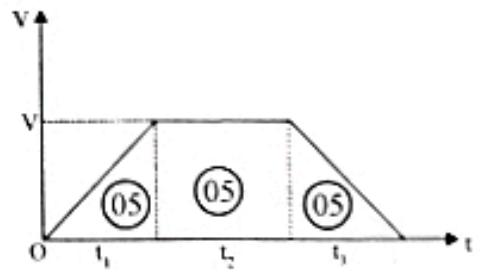
G.C.E. (Advanced Level) Examination - August 2007
10 - Combined Mathematics - II
Marking Scheme

(01) (*a*) Velocity - time graph



15

Acceleratio = $\frac{1}{3}=\frac{V}{t_1}\Rightarrow t_1=3V$ sec. (05)

Retardation = $1=\frac{V}{t_3}\Rightarrow t_3=V$ sec (05)

Motion with constant velocity lasts for a time

$t_2 = 60 - 4V$ sec. (05)

Total distance $=\frac{1}{2}\,3V\cdot V+V(60-4V)+\frac{1}{2}\,V\cdot V=432$ (05)

meters

$V^2 - 30V + 216 = 0$; $(V - 12)(V - 18) = 0$ (05)

Since $t_2 > 0$ the value of $V = 12$ ms^{-1} (05)

Distance traveled in the three stages of motion are in the ratios.

$=\frac{1}{2}\,3V\cdot V+V(60-4V):\frac{1}{2}\,V\cdot V$, where $V = 12$ (05)

$= 3 : 2 : 1$

35

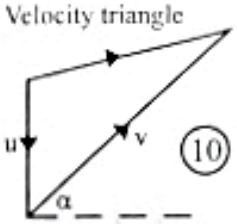
(*b*) Vel. of S, E = U ↑ and Vel. of B, E = V ∠α

Vel. of B, S = B, E + E, S (05)

= V ∠α + U ↓ (05)

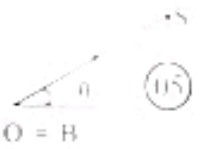
and its direction makes an angle $\theta = \tan^{-1}\left(\frac{2d}{6d}\right)$ with the East. (05)

Velocity triangle



10

Path of B w.r.t. S



05

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Hence $\frac{V\sin\alpha - U}{V\cos\alpha}=\frac{1}{3}$ (05)

$3\left(\frac{3}{5}V - U\right)=\frac{4}{5}V\Rightarrow V = 3U$ (05)

time taken $=\frac{6d}{V\cos\alpha}=\frac{6d}{3U\left(\frac{4}{5}\right)}=\frac{5d}{2U}$ (05)

(05)

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(02) Accn. of life, E = F ↑

Accn. of m, lift = f↑, (05)

Accn. of km, lift = f↓

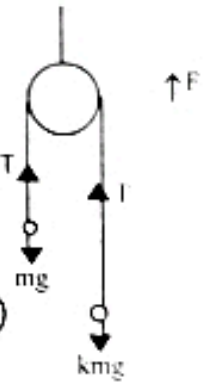
Accn. of m, E = (f + F)↑, (05)

Accn. of km, E = (f - F)↓

Equations of motion :

↓kmg - T = km (f - F) →(01) (05)

↑T - mg = m (f + F)→(02) (05)



(01) + (02) gives (k - 1) mg = (k + 1) mf - (k - 1) mF (05)

Accn. of each mass, relative to the lift is of magnitude

$$f=\frac{k-1}{k+1}(g+F)$$ (05)

30

Also, (01) - k x (02) gives $2kmg(1+k)T = -2kmF$ (05)

$$T = \frac{2km}{k+1}(g+F)$$ (05)

Since the system starts from rest, the heavier mass km remains stationary if f - F = 0 and using(01) ⇨ kmg - T = 0 ⇨ (05)

(k + 1)g = 2 (g + F), using the value of T.

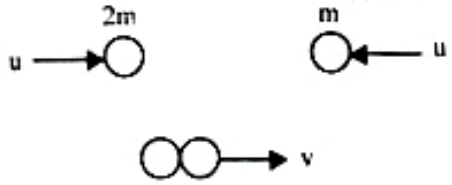
Hence $F = \frac{9}{2}(k-1)$ (05)

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(b) Just before the 1st impact, each spehre will get a speed u given by

$\frac{1}{2}mu^2 = mga \Rightarrow U^2 = 2ga$ (05)

and they move to wards each other



Just after the 1st impact, since the heavier sphere comes to rest,

conservation of momentum ⇨ →mv = 2mu - mu →(01) (05)

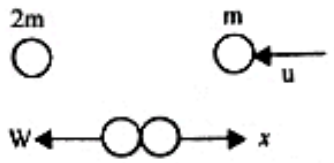
Where v is the speed of the lighter sphere.

