta - 2g + \frac{u^2}{\ell}\right)$ [30]

(ii) Particle P can subsequently reach the horizontal level of O, provided that

$v^2 \geq 0$, *when* $\theta = \frac{\pi}{2}$ (05)

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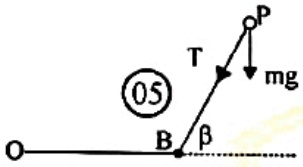
$\Rightarrow u^2 - 2g\ell \geq 0,\ u \geq \sqrt{2g\ell}$ (05)

The least possible value of u, so that P reaches the level of O is $\sqrt{2g\ell}$ (05) [15]

(iii) For circular motion of particle P, about centre B applying the principle of conservation of energy:

$$\frac{1}{2}mv_1^2 + mg\left(\frac{\ell}{2}\right)\sin\beta = \frac{1}{2}m\left(u^2 - 2g\ell\right); \text{ (10)}$$

$$v_1^2 = u^2 - 2g\ell - g\ell\sin\beta \text{ (05)}$$



Applying $\underline{F} = m\underline{a}$, for particle P along PB,

$$T_1 + mg\sin\beta = m\left(\frac{v_1^2}{\frac{\ell}{2}}\right) \text{ (10)}$$

$$T_1 = -mg\sin\beta + \frac{2m}{\ell}\left\{u^2 - 2g\ell - g\ell\sin\beta\right\} \text{ (05)}$$

[35]

The motia lasts as long as the string is taut, or $T_1 \geq 0$

i.e. $2\left(u^2 - 2g\ell\right) \geq 3g\ell\sin\beta$ (05)

given $2g\ell < u^2 < \frac{7}{2}g\ell$;

(i) First inequality ensures that the portion BP of the string rises above the level of OB.

(ii) Second inequality implies that,

$$\sin\beta < \frac{2}{3g\ell}\left(\frac{7}{2} - 2\right)g\ell = 1 = \sin 90^0 \text{ (05)}$$

$$\beta < 90^0 \text{ (05)}$$

Hence, under the given conditions, the string becomes slack before portion BP becomes vertical. [20]

Alternative Method (last Part)

When the string becomes slack.

$$T_1 = 0 \Rightarrow 2\left(u^2 - 2g\ell\right) - 3g\ell\sin\beta = 0 \text{ (05)}$$

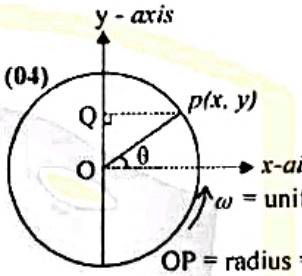
$$\sin\beta = \frac{2}{3g\ell}\left(u^2 - 2g\ell\right)$$

Since $0 < \sin\beta < 1$, $0 < \frac{2}{3g\ell}\left(u^2 - 2g\ell\right) < 1$ (05)

$\Rightarrow$ The required inequalities

$$2g\ell < u^2 < \frac{7}{2}g\ell$$

(05) (05) [20]



(05)

ω = uniform angular speed = $\dot{\theta} = \frac{d\theta}{dt}$

OP = radius = a, $P\hat{O}X = \theta$ and $0 \leq \theta < 2\pi$

Now, OQ = y = $a\sin\theta$ and $\theta = \omega t$ (05)

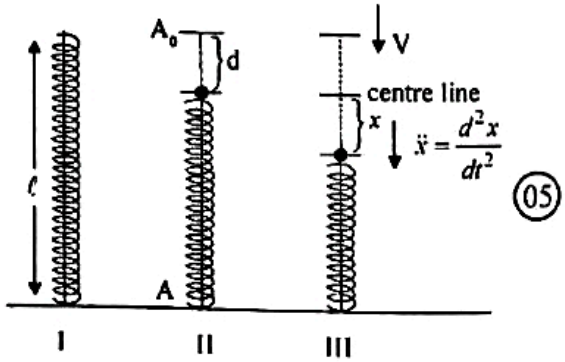
$$\Rightarrow \frac{dy}{dt} = a\omega\cos\theta \text{ (05)}$$

$$\Rightarrow \frac{d^2y}{dt^2} = -a\omega^2\sin\theta = -\omega^2 y \text{ (05)}$$

i.e. $\ddot{y} = -\omega^2 y; \quad |y| \leq a$

Thus the motion of Q is SHM on the y - axis, while P performs a circular motion with constant angular speed $\omega = \dot{\theta}$ (05)

