Combined Mathematics II – 2007 Advanced Level Examination

❖ Answer six questions only.

(01) (a) A train starting from rest moves in a straight track with uniform acceleration $\frac{1}{3}\ ms^{-2}$ and acquires a velocity $V\ ms^{-1}$. Then it moves with uniform velocity V for a certain interval of time. Finally, the train moves with uniform retardation $1\ ms^{-2}$, and comes to rest. The total time taken is one minute and the total distance travelled is 432 metres. Sketch the velocity-time graph for the motion of the train, and find the value of V.

Hence, show that the distances travelled in the three stages of motion are in the ratio $3: 2: 1$.

(b) A motor boat sights a ship travelling due north with constant velocity $U\ kmh^{-1}$. The coordinates of the ship at the time of sighting are $(6d, 2d)$ with respect to Cartesian axes Ox, Oy in the east and north directions respectively, where the origin O is taken in the boat and distances are measured in kilometres. The boat immediately begins to move with constant velocity $V\ kmh^{-1}$, in a direction making an acute angle α north of east, so as to intercept the ship. Given that $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$, sketch the path of the boat relative to the ship.

Hence, find the value of V, in terms of U, and show that the time taken for the interception is $\frac{5d}{2U}$ hours.

(02) (a) A light inextensible string passes over a light smooth pulley fixed to the ceiling of a lift and carries particles of masses m and $Km(K > 1)$ at its ends. The lift is made to move vertically upwards with constant acceleration F and at the same time, the particles are released from rest. Find the acceleration of each particle relative to the lift, and show that the tension in the string is $\frac{2Km}{K+1}(g + F)$.

Find the value of F in order that the heavier particle is stationary.

(b) Two small smooth spheres of masses m and $2m$ are connected by a light inextensible string of length $2a$. The middle point of the string is tied to a fixed horizontal long pin and the two spheres are held at a distance $2a$ apart with the two portions of the string taut and horizontal. The spheres are now released from rest, simultaneously. Given that the first impact brings the heavier sphere to rest, find the coefficient of restitution.

Show further that

(i) the second impact brings the lighter sphere to rest,

(ii) the total mechanical energy of the system between the second and the third impacts is $\frac{1}{2}mga$, if the potential energy at the level of the point of impact is zero.

Maths 4U .com

(03) A smooth sphere with centre O and radius a, is fixed on to the horizontal surface of a table. A smooth particle P is placed on the outer surface of the sphere at a point A where OA makes an acute angle α with the upward vertical. The particle is released from rest.

(i) Show that, when OP makes an angle θ with the upward vertical and the particle is still in contact with the surface of the sphere $a\dot{\theta}^2 = 2g(\cos\alpha - \cos\theta)$.

(ii) Find the magnitude of the reaction between the sphere and the particle,

(iii) Show that the particle leaves the sphere at a height $a\left(1 + \frac{2}{3}\cos\alpha\right)$ above the surface of the table.

(iv) Let d be the distance of the point where the particle hits the table, from the vertical diameter of the sphere. Given that $\cos\alpha = \frac{3}{4}$ find d.

(04) One end of a light elastic string of natural length l is attached to a fixed point O, while the other end of the string is attached to a particle of mass m. When the particle hangs in equilibrium, the length of the string is $\frac{3l}{2}$. Find the modulus of elasticity of the string.

The particle is pulled a distance a vertically down from its equilibrium position, and released from rest there. The displacement at time t, of the particle, measured downwards from the equilibrium position is x. Show that $\frac{d^2x}{dt^2} + \omega^2 x = 0$, where $\omega^2 = \frac{2g}{l}$, so long as the string is taut.

(i) In the case when $a < \frac{1}{2}$, find the period and the amplitude of the ensuring motion.

(ii) In the case when $a = \frac{l}{2} + b$, where $b > 0$, show that the time taken for the string to first become slack is $\sqrt{\frac{l}{2g}}\left(\pi - \cos^{-1}\left(\frac{l}{l+2b}\right)\right)$.

[you may assume that the solution of the equation $\frac{d^2x}{dt^2} + \omega^2 x = 0$ is $x = A\cos\omega t + B\sin\omega t$ where A and B are constants to be determined.]

(05) (a) $ABCDEF$ is a regular hexagon of centre O and length of a side a metres. Five forces $P, 2P, 3P, 4P, 5P$ newtons act along the sides AB, BC, CD, DE, EF respectively, in the directions indicated by the order of the letters. There new forces Q, R, S newtons acting along the sides AF, FO, OA respectively, of the triangle AFO are added to the system. Find the values of Q, R, S in terms of P in order that the combined system is

(i) in equilibrium,

(ii) equivalent to a couple of moment $Pa\sqrt{3}\ Nm$ in the sense ABC.

(b) A uniform rod of length a and weight W rests in a vertical plane inside a fixed rough hemispherical bowl of radius a. The rod is in limiting equilibrium inclined at an angle θ to the horizontal, and the coefficient of friction is $\mu(< \sqrt{3})$. Show that the reaction at the lower end of the rod is $\frac{W\cos\theta}{\sqrt{3}-\mu}$, and find the reaction at the upper end.

