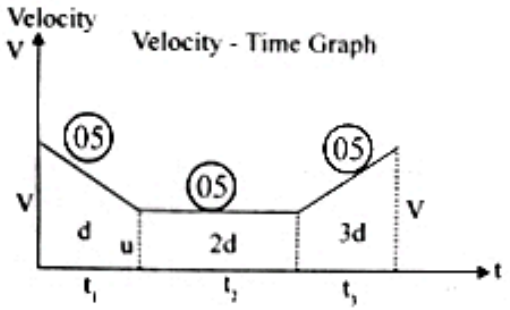
G.C.E. (Advanced Level) Examination - August 2008
10 - Combined Mathematics - II
Marking Scheme

(01) (a)



15

Displacement at uniform retardation is $= \frac{t_1}{2}(V+u) = d$

Displacement at uniform Velocity u is $= t_2 u = 2d$

Displacement at uniform acceleration $= \frac{t_3}{2}(V+u) = 3d$ (05)

$$t_1 + t_2 + t_3 = \frac{2d}{V+u} + \frac{2d}{u} + \frac{6d}{u+V}$$ (05)

Total time taken

$$= \frac{2d}{u(u+v)}\{4u + (u+v)\} = \frac{2d + (5u+v)}{u(u+v)} \text{ hours}$$ (05)

Time taken is no track repair is done $= \frac{6d}{v}$ hours

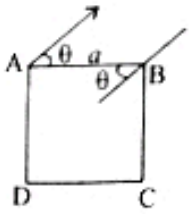
Time lost due to track - repair

$$= \frac{2d(5u+v)}{u(u+v)} - \frac{6d}{v}$$ (05)

$$= 2d\frac{(5uv + v^2 - 3u^2 - 3uv)}{uv(u+v)}$$

$$= \frac{2d(v^2 + 2uv - 3u^2)}{uv(u+v)}$$ (05)

$$= \frac{2d(v-u)(v+3u)}{uv(u+v)} \text{ hours}$$ 25



Velocity of H, W = V

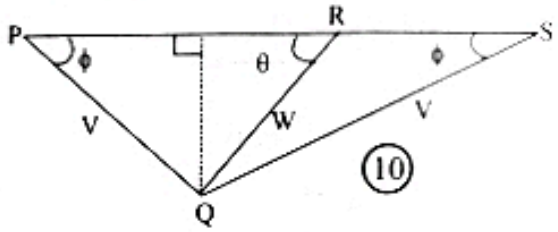
W, E = W ↗

Vector addition ⇨ Vel of

H, E = V + W ↗

(05)

Velocity triangles.



10

For the flight from A to B, in the direction $\overline{AB}$ Velocity of H, E = $\overline{PR}$

For the flight from C to D, in the direction $\overline{CD}$. Velocity of H, E = $\overline{SR}$

From the two velocity-triangles

$$PR = \sqrt{V^2 - \omega^2 \sin^2\theta} + W\cos\theta$$ (05)

$$SR = \sqrt{V^2 - \omega^2 \sin^2\theta} - \omega\cos\theta$$ (05)

Time from A to B $= \frac{a}{PR}$

Time from C to D $= \frac{a}{SR}$

Their sum $T_1 = a\left[\frac{SR + PR}{(PR)(SR)}\right]$

$$= \frac{2a\sqrt{V^2 - \omega^2\sin^2\theta}}{V^2 - \omega^2} \text{ hours}$$ (05) 30

Sum of the time from B to C and from D to A

(Replacing θ by $\frac{\pi}{2} - \theta$) $= \frac{2a\sqrt{V^2 - \omega^2\cos^2\theta}}{V^2 - \omega^2} = T_2$ (05)

(05)

Total time

$$T = T_1 + T_2 = A\left(\sqrt{V^2 - \omega^2 \sin^2\theta} + \sqrt{V^2 - \omega^2 \cos^2\theta}\right)$$ (05)

Where A is a constant $= 2a / v^2 - \omega^2$ [15]