Hence the period of the SHM is $\frac{2\pi}{\omega}$ [25]



(05)

In the situation II, $mg = T_0 = \frac{\lambda}{\ell}d \rightarrow (01)$ (05)

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Where λ is the modulus of the spring.

In the situation (III), $mg - T = m\ddot{x} \rightarrow (02)$ (05)

Where $T = \frac{\lambda}{\ell}(d + x)$

By (01) and (02), $\ddot{x} = -\frac{\lambda}{m\ell}x = -\frac{g}{d}x \rightarrow (03)$ (05)

Hence the particle executes a SHM with $\omega = \sqrt{\frac{g}{d}}$ (05)

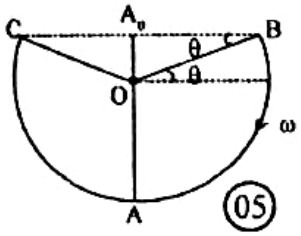
write $\ddot{x} = v\frac{dv}{dx}$, where $v = \dot{x} = \frac{dx}{dt}$

Then we get $v^2 = -\frac{g}{d}x^2 + g(d + 2h)$ (10)

$\Rightarrow x_{max} = \sqrt{d^2 + 2dh} = a = amplitute$

During the motion, the maximum compression of the spring mist be less then or equal to ℓ.

There fore, $a + d \le \ell$ (05) 40



(05)

$B\hat{O}A = \theta + \frac{\pi}{2}$

$OA = a = \sqrt{d^2 + 2dh}$

$OA_0 = d$; $A_0B = \sqrt{2dh}$

$\tan\theta = \sqrt{\frac{d}{2h}}$ (05)

Using the reverse geometrical interpretation of the SHM, the time where the particle remains on the spring is given by,

$T = 2\left(\frac{B\hat{O}A}{\omega}\right) = 2\left(\frac{\theta + \pi/2}{\omega}\right)$ (10)

$\Rightarrow T = \sqrt{\frac{d}{g}}\left[2\tan^{-1}\sqrt{\frac{d}{2h}} + \pi\right]$ (05)

It is given that $T \ge \frac{3\pi}{2}\sqrt{\frac{d}{g}}$

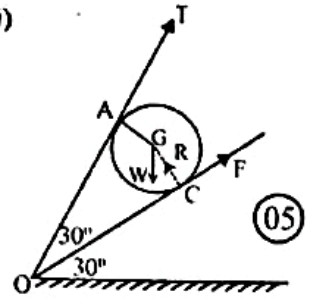
therefore, $\tan^{-1}\sqrt{\frac{d}{2h}} \ge \frac{\pi}{4}$ (05)

$\Rightarrow \sqrt{\frac{d}{2h}} \ge 1$

$\Rightarrow \frac{h}{d} \le \frac{1}{2}$ (05)

$\Rightarrow \left(\frac{h}{d}\right)_{max} = \frac{1}{2}$ 35

(05) (a)



(05)

G ↺ $Fa = Ta \rightarrow (01)$ (05)

C ↻ $Ta(1 + \cos 30^0) = Wa \sin 30^0 \rightarrow (02)$ (10)

O ↺ $Ra \cot 15^0 = W\,a\,\text{cosec}\,15^0 \cos 45^0 \rightarrow (03)$ (10)

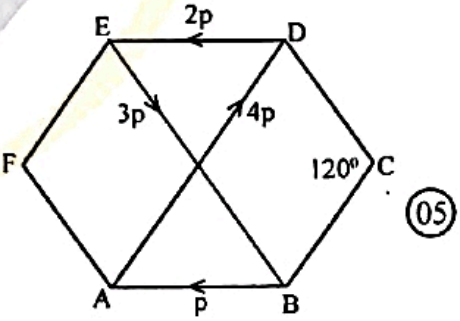
From (01) and (02), $F = T = W\left(2 - \sqrt{3}\right)$ (05)