Newton's law of restitution ⇨ v + o = e(u + u) →(02) (05)

Solving (01) and (02), $e = \frac{1}{2}$ and v = u (05)

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Since mechanical energy is conserved between 1st and 2nd impacts the lighter sphere returns to the point of impact with the same speed u, but moving towards the other.



For the 2nd impact :

Conservation of momentum ⇨ ←2mw - mx = mu (05)

Newton's law of restitution : $w + x = \frac{1}{2}u$ (05)

solving x = 0; so the lighter sphere comes to rest (05)

and $w = \frac{1}{2}u$ (05)

kinetic energy in the system, just after 2nd impact

$$= \frac{1}{2}(2m)\left(\frac{u}{2}\right)^2 = \frac{1}{4}mu^2$$

$$= \frac{1}{2}mga$$ (05)

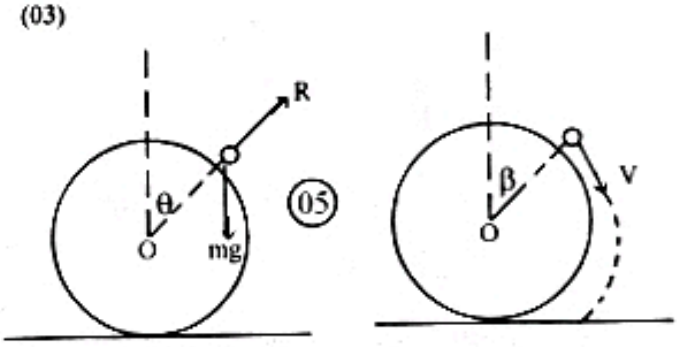
Since the potential energy is zero at the level of the point of impact,

Total mechanical energy between the 2nd and 3rd impacts

$= \frac{1}{2}mga$ (05)

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(03)

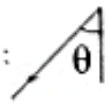


(05)

(i) Conservation of Energy

$\frac{1}{2}m(a\dot{\theta})^2 + mga\cos\theta = mga\cos\alpha$ (10)

$a\dot{\theta}^2 = 2g(\cos\alpha - \cos\theta)$

(ii) Equation of motion :  $mg\cos\theta - R = ma\dot{\theta}^2$ (10)

The reaction from the sphere

R = mgcosθ - 2mg (cosα - cosθ)

= mg (3cosθ - 2cosα) (05)

(iii) Particle leaves the sphere when R = 0 ⇨

$\cos\theta = \frac{2}{3}\cos\alpha = \cos\beta$, (05)

and its height from the surface of the table then is

$= a + a\cos\beta = a\left(1 + \frac{2}{3}\cos\alpha\right)$ (05) 40

(iv) At the instant of leaving the sphere, Velocity of the particle is of magnitude V given by

$$V^2 = 2ga(\cos\alpha - \cos\beta) = 2ga\cdot\frac{3}{4}\left(1-\frac{2}{3}\right)$$ (05)

$$\Rightarrow V = \sqrt{ga/2}$$ (05)

Horizontal component of velocity $= V\cos\beta = \frac{V}{2}$ (05)

Vertical component of velocity $= V\sin\beta = \frac{V\sqrt{3}}{2}$ (05)

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Time T to reach the table is given by

$$\downarrow \quad a(1+\cos\beta) = T\cdot V\,\frac{\sqrt{3}}{2} + gT^2/2$$ (10)

$$\Rightarrow \quad gT^2 + T.V.\sqrt{3} - 3a = 0$$ (05)

$$\Rightarrow \quad T = \frac{-V\sqrt{3} + \sqrt{3V^2 + 12ga}}{2g}$$ (05)

$$T = \frac{\sqrt{3}\,V}{g}$$ (05)

Hosizontal distance traveled, freely under gravity,

$$x = \frac{V}{2}\cdot T = \frac{V^2\sqrt{3}}{2g}$$ (05)