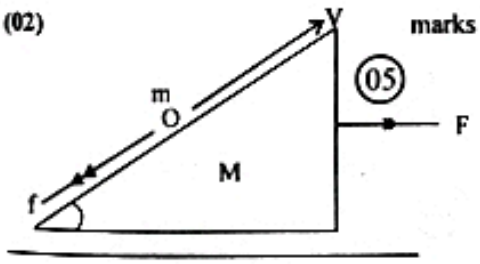
$$\frac{dT}{d\theta} = A\left[\frac{-\omega^2 \sin\theta\cos\theta}{\sqrt{V^2 - \omega^2 \sin^2\theta}} + \frac{\omega^2 \sin\theta \cdot \cos\theta}{\sqrt{V^2 - \omega^2 \cos^2\theta}}\right]$$

$$= \left[\frac{A\omega^2 \sin\theta\cos\theta}{\sqrt{V^2 - \omega^2 \sin^2\theta}\sqrt{V^2 - \omega^2 \cos^2\theta}\left[\sqrt{V^2 - \omega^2 \sin^2\theta} - \sqrt{V^2 - \omega^2 \cos^2\theta}\right]}\right]$$ (05)

When $0 < \theta < \frac{\pi}{4}$, $\frac{dT}{d\theta} > 0$ (05)

$\frac{\pi}{4} < \theta < \frac{\pi}{2}$, $\frac{dT}{d\theta} < 0$

$\therefore$ *T is a maximum When* $\theta = \frac{\pi}{4}$ (05) [15]

(02) 

marks for the Figure (05)

Acceleration of wedge M, T, = F →

acceleration of m, M = f ↙

Vector Addition ⇨ acceleration of m.T (05)

= (F - f cos α) → + ↓ f sin α

For the system, → m(F - f cosα) + MF = 0 (10)

(M + m) F = mfcosα

$$\frac{F}{f} = \frac{m\cos\alpha}{M+m} = constant \rightarrow (01)$$ (05) [25]

For motion of the particle, along the inclined face

↙ mg sin α = m (f - F cos α) (10)

$$g\sin\alpha = f - \frac{mf\cos^2\alpha}{M+m}, \text{ by making use of (01).}$$

$$= \frac{f}{M+m}\left(M + m\sin^2\alpha\right)$$

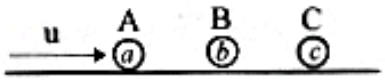
$$f = \frac{(M+m)g\sin\alpha}{M + m\sin^2\alpha}$$ (05)

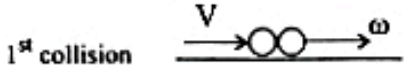
This is the constant retardation of m, relative to the wedge M. (05)

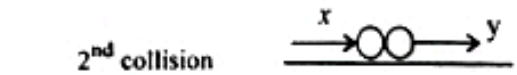
Using $S = ut - \frac{1}{2}ft^2$, for motion of the particle relative to wedge, ↗

Time taken to come back to the ponit of pro jection is

$$= \frac{2V}{f} = \frac{2V\left(M + m\sin^2\alpha\right)}{(M+m)g\sin\alpha}$$ (05) [25]

(02) (b) 





1st collision.

Conservation of momentum : av + bω = au (05)

Newton's law of restitution : ω - v = eu (05)

$$V = \frac{(a-be)u}{a+b} \rightarrow (01)$$ (05)

$$W = \frac{(1+e)au}{a+b} \rightarrow (02)$$ (05)

Similarly, for second collision, replacing (a, b) by (b, c), (v, w) by (x, y) and u by w.

$$x = \frac{(b-ce)W}{b+C} \rightarrow (03)$$ (05) $$y = \frac{(1+e)bW}{b+C} \rightarrow (04)$$ (05)

from (02), (04), $$y = \frac{abu(1+e)^2}{(a+b)(b+c)} = \frac{(1+e)^2 u}{\left(1+\frac{b}{a}\right)\left(1+\frac{c}{b}\right)}$$ (05) [35]

Given $v = 0, x = 0 \Rightarrow \frac{a}{b} = e = \frac{b}{c}$ (05)

$\therefore \quad a : b : c = e^2 : e : 1$

Initial KE $T_0 = \frac{1}{2}au^2$

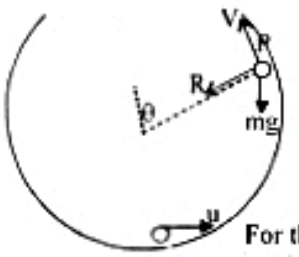
Final KE (retained in the system) $T = \frac{1}{2}cv^2$

$\Rightarrow \quad \frac{T}{T_0} = \frac{c}{a}\left(\frac{v}{u}\right)^2$ (05)

$$\frac{T}{T_0} = \frac{c}{a} \frac{(1+e)^4}{\left(1+\frac{1}{e}\right)^2\left(1+\frac{1}{e}\right)^2} = \frac{1}{e^2} e^4 = e^2$$ (05)

15

(03) (i)



(05)

For the figure

Let m be the mass of the particle.