From (03), $R = \frac{W}{\sqrt{2}\cos 15^0}$ (05)

Since $F \le \mu R$, $\mu \ge \frac{\sqrt{2}\cos 15^0}{2 + \sqrt{3}}$ (05)

$\mu \ge \sqrt{2}\left(2 - \sqrt{3}\right)\cos 15^0$ (05) 50

(b)



(05)

$\rightarrow (along\ AB)\ x = -p - 2p + \frac{1}{2}(4p + 3p) = \frac{1}{2}p$ (05)

$\uparrow (along\ AE)\ y = (4 - 3)p\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}p$ (05)

Magnitude of $R = \sqrt{x^2 + y^2} = p$ (05)

Resultant R makes an angle $\tan^{-1}\sqrt{3}\left(= \frac{\pi}{3}\right)$ *with* $\overrightarrow{AB}$ (05)

Thus resultant R is parallel to the side $\overrightarrow{BC}$

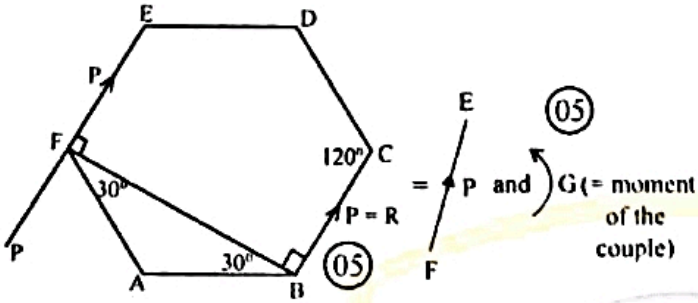
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D ↻ $3p\frac{a\sqrt{3}}{2} - pa\sqrt{3} = p\left(\frac{a\sqrt{3}}{2}\right)$ (05)

= p × prependicular distances to BC from D (05)

Therefore the line of action of the resultant R is along BC

35



(05)

= P and ↻ G (= moment of the couple) (05)

Introducing a couple of moment

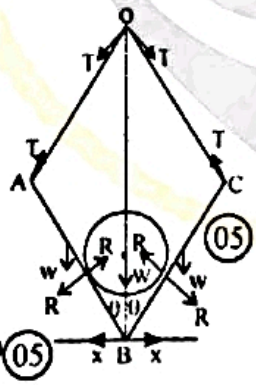
$G = p \times FB = p \times 2a\frac{\sqrt{3}}{2} = Pa\sqrt{3}$ (05)

in the clockwise sense, the system is reduced to a single force P along $\vec{FE}$

15

(06) (a)

Resolving vertically for the sphere.



(05)

↑ $2R\sin\theta = W \rightarrow (01)$ (05)

$R = \frac{W}{2}\operatorname{cosec}\theta$

For the two rods AB and BC

↑ $2T\cos\theta = 2W + 2R\sin\theta \rightarrow (02)$ (05)

$T = \frac{3W}{2}\sec\theta$

Taking moments about B, for the rod BA.

B ↻ $W \cdot a\sin\theta + R \cdot \frac{a}{3}\cot\theta - T(2a\sin 2\theta) = 0 \rightarrow (03)$ (05)

Using (01), (02) and (03) we get

$W \cdot a\sin\theta + \frac{a}{3}\cdot\frac{W}{2}\operatorname{cosec}\theta \cdot \cot\theta - 3W\sec\theta \cdot a\sin 2\theta = 0$

$6\sin\theta + \operatorname{cosec}\theta\cot\theta - 18\sec\theta\sin 2\theta = 0$

$6 + \operatorname{cosec}^2\theta\cot\theta - 36 = 0$

$6 + (1 + \cot^2\theta)\cot\theta - 36 = 0$

$\cot^3\theta + \cot\theta - 30 = 0$ (05)