Hence, show that $\tan\theta = \frac{4\mu}{3-\mu^2}$.

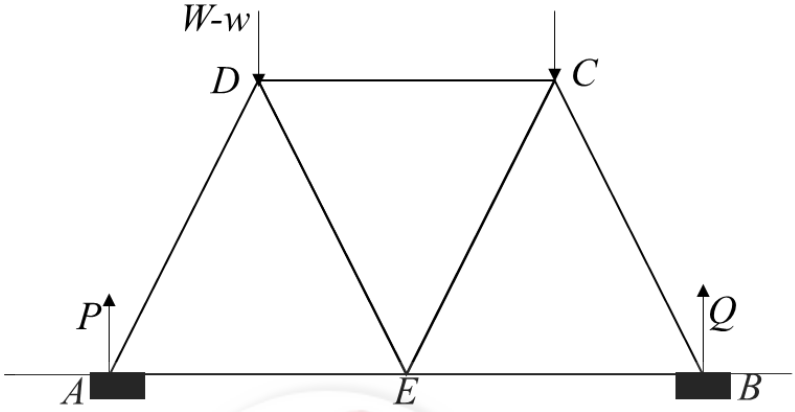
(06) (a) A rhombus $ABCD$ is formed of four equal uniform rods, each of length $2a$ and weight W freely jointed together at their ends. The rhombus is subtended from the joint A, and its shape is maintained by a light rod of length $2a\sin\alpha$ joining the mid-points of BC and CD. Show that the thrust in the light rod is $4W\tan\alpha$, and find the reaction at the joint C.

(b) A framework is made up of seven light rods of equal length freely jointed, as shown in the figure below. A and B rest on smooth supports and there are loads of $W - w$ at D and w at C. Show that the reaction on the framework at A is $P = \frac{3W}{4} - \frac{w}{2}$.

Given that $W > 2w$, draw a suitable stress diagram using Bow's notation and find the stresses in the rods AE, DE and DC.

Indicate whether they are tensions or thrusts.

Show that the stress in DC is independent of w.



(07) A hollow baseless cone of vertex O, semi-vertical angle α and height is made up of a uniform thin metal sheet of mass σ per unit area. Show that its mass is $\pi\sigma h^2 \sec\alpha \tan\alpha$, and find the position of its centre of mass.

A uniform circular disc, of centre B and radius $h \tan\alpha$, made up of the same type of metal sheet is now fixed as the base of the above cone. Show that the distance of the centre of mass of the composite body from O is $\frac{h\left(\frac{2}{3}\sec\alpha+\tan\alpha\right)}{\sec\alpha+\tan\alpha}$.

The composite body is suspended from a point A of the rim of the base. If AO and AB make equal angles with the downward vertical, show that $\sin\alpha = \frac{1}{3}$.

(08) (a) Let A and B be two events. Define $P(A|B)$, the conditional probability of A given B.

State the relationship between A and B when

(i) $P(A|B) = 0$

(ii) $p(A|B) = P(A)$.

(b) Three friends, Wimal, Nimal and Piyal go to a restaurant to buy lunch packets. Packets of rice with meat or with fish or with vegetables are available in the restaurant. Wimal, who does not eat meat, tosses a fair coin to decide whether to buy a packet of rice with fish or vegetables. Observing this, Nimal also tosses a fair coin to decide between meat and fish. Piyal tosses a

fair coin to decide whether to buy a vegetable packet or a packet of other two kinds. In the l later case, he tosses the coin again to decide between meat and fish.

Find the probability that

(i) Wimal and Nimal buy the same kind of packets,

(ii) Nimal and Piyal buy the same kind of packets,

(iii)All three buy the same kind of packets,

(iv)Wimal, Nimal and Piyal buy different kinds of packets.

(c) A student sits a multiple choice examination in which each question has 5 possible answers, out of which only one is correct. If the student knows the answer he selects the correct answer. Otherwise, he selects one answer at random from the 5 possible answers. Suppose that the student knows the correct answer to 70% of the questions.

(i) Find the probability that, for a given question, the student selects the correct answer.

(ii) If the student selects the correct answer to a question, find the conditional probability that he knows the answer.

(09) (a) The speeds of private buses travelling towards Colombo along Galle road were observed to the nearest kilometre per hour at Kalutara bridge. The data collected are given in the following table:

Mid-value of the class interval	15	30	45	60	75	90
Frequency	10	-	25	30	-	10

If the median of the distribution is 49.5 and the mode of the distribution is 55, estimate the two missing frequencies.

Hence, find the mean and the variance of the distribution.

(b) A set of 12 numbers has mean 4 and standard deviation 2. A second set of 20 numbers has mean 5 and standard deviation 3. Find the mean and the standard deviation of the combined set of 32 numbers.