Conservation of energy.

$\frac{1}{2}mv^2 + mga\cos\theta = \frac{1}{2}mu^2 - mga$ (15)

$V^2 = U^2 - 2ga(1+\cos\theta)$ (05)

$\Rightarrow \quad V = \sqrt{u^2 - 2ga(1+\cos\theta)}$ (05)

30

For particle P

$\swarrow \quad R + mg\cos\theta = \frac{mv^2}{a}$ (10)

$R = -mg\cos\theta + m\left[\frac{u^2}{a} - 2g(1+\cos\theta)\right]$ (05)

$= m\left[\frac{u^2}{a} - g(2+3\cos\theta)\right]$ (05)

20

(ii) The particle will leave the bowl at the [illegible] provided that $R > 0$, when

$\theta = \cos^{-1}\left(\frac{1}{4}\right) = \alpha$, an acute angle (05)

i.e. provided that $u^2 - ga\left(2+\frac{3}{4}\right) > 0$ (05)

i.e. provided that $u^2 > \frac{11ga}{4}$

10

(iii) When leaving the bowl at the edge where $\theta = \alpha$, the speed V of the particle is given by

$v^2 = u^2 - 2ga\left(1+\frac{1}{4}\right)$ (05)

$= u^2 - \frac{5ga}{2}$

In free motion under gravity

$\leftarrow \quad x = t.V\cos\alpha$ (05)

$\uparrow \quad O = t.v\sin\alpha - \frac{1}{2}gt^2$ (05)

$t = \frac{2v\sin\alpha}{g}$

$x = \frac{2v^2}{g}\sin\alpha\cos\alpha$ (05)

Particle will not fall back into the bowl, provided that $x > 2a\sin\alpha$. (05)

i.e. Provided that $\frac{2v^2}{g}\sin\alpha\cos\alpha > 2a\sin\alpha$ (05)

i.e. $v^2 > \frac{ga}{\cos\alpha}$ (05)

$u^2 - \frac{5}{2}ga > 4ga$ (05)

i.e. $u^2 > \frac{13}{2}ga$

40

(04) $T_0 = 4mg$ (05) $= \lambda.\frac{4a}{\ell}$ (05) $\Rightarrow \frac{\lambda}{\ell} = \frac{mg}{a}$ (05)

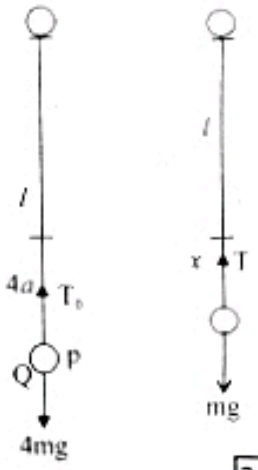
At time t

$T = \frac{\lambda x}{\ell} = \left(\frac{mg}{a}\right)x, \qquad x > 0$ (05)

$\downarrow$ $mg - T = m\ddot{x}$ (05)

$mg - \frac{mg}{a}x = m\ddot{x}$

$\frac{d^2x}{dt^2} + \frac{g}{a}(x - a) = 0, \; x > 0$ (05)



25

Given solution.