$(\cot\theta - 3)(\cot^2\theta + 3\cot\theta + 10) = 0$

$\therefore \cot\theta = 3$ or $(\cot^2\theta + 3\cot\theta + 10) = 0 \rightarrow (04)$

Since Δ (discriminant) < 0, the quadratic $(\cot^2\theta + 3\cot\theta + 10) = 0$ has no, real roots, and the only possible calue is $\cot\theta = 3$ (05)

35

By the symmetry, the reaction X at B is horizontal Resolving horizontally for AB →; (05)

$-X - R\cos\theta + T\sin\theta = 0$ (05)

$\therefore X = -R\cos\theta + T\sin\theta$

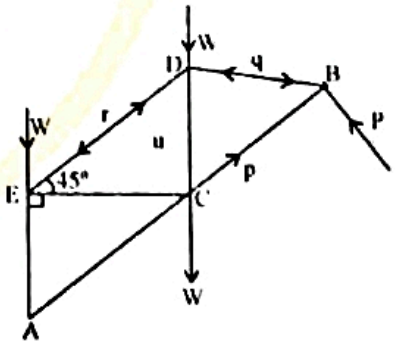
$= \frac{-W}{2}\cot\theta + \frac{3W}{2}\tan\theta$

$= \frac{-3W}{2} + \frac{W}{2} = -W.\rightarrow$ (05)

(Thus the reaction X at the joint B is W and is in the outward direction, ←)

15

(b)



taking moments about, for the framework.

A ↻ $p \times 4a = W \times 2a\sin 45^0 + W \times 2a\sin 45^0$ (05)

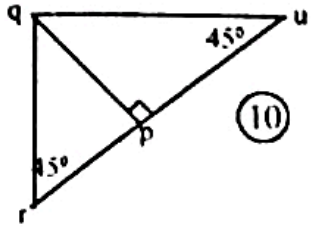
where BC = 2a

$P = 4Wa \cdot \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{2}\,W}{2}$ (05)

10

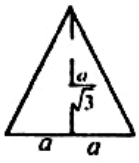
Stress diagram



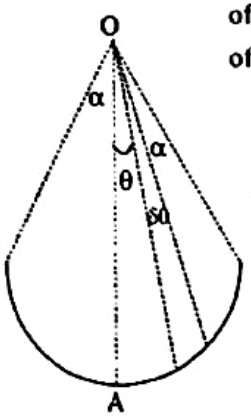
(10)

Rod	Stress	Magnitude
BD	Thrust	W
BC	Tension	$\frac{W}{\sqrt{2}}$
DE	Thrust	$W\sqrt{2}$

(15) (15) 40

Object	Mass	Distance from C to the centre of mass
Semi-circular lamina 	$\frac{1}{2}\pi a^2\sigma$ (05)	$\frac{a}{\sqrt{3}}+\frac{2a}{3}\left(\frac{\sin\pi/2}{\pi/2}\right)$ $=\frac{a}{\sqrt{3}}+\frac{4a}{3\pi}$ (05)
Triangular lamina 	$\frac{1}{2}(2a)\cdot\left(\frac{a}{\sqrt{3}}\right)\sigma$ $=\frac{a^2\sigma}{\sqrt{3}}$ (05)	$\frac{2}{3}\cdot\frac{a}{\sqrt{3}}$ (05)
Sector of angle $\frac{2\pi}{3}$ 	$\frac{1}{2}\left(\frac{2\pi}{3}\right)\left(\frac{2a}{\sqrt{3}}\right)^2\sigma$ $=\left(\frac{4\pi a^2}{9}\right)\sigma$ (05)	$\frac{2}{3}\left(\frac{2a}{\sqrt{3}}\right)\cdot\frac{\sin\pi/3}{(\pi/3)}$ $=\frac{2a}{\pi}$ (05)
crescent 	$\frac{\pi a^2\sigma}{2}+\frac{a^2\sigma}{\sqrt{3}}-\frac{4\pi a^2\sigma}{9}$ $=a^2\left(\frac{\pi}{18}+\frac{1}{\sqrt{3}}\right)\sigma$ (05)	$\bar{x}$