Distance of point of impact from the vertical diameter is

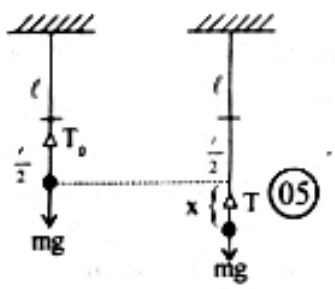
$$d = a\sin\beta + x$$ (05)

$$= \frac{\sqrt{3}a}{2} + \frac{\sqrt{3}a}{4}$$

$$= \frac{3\sqrt{3}a}{4}$$ (05)

40

(03)



(05)

In the equilibrium position, tension $T_0 = mg$ (05)

$$= \lambda\frac{(\ell/2)}{\ell}$$

$$\Rightarrow \lambda = 2mg$$ (05)

equation of motion is $mg - T = m\ddot{x} \rightarrow (01)$ (10)

Where the tension $T = 2mg\left(\frac{x+\frac{\ell}{2}}{\ell}\right) \rightarrow (02)$ (10)

$$\frac{(01)}{m} + \frac{(02)}{m} \Rightarrow \quad \ddot{x} + \left(\frac{2g}{\ell}\right)x = 0$$ (05)

i.e. $\ddot{x} + \omega^2 x = 0$, where $\omega^2 = \frac{2g}{\ell}$

40

In both cases (01) and (02) the solution is

$x = A\cos\omega t + B\sin\omega t$ (given)

$\Rightarrow \dot{x} = -A\omega\sin\omega t + B\omega\cos\omega t$ (05)

Since $x = a$ at $t = 0$, $a = A$ (05)

Since $\dot{x} = 0$ (released from rest) at $t = 0$, $B = 0$ (05)

Hence, as long as $T > 0$, $x = a\cos\omega t$ (05)

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Case I

Since $a < \frac{\ell}{2}$, the extension

$= \frac{\ell}{2} + x = \frac{\ell}{2} + a\cos\omega t > 0$ $(T > 0)$, for all t. (05)

Therefore, the period $\frac{2\pi}{\omega}$ and the amplitude $= a$ (05) (05)

15

Case II

$$a = \frac{\ell}{2} + b \Rightarrow x = \left(\frac{\ell}{2} + b\right)\cos\omega t$$ (05)

and the string becomes slack when $T = 0$ (05)

$$\Rightarrow \frac{\ell}{2} - \left(\frac{\ell}{2} + b\right)\cos\omega t = 0$$ (05)

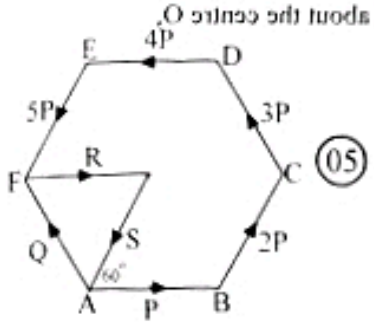
$$\Rightarrow \omega t = \cos^{-1}\left(-\frac{\ell}{\ell + 2b}\right)$$ (05)

This happens for the first time, at time

$$t_1 = \sqrt{\frac{\ell}{2g}}\left[\pi - \cos^{-1}\left(\frac{\ell}{\ell + 2b}\right)\right]$$ (05)

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(05)



(05)

In bothe cases (01) and (02) sum of resolures of forces in any direction is zoro :

$\rightarrow$ $P - 4P + \frac{1}{2}(2P - 3P - 5P) + R - \frac{1}{2}(Q + S) = 0$ (05)

$\Rightarrow R = 6P + \frac{1}{2}(Q + S)$ $\rightarrow (01)$ (05)

$\uparrow$ $\frac{\sqrt{3}}{2}(Q - S) = 0 \Rightarrow S = Q \rightarrow (02)$ (05) **20**

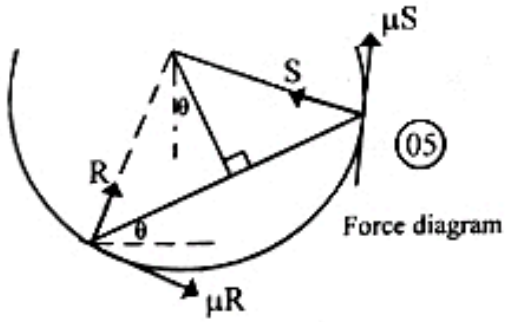
Takimg moments about the centre O,

Case (01) (Equilibrium) $\frac{\sqrt{3}a}{2}(15P - Q) = 0$ (05)