$x = a + b \sin \omega t + C \cos \omega t \rightarrow (01) \quad \omega^2 = \frac{g}{a}$

$\frac{dx}{dt} = b\omega \cos \omega t - c\omega \sin \omega t$ (05)

When $t = 0, x = 4a$ (05)

$4a = a + C$

$C = 3a$ (05)

When $t = 0, \; \frac{dx}{dt} = 0$ (05)

$0 = b\omega \Rightarrow b = 0$ (05)

25

When the string becomes slack $x = 0$, (05)

$\Rightarrow 0 = a + 3a \cos \omega t$ (05)

$\Rightarrow \cos \omega t = -\frac{1}{3}$

Moving under SHM, time t_1 at which the string becomes slack is given by $t_1 = \frac{1}{\omega}(\pi - \alpha)$ where α is the acute angle $\cos^{-1}\left(\frac{1}{3}\right)$ (05)

15

Then, velocity of P, just before the string becomes slack is

$\frac{dx}{dt} = -c\omega \sin \omega t_1 \quad where \quad \sin \omega t_1 = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$ (05)

$\frac{dx}{dt} = -3a\sqrt{(g/a)} \cdot \frac{2\sqrt{2}}{3} = -2\sqrt{2ga} = -V$

(05) (05)

15

In free motion under gravity maximum height reached is

$h = \frac{v^2}{2g} = \frac{8ga}{2g} = 4a$ (05)

Total maximum height, from the initial position

$= 4a + 4a = 8a$ (05)

10

In free motion under gravity, time $t_2 = \frac{v}{g}$

$= \frac{2\sqrt{2ga}}{g} = 2\sqrt{2}\sqrt{\frac{a}{g}}$ (05)

Total time to reach the highest position.

$= t_1 + t_2$

$= \frac{1}{\omega}(\pi - \alpha) + \frac{1}{\omega}2\sqrt{2}$ (05)

10

Alternative Method : Last part of (04)

with $X = x - a$, the equation of motion reads

$\ddot{x}^2 = \omega^2 X = 0$

This equation has a solution in the form

$\dot{x} = \omega^2\left(A^2 - X^2\right)$

When $t = 0, X = 3a, \; \dot{x} = 0$ (05)

$\Rightarrow$ $A = 3a$ (Amplitude of SHM) (05)

When the string becomes slack $x = 0, X = -a$ and

$\dot{X}^2 = \omega^2\left(9a^2 - (-a)^2\right)$ (05)

$\dot{X} = \dot{x} = -\omega a\left(2\sqrt{2}\right) = -V$ (05)

20

In free motion under gravity, maximum height reached is

$h = \frac{v^2}{2g} = \frac{8ga}{2g} = 4a$ in (05)

Total maximum height, from the initial position.

$= 4a + 4a = 8a$ (05)

10

Time taken by the particle in SHM (using the circular arc in the figure below) $t_1 = \frac{1}{\omega}(\pi - \alpha)$ (05)



Figure (05)

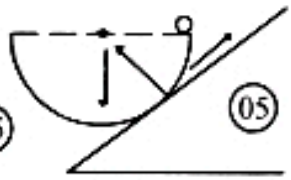
In free motion under gravity, time $t_2 = \frac{v}{g}$ (05)

$= \frac{2\sqrt{2ga}}{g} = 2\sqrt{2}\sqrt{\frac{a}{g}}$

Total time to reach the highest position = $t_1 + t_2$

$= \frac{1}{\omega}(\pi - \alpha) + \frac{1}{\omega}(2\sqrt{2})$ (05) [20]

(05) (a)



for the figure (05)

Resolving horizontally

→ $F\cos\alpha = R\sin\alpha$ →(01) (05)

Also, $F = \mu R$ →(02) (05)

Dividing (02) by (01), $\mu = \tan\alpha$ (05)

Resolving along AO : $R = (W + \omega)\cos\alpha$ →(03) (05)

Taking moments about O $Fa = Wa$ →(04) (05)

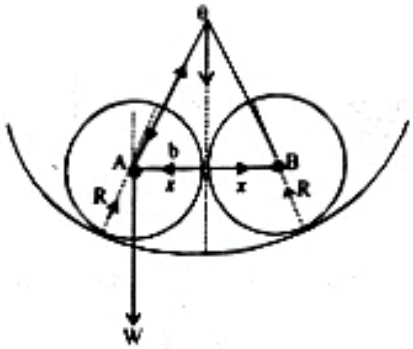
Dividing (04) by (03) $\mu = \frac{\omega}{W + \omega}\sec\alpha$ (05)

$\left(\frac{W + \omega}{\omega}\right)^2 = \frac{1 + \mu^2}{\mu^2}$

$\left(\frac{W + \omega}{\omega}\right)^2 - 1 = \frac{1}{\mu^2}$

$\mu = \frac{\omega}{\sqrt{(W + \omega)^2 - \omega^2}} = \frac{\omega}{\sqrt{W(W+2\omega)}}$ (05) [40]