(07)

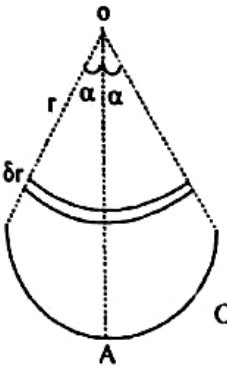


Let λ be the mass per unit length of are By symmentry the centre of mass G lies on OA, and

$$OG = \frac{\int_{-\alpha}^{\alpha} \lambda r\cos\theta \cdot r\, d\theta}{\int_{-\alpha}^{\alpha} \lambda r\, d\theta} \quad (10)$$

$$= \frac{r\sin\theta\big|_{-\alpha}^{\alpha}}{2\alpha} = \frac{r\sin\alpha}{\alpha} \quad (05)$$

15



Let σ be the mass per unit area. By symmentry, the centre of mass G_1 lies on OA, and

$$OG_1 = \frac{\int_0^a \sigma\cdot 2\alpha\cdot r\cdot\left(r\frac{\sin\alpha}{\alpha}\right)dr}{\int_0^a \sigma 2\,\alpha r\, dr} \quad (10)$$

$$= \left(\frac{\sin\alpha}{\alpha}\right)\frac{\frac{r^3}{3}\Big|_0^a}{\frac{r^2}{2}\Big|_0^2} = \frac{2a}{3}\left(\frac{\sin\alpha}{\alpha}\right) \quad (05)$$

15

Taking moments, about C

$$\frac{a^2}{18\sqrt{3}}\left(\sqrt{3}\pi+18\right)\bar{x}$$

$$= \frac{1}{2}\pi a^2\left(\frac{1}{\sqrt{3}}+\frac{4}{3\pi}\right)a + \frac{a^2}{\sqrt{3}}\cdot\frac{2}{3}\cdot\frac{a}{\sqrt{3}} - \frac{4\pi a^2}{9}\cdot\frac{2a}{\pi} \quad (10)$$

$$= a\left\{\frac{\pi}{2\sqrt{3}}+\frac{2}{3}+\frac{2}{9}-\frac{8}{9}\right\}$$

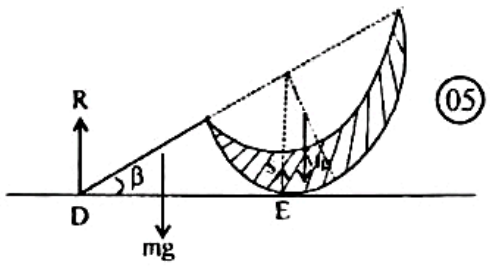
$$\because \bar{x} = \frac{a\pi}{2\sqrt{3}}\cdot\frac{18\sqrt{3}}{\left(\sqrt{3}\cdot\pi+18\right)} \quad (05)$$

$$= \frac{9\pi}{\left(\sqrt{3}+18\right)}a = \frac{3\sqrt{3}\pi}{\left(\pi+6\sqrt{3}\right)}a = ka$$

50

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05

Taking moments, about E for the gomposite body.

E ↷

$R \cdot 3a\cos\beta + Mg\cdot\left(ka - \frac{a}{\sqrt{3}}\right)\sin\beta - mg\cdot 2a\cos\beta = 0$ (05)

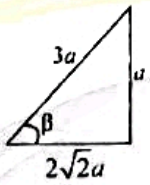
For equilibrium with the end D touching the floor, R > 0

Hence, We must have (05)

$mg \times 2a\cos\beta > Mg\left(ka - a/\sqrt{3}\right)\sin\beta$

$\sqrt{3}\ \frac{\cot\beta}{2\sqrt{2}}\ 2m > M\left(\sqrt{3}k - 1\right)a$ (05)

$M\left(\sqrt{3}\ k - 1\right) < 4\sqrt{6}\ m$



$\cos\beta = \frac{2\sqrt{2}}{3}$

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(08) Conditional Probability of B given A is defined to be