Using (01) and (02) above S = Q = 15 P and R = 21P (05) (05)

15

(b)



(05)

Force diagram

Taking moments about the upper end B,

$R \cdot \frac{\sqrt{3}a}{2} - \mu R \cdot \frac{a}{2} = W \cdot \frac{a}{2}\cos\theta$ (10)

$\Rightarrow R = \frac{W\cos\theta}{\sqrt{3} - \mu}$ $\rightarrow (01)$

Taking moments about the lower end A,

$S \cdot \frac{\sqrt{3}a}{2} + \mu S \cdot \frac{a}{2} = W \cdot \frac{a}{2}\cos\theta$ (10)

$\Rightarrow S = \frac{W\cos\theta}{\sqrt{3} + \mu}$ $\rightarrow (02)$ (05) **30**

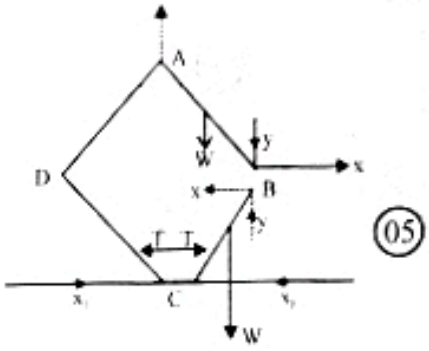
$(\mu R + \mu S) \cdot a = W \cdot \frac{\sqrt{3}a}{2}\sin\theta$ (10)

Also, (01) + (02) $\Rightarrow R + S = \frac{2\sqrt{3} \cdot W\cos\theta}{3 - \mu^2}$ (05)

$\mu = \frac{3 - \mu^2}{4}\tan\theta$ (05)

$\tan\theta = \frac{4\mu}{3 - \mu^2}$ **20**

(06)



(05)

By symmetry, the reaction X_1 at C is horizontal (05)

Resolving vertically for BC

$\uparrow$ $y = W$ (05)

Taking moments about A, for AB

$x \cdot 2a\cos\alpha = y \cdot 2a\sin\alpha + W \cdot a\sin\alpha$

$\Rightarrow X = \frac{3}{2}W\tan\alpha$ (10)

Taking moments about C, for CB

$T \cdot a\cos\alpha + W \cdot a\sin\alpha = X \cdot 2a\cos\alpha + y \cdot 2a\sin\alpha$ (10)

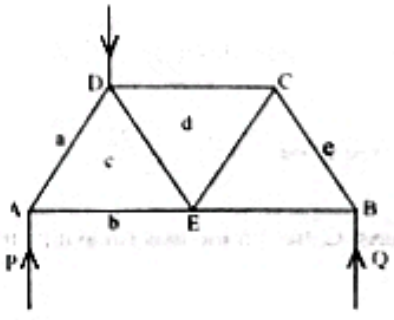
$T = 2X + (2y - W)\tan\alpha$ (05)

$= 4W\tan\alpha$

Resolving horizontally for BC : $\leftarrow X_1 = T - X$ (05)

$X_1 = \frac{5}{2}W\tan\alpha$ (05) **50**

(b)



Taking moments about B for the framework

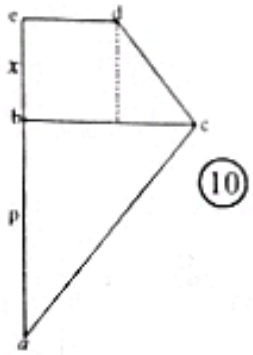
$$P \cdot 2a = \omega \cdot \frac{a}{2} + (W - \omega) \cdot \frac{3a}{2} \quad (05)$$

$$P = \frac{3W}{4} - \frac{\omega}{2}$$

Stress Diagram is as shown below, provided that

$$W - \omega > P \text{ or } \frac{W}{4} - \frac{\omega}{2} > 0 \quad (05)$$

i.e. $W > 2\omega$



(10)

20

Stress in AE is a tension of magnitude

$$= \frac{P}{\sqrt{3}} = \frac{1}{\sqrt{3}}\left(\frac{3W}{4} - \frac{\omega}{2}\right) \quad (05)\ (05)$$