(b)



↑ $2R\cos\theta = 2W$; $R = W\sec\theta$ (05)

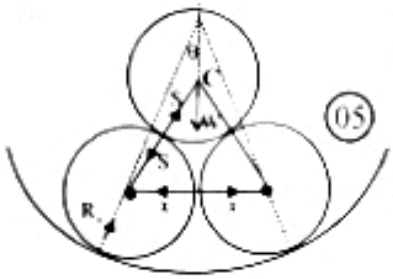
← $x = R\sin\theta$

$= W\tan\theta$ (05)

$= W\frac{b}{\sqrt{(a-b)^2 - b^2}}$ (05)

$= \frac{Wb}{\sqrt{a(a-2b)}}$ (05) [20]

For the Figure



(05)

↑ $2S\frac{\sqrt{3}}{2} = W$ (05)

$2R_1\cos\theta = 3W$ (05)

$\tan\theta = \frac{b}{\sqrt{a(a-2b)}}$

$x + \frac{S}{2} = R_1\sin\theta$ (05)

$x = \frac{3W}{2}\tan\theta - \frac{W}{2\sqrt{3}} > 0$ (05)

$\tan\theta > \frac{1}{3\sqrt{3}}$

$\frac{b^2}{a^2 - 2ab} > \frac{1}{27}$ (05)

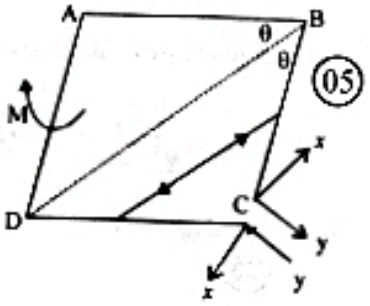
$a^2 - 2ab < 27b^2$

$(a-b)^2 < 28b^2$ (05)

$a - b < 2\sqrt{7}\,b$

$a < b(1 + 2\sqrt{7})$ (05) [40]

(06) (a)



(05)

Let (x, y) be the components of the reaction at C parallel and perpendicular to the string, respectively.

Taking moments about B for BC

$x \cdot 2a \sin\theta + y \cdot 2a \cos\theta = T \cdot a \sin\theta \rightarrow (01)$ (10)

Taking moments about D for DC;

$x \cdot 2a \sin\theta - y \cdot 2a \cos\theta = T \cdot a \sin\theta \rightarrow (02)$ (10)

(02) - (01) $\Rightarrow$ $y = 0$ There action at C is parallel to the string (05)

and $x = \frac{1}{2}T$ (05)

Taking moments about A for AD, DC

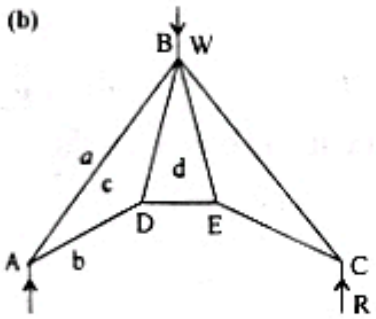
$M + x \cdot 4a \sin\theta = T \cdot 3a \sin\theta$ (10)

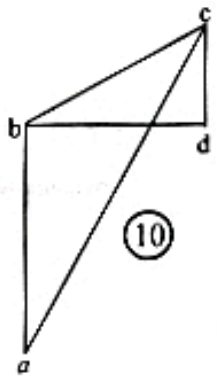
Eliminating x between the last two equations we get

$M = T(3 - 2)\sin\theta$

$\Rightarrow T = \frac{M}{a\sin\theta}$ (05) 50

(b)





(10)

From Δ abc, $\frac{ab}{\sin 30} = \frac{bc}{\sin 45} = \frac{ca}{\sin 105}$

$\Rightarrow bc = \frac{W}{\sqrt{2}}$; (05) Tension in AD (05)