$P(B/A) = \frac{P(A\cap B)}{P(A)}$, provided that P(A) > 0 (10)

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provided that P(A) > 0 , and $P(A\cap B) > 0$

$P(A)\ P(B/A)\ P\{C/(A\cap B)\}$

$= P(A)\left\{\frac{P(A\cap B)}{P(A)}\right\}\ \left\{\frac{P\{A\cap(B\cap C)\}}{P(A\cap B)}\right\} = P(A\cap B\cap C)$ (10)

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Since $\{B_1, B_2, B_3\}$ is a partition of event Ω

Event $A = (A\cap B_1)\cup(A\cap B_2)\cup(A\cap B_3)$ (05)

$P(A) = P\{(A\cap B_1)\cup(A\cap B_2)\cup(A\cap B_3)\}$

$= P(A\cap B_1) + P(A\cap B_2)\cup P(A\cap B_3)$ (05)

$\therefore$ $(A\cap B_1)$ $(A\cap B_2)$ $(A\cap B_3)$ are mutually exclusive events

$= P(B_1)\,P(A/B_1) + P(B_2)\,P(A/B_2) + P(A/B_3)$ (05)

for i = 1, 2, 3.

$P(B_1/A) = \frac{P(A\cap B_1)}{P(A)}$

$= \frac{P(B_1)P(A/B_1)}{P(B_1)\,P(A/B_1) + P(B_2)P(A/B_2) + P(B_3)P(A/B_3)}$ (05)

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P(vehicle turning left) = 0.5 P(vehicle turning right) = 0.2 and P(vehicle going straght on) = 0.3

Let V_1, V_2 and V_3 be the three consecutive vehicles.

Let A_i (i = 1, 2, 3) be the event that the vehicle V_i turus left.

Let B_i (i = 1, 2, 3) be the event that the vehicle V_i turus right.

Let C_i (i = 1, 2, 3) be the event that the vehicle V_i goes straight on.

Then for i = 1, 2, 3 $P(A_i) = 0.5$, $P(B_i) = 0.2$ and $P(C_i) = 0.3$.

(i) $P(c_1\ c_2\ c_3) = p(c_1)\ p(c_2)\ p(c_3) = 0.3 \times 0.3 \times 0.3$ (05)

$= 0.027$ (05)

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(ii) $p\{(A_1\ A_2\ A_3)\cup(B_1\ B_2\ B_3)\cup(C_1\ C_2\ C_3)\}$ (05)

$= P(A_1\ A_2\ A_3) + p(B_1\ B_2\ B_3) + P(C_1\ C_2\ C_3)$

$= (0.5)^3 + (0.2)^3 + (0.3)^3$

$= 0.125 + 0.008 + 0.027$

$= 0.16$ (05)

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(iii) $P\{(B_1B_2A_3)\cup(B_1A_2B_3)\cup(A_1B_2B_3)\}$ (05)

$= P(B_1B_2A_3) + P(B_1A_2B_3) + P(A_1B_2B_3)$

$(0.5)\,(0.2)^2 + (0.2)^2\,(0.5) + (0.2)^2\,(0.5)$

$= 3 \times 0.02$

$= 0.06$ (05)

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(iv) $P\{(A_1B_2C_3)\cup(B_1C_2A_3)\cup(C_1A_2B_3)$

$= P(A_1B_2C_3) + P(B_1C_2A_3) + P(C_1A_2B_3)$

$= 6\,(0.5)\,(0.2)(0.3)$ (05)

$= 0.18$ (05)

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Let D be the event that all three vehicles go in the same direction

Then $D = (A_1A_2A_3) \cup (B_1B_2B_3) \cup (C_1C_2C_3)$

$$P\{(A_1A_2A_3)/D\} = \frac{P\{(A_1A_2A_3) \cap D\}}{P(D)}$$

$$= \frac{P(A_1A_2A_3)}{P(D)} = \frac{0.125}{0.16} = 0.78125 \quad (05)$$

$$P\{(B_1B_2B_3)/D\} = \frac{P\{(B_1B_2B_3) \cap D\}}{P(D)}$$

$$= \frac{P(B_1B_2B_3)}{P(D)} = \frac{0.008}{0.16} = 0.05 \quad (05)$$

$$P\{(C_1C_2C_3)/D\} = \frac{P\{(C_1C_2C_3) \cap D\}}{P(D)}$$

$$= \frac{P(C_1C_2C_3)}{P(D)} = \frac{0.027}{0.16} = 0.16875 \quad (05)$$

$P\{(A_1A_2A_3)/D\}$ is the highest among the three conditional probabilities.