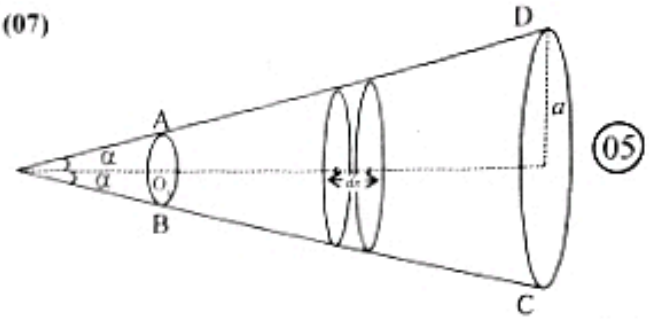
Also $\Rightarrow$ $ca = W \sin 75^0$, (05) Thrust in AB (05)

From Δ bde, $\frac{cb}{\sin 105^0} = \frac{bd}{\sin 60^0} = \frac{dc}{\sin 15^0}$

$\Rightarrow bd = \frac{W\sqrt{3}}{2\sqrt{2}\sin 75}$; *Tension in* DE (05) (05)

Also $\Rightarrow dc = \frac{W}{\sqrt{2}}\tan 15^0$; *Tension in* DB (05) (05) 50

(07)



(05)

Height of the frustum $h = a(1 - \lambda)\cot\alpha$. Where α is the semi-vertical angle.

$\Rightarrow a\cot\alpha = \frac{h}{1-\lambda}$ (05)

Mass of the frustum $M = \int_{\lambda a\cot\alpha}^{a\cot\alpha} \rho\pi(x\tan\alpha)^2\,dx$ (05)

$= \pi\rho\tan^2\alpha.\frac{1}{3}(1-\lambda^3)a^3\cot^3\alpha$

$= \pi\rho.\frac{1}{3}(1-\lambda^3)a^3\cot\alpha$

$= \pi\rho.\frac{1}{3}(1-\lambda^3)a^2h(1-\lambda)$ (05)

$= \pi\rho\frac{h}{3}(1+\lambda+\lambda^2)a^2$ 20

The distance $\bar{x}$ of the centre of mass, G, from the vertex O of the cone is given by

$M\bar{x} = \int_{\lambda a\cot\alpha}^{a\cot\alpha} x.\rho\pi(x\tan\alpha)^2\,dx$ (05)

$= \pi\rho\tan^2\alpha.\frac{1}{4}(1-\lambda^4)a^4\cot^4\alpha$ (05)

$= \pi\rho\frac{1}{4}(1-\lambda^4)a^4\cot^2\alpha$ (05)

Hence $\bar{x} = \frac{3}{4}\frac{(1-\lambda^4)}{(1-\lambda^3)}a\cot\alpha$

Distance of G from the centre O_1 of the smaller face,

$O_1G = \bar{x} - \lambda a\cot\alpha$ (05)

$= \frac{h}{(1-\lambda)}\left\{\frac{3(1-\lambda^4)}{4(1-\lambda^3)} - \lambda\right\} = \frac{h}{4}\cdot\frac{3(1-\lambda^4)-4\lambda(1-\lambda^3)}{(1-\lambda)}$

$= \frac{h}{4}\cdot\frac{3-4\lambda+\lambda^4}{(1-\lambda)^2(1+\lambda+\lambda^2)} = \frac{h}{4}\cdot\frac{3+2\lambda+\lambda^2}{1+\lambda+\lambda^2}$ (05)

25

Letting $\lambda \to 0^+$, We get $M = \frac{1}{3}\pi a^2 h\rho$. (05)

and $\bar{x} = \frac{3}{4}h$ (05) [10]

The distance $O_1G_1 = \bar{x}$ of the centre of mass of the body J is given by

$$\bar{x} = \frac{\frac{1}{3}\pi a^2 h\rho.\frac{h}{4}\left(3+2\lambda+\lambda^2\right)-\frac{1}{3}\pi a^2\lambda^2.\frac{1}{2}h\rho.\frac{1}{4}.\frac{h}{2}}{\frac{1}{3}\pi a^2 h\rho\left(1+\lambda+\lambda^2\right)-\frac{1}{3}\pi a^2\lambda^2.\frac{1}{2}h\rho} \quad (20)$$

$$= \frac{\frac{h}{48}\left[4\left(3+2\lambda+\lambda^2\right)-\lambda^2\right]}{\frac{1}{6}\left(2+2\lambda+\lambda^2\right)} \quad (05)$$

$$= \frac{h}{8}\cdot\frac{\left(12+8\lambda+3\lambda^2\right)}{2+2\lambda+\lambda^2} \quad (05) \qquad [30]$$

Suppose $G_1 \equiv V$ so that

$$\bar{x} = \frac{h}{2} \Rightarrow 12+8\lambda+3\lambda^2 = 8+8\lambda+4\lambda^2$$

$\Rightarrow \lambda^2 = 4 \quad \Rightarrow \lambda = 2 > 1$ We arrive at a contradiction (05)

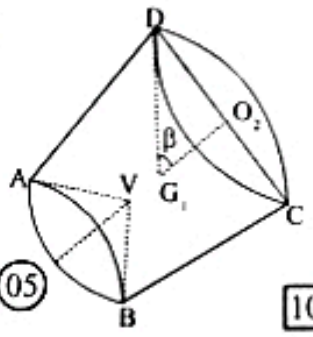
There fore G_1 and V cannot coincide. [05]

When suspended from a point D, the axis of symmetry of J is inclined at an angle β to the vertical DG_1, given by,

$$\tan\beta = \frac{O_2C}{O_2G_1} = \frac{a}{h-\bar{x}} \quad (05)$$

$$= \frac{a}{\frac{h}{8}\cdot\frac{\left(4+8\lambda+5\lambda^2\right)}{\left(2+2\lambda+\lambda^2\right)}}$$

$$= \frac{8a}{h}\cdot\frac{\left(2+2\lambda+\lambda^2\right)}{\left(4+8\lambda+5\lambda^2\right)} \quad (05) \qquad [10]$$



(08) (a)

(i) Two events A and B are said to be independent if $P(A\cap B) = P(A).P(B)$ (05)

(ii) Two events A and B are said to be mutually exclusive if $A\cap B = \phi$ (05)

(iii) Two events A and B are said to be exhaustive if $A\cup B = S$ (05) [15]

Since the events $A\cap B$ and $A\cap B'$ are mutually exclusive.

$$P\left\{(A\cap B)\cup\left(A\cap B'\right)\right\} = P(A\cap B) + P\left(A\cap B'\right) \quad (05)$$

Also, since $(A\cap B)\cup\left(A\cap B'\right) = A$ (05)

$$P\left((A\cap B) \cup P(A\cap B)\right) = P(A)$$

Hence $P(A\cap B) + P(A\cap B') = P(A) \rightarrow (01)$ [10]

Using (01) $P(A\cap B) + \frac{1}{2} = \frac{1}{2} \Rightarrow P(A\cap B) = 0$ (05)

Also, using (01), $P(A\cap B) + P(A'\cap B) = P(B)$ (05)

$$0 + P\left(A'\cap B\right) = \frac{1}{3} \quad (05)$$

$$P\left(A'\cap B\right) = \frac{1}{3} \qquad [15]$$

$$P\left(A'\cap B'\right) = P\left((A\cup B)'\right)$$

$$= 1 - P(A\cup B)$$

$$= 1 - \left(P(A)+P(B)\right) \quad (05)$$

$$= 1 - \left(\frac{1}{2}+\frac{1}{3}\right) = \frac{1}{6} \quad (05) \qquad [10]$$

(b) The conditional probability of A given B, is defined by

$$P(A/B) = \frac{P(A\cap B)}{P(B)} \quad (05)$$

Let C be the event: 'the student cycles to school' Then C is the event : 'the student goes to school by bus' (05)

Let T be the event : 'the student arrives at the school early or on time'

Then T is the event: ' the student arrives late'

Since $P(T) = \frac{19}{28}$ it follows that $P(T^1) = 1 - \frac{19}{28} = \frac{9}{28}$ (05)

Since $P(T/C^1) = \frac{3}{4}$, it follows that $P(T^1/C^1) = 1 - \frac{3}{4} = \frac{1}{4}$ (05)

Hence $P(T^1/C) = 2P(T^1/C^1) = 2 \times \frac{1}{4} = \frac{1}{2}$ (05)

Now $P(T^1) = P(T^1 \cap C) + P(T^1 \cap C^1)$

$= P(T^1/C) \cdot P(C) + P(T^1/C^1) \cdot P(C^1)$ (05)