Thus if the three consecutive vehicles all go in the same direction, then most of the time they all turn left. (05)

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(09) (a) $\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n}\left(x_i^2 - 2\bar{x}\,x_i + \bar{x}^2\right)$ (05)

$$= \sum_{i=1}^{n} x_i^2 - 2\bar{x}\sum_{i=1}^{n} x_i - n\bar{x}^2 \quad (05)$$

$$= \sum_{i=1}^{n} x_i^2 - n\bar{x}^2 \quad \therefore \sum_{i=1}^{n} x_i^2 = n\bar{x} \quad (05)$$

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Let x_i be the number of printing mistakes in the i^{th} page of the book.

Then $\sum_{i=1}^{200} x_i = 920$ *and* $\sum_{i=1}^{200} x_i^2 = 5032$ (05)

$$Mean = \frac{1}{200}\sum_{i=1}^{200} x_i = \frac{920}{200} = 4.6 \quad (05)$$

$$\text{Variance} = \frac{1}{200}\sum_{i=1}^{200} x_i^2 - \bar{x}^2 = \frac{5032}{200} - 4.6^2 \quad (05)$$

$$= 25.16 - 21.16$$

$$= 4 \quad (05)$$

$\therefore$ the standard deviation = 2 (05)

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$$\frac{1}{50}\sum_{i=201}^{250} x_i = 4.4 \Rightarrow \sum_{i=201}^{250} x_i = 220 \quad (05)$$

$$\therefore mean = \frac{1}{250}\sum_{i=1}^{250} x_i = \frac{920 + 220}{250} = 4.56 \quad (05)$$

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$$\frac{1}{50}\sum_{i=201}^{250} x_i^2 - (4.4)^2 = (2.2)^2 \Rightarrow \sum_{i=201}^{250} x_i^2 = 1210 \quad (05)$$

$$\text{Veriance} = \frac{1}{250}\sum_{i=1}^{250} x_i^2 - (4.56)^2$$

$$= \frac{5032 + 1210}{250} - 20.7936$$

$$= 24.9680 - 20.7936$$

$$= 4.1744 \quad (05)$$

Standard deviation $= \sqrt{4.1744} = 2.04$ (05)

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(b) Let $\bar{x}$ and S_x be the mean and the standard deviation of the original marks.

Let $\bar{y}$ and S_y be the mean and the standard deviation of the scaled marks.

(i) Let $y = \alpha + \beta x$, where α and β are constant, be the liner transtermation from an original mark to the correspond -ing scaled mark.

Then, from given data, $80 = \alpha + 70\,\beta$

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Also, from the transformation equation, it follows that.

$\bar{y} = \alpha + \beta\bar{x} \Rightarrow 50 = \alpha + 45\beta$ (05)

$\Rightarrow \beta = 1.2$ and $\alpha = -4$ (05)

$\therefore y = -4 + 1.2x$

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When $y = 2$, $2 = -4 + 1.2x$ $\Rightarrow x = 5$ (05)

When $y = 92$, $92 = -4 + 1.2x$ $\Rightarrow x = 80$ (05)

Therefore the corresponding original marks are 5 and 80 respectively.

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(ii) Since

$S_y = 1.2\, S_x \Rightarrow 15 = 1.2\, S_x \Rightarrow S_x = 12.5$ (05)

(05)

The standard deviation of the original marks is 12.5

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(iii) When a mark is not ohanged, $y = x$

$\therefore x = -4 + 1.2x \Rightarrow x = 20$ (05)

The original mark of 20 is not changed under the scaling.

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