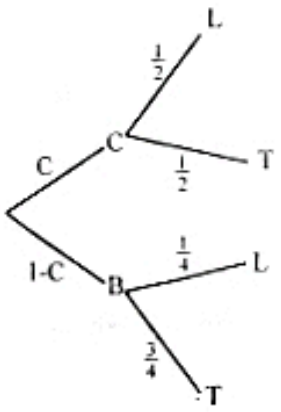
$= \frac{1}{2}P(C) + \frac{1}{4}\{1 - P(C)\} = \frac{9}{28}$ (05)

Hence $P(C) = \frac{2}{7}$ and $P(C^1) = \frac{5}{7}$ (05) [40]

(ii) $P(C^1/T^1) = \frac{P(C^1 \cap T^1)}{P(T^1)} = \frac{P(T^1/C^1) \cdot P(C^1)}{P(T^1)}$ (05)

$= \frac{\left(\frac{1}{4}\right) \times \left(\frac{5}{7}\right)}{\frac{9}{28}} = \frac{5}{9}$ (05) [10]

Alternative Method for 8(b)



(With all probabilities expressed numerically, or in terms of C)

(i) Here C denotes the probability cycling to school, P(c)

Then the probability of the student coming to school on time is

$\frac{1}{2}C + \frac{3}{4}(1 - C) = \frac{19}{28}$ (05)

$C = \frac{2}{7}$ (05) [10]

Hence $P(B) = 1 - C = 1 - \frac{2}{7} = \frac{5}{7}$

(ii) $P(B/L) = \frac{P(B \cap L)}{P(L)} = \frac{P(L/B)}{P(B)} = \frac{\left(\frac{1}{4}\right)\left(\frac{5}{7}\right)}{\left(\frac{9}{28}\right)} = \frac{5}{9}$ (05) (05) [10]

Here the event $B = C^1$, and the event $L = T^1$

(09) (i) Mean $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ (05)

Using the given transformation

$\sum_{i=1}^{n} y_i = \sum_{i=1}^{n}(ax_i + b) = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} 1$

$= an\bar{x} + bn$ (05)

(since a, b are constants)

Hence $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = a\bar{x} + b$ (05) [15]

(ii) Standard deviation $S_x = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$ (05)

Also, $\sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n}(ax_i + b - a\bar{x} - b)^2$

$= a^2\sum_{i=1}^{n}(x_i - \bar{x})^2$ (05)

$= a^2 \cdot n \cdot S_x^2$

Hence $S_y = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2}$

$= \sqrt{\frac{a^2 n S_x^2}{n}}$ (05)

$= |a|\, S_x$

(Since $S_x \geq 0$) [15]

Consider Geography

Let x_i denote the origigal mark of i^{th} the candidate and

Let $y_i = ax_i + b$ be his / her scaled mark (i = 1, 2,.....n) (05)

Then $40 = 40a + b \rightarrow (01)$ (05)

Also, by result (01) $50 = am + b \rightarrow (02)$ (05)

and by (02) $15 = 12|a| \Rightarrow |a| = \frac{5}{4}$ (05)

(02) - (01) $\Rightarrow$ $10 = a(m - 40)$

$\Rightarrow m = 40 + \frac{10}{a}$ (05)

$m = 48$ or 32 (05) [30]

Next consider History

Let x_j^1 denote the original mark of the j^{th} candidate, and

Let $y_j^1 = cx_j^1 + d$ be his/ her scaled mark $(j = 1, 2, \ldots\ldots N)$

Then $56 = 61C + d \rightarrow (03)$ (05)

Also, by (i) $50 = 53C + d \rightarrow (04)$ (05)

(03) - (04) $\Rightarrow C = \frac{3}{4}$ (05)

By result (02) $15 = |C|\ S$ (05)

$S = 20$ (05) [25]

Let N denote the total number of candidates sat for History.

The sum of marks of all candidates before re-scaling $= 53N$ (05)

The number of candidates whose marks were changed

$= \frac{N}{1000}$

The sum of marks of all candidates after re-scaling

$= 53N + \frac{N}{1000}(68 - 65) = \frac{53003N}{1000}$ (05)

The mean mark of all candidates after re-scaling

$= \frac{53003N}{1000N} = 53.003$ (05) [15]