Stress in DE is a thrust of magnitude

$$= \frac{2X}{\sqrt{3}} = \frac{2}{\sqrt{3}}\left[(W-\omega) - P\right] \quad (05)$$

$$= \frac{1}{\sqrt{3}}\left(\frac{W}{2} - \omega\right) \quad (05)$$

Stress in DC is a thrust of magnitude

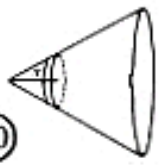
$$= \frac{P}{\sqrt{3}} - \frac{X}{\sqrt{3}} = \frac{W}{2\sqrt{3}} \text{ independent of } \omega$$

(05) (05)

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(07)

For the baseless uniform hollow cone



Mass $M = \int_0^h \sigma \cdot 2\pi\,(x \tan\alpha)\,\sec\alpha \cdot dx$ (10)

$$= 2\pi r\,\sec\alpha \cdot \tan\alpha \cdot \left[\frac{h^2}{2}\right] \quad (05)$$

The centre of mass, G, lies on the axis (of symmetry) of the cone, (05)

and its position is given by

$$M(OG) = \int_0^h \sigma \cdot 2\pi\left(x^2 \tan\alpha\right)\sec\alpha \cdot dx \quad (05)$$

$$= 2\pi\sigma \cdot \sec\alpha \cdot \tan\alpha\left(\frac{h^3}{3}\right) \quad (05)$$

Hence $OG = \frac{2h}{3}$ (05)

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When the base is attached, the CM of the composite body, G_1 is such that

$$(OG_1) = \left[\left(\sigma\pi h^2 \sec\alpha \tan\alpha\right) + \sigma\pi\left(h\tan\alpha\right)^2\right]$$

$$= \frac{2h}{3}\left[\left(\sigma\pi h^2 \sec\alpha \tan\alpha\right) + h\sigma\pi\left(h\tan\alpha\right)^2\right]$$

$$OG_1 = h\left[\frac{\frac{2}{3}\sec\alpha + \tan\alpha}{\sec\alpha + \tan\alpha}\right]$$

If AO and AB make equal angles with the downward vertical AG_1. (05)

Then $\frac{OA}{AB} = \frac{OG_1}{G_1B} \Rightarrow \frac{OA}{OA+AB} = \frac{OG_1}{OG_1+G_1B}$ (05)

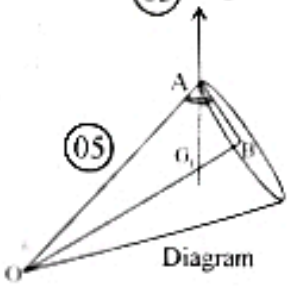
(05)

$$\Rightarrow \frac{h\sec\alpha}{h(\sec\alpha + \tan\alpha)} = \frac{OG_1}{h} = \left[\frac{\frac{2}{3}\sec\alpha + \tan\alpha}{\sec\alpha + \tan\alpha}\right]$$

(05) (05)

$$\frac{1}{3}\sec\alpha = \tan\alpha \quad (05)$$

$$\frac{1}{3} = \sin\alpha \quad (05)$$



Diagram

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(08)

(a) $P(A/B) = \frac{P(A \cap B)}{P(B)}$, Provided $P(B) > 0$ (05)

(i) $P(A/B) = 0 \Rightarrow P(A \cap B) = 0$ (05)

$\because P(B) > 0 \Rightarrow A \cap B = \phi$

i.e A and B are mutually exclusive. (05)

(ii) $P(A/B) = P(A) \Rightarrow P(A \cap B) = P(A)\ P(B)$ (05)

i.e. A and B are independent. (05)

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(b) Let WF = Wimal buy a packet of rice with fish, WV = Wimal buy a packet of rice with Vegetable, NM = Nimal buy a packet of rice with meat, NF = Nimal buy a packet of rice with fish, PV = Piyal buy a packet of rice with vegetable, PM = Piyal buy a packet of rice with meat and PF = Piyal buy a packet of rice with fish.

Then

$P(WF) = \frac{1}{2}$, $P(WV) = \frac{1}{2}$, $P(NM) = \frac{1}{2}$, $P(NF) = \frac{1}{2}$

$P(PV) = \frac{1}{2}$, $P(PM) = \frac{1}{4}$ and $P(PF) = \frac{1}{4}$, (15)

(i) P(Wimal and Nimal buy the same kind of Packets)

$P\{WF \cap NF\} = P(WF)\ P(NF) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ (05)

(ii) P(Nimal and Piyal buy the same kind of packets)

$= P[\{NF \cap PF\} \cup \{NM \cap PM\}]$

$= P\{NF \cap PF\} + P\{NM \cap PM\}$

$= P(NF)P(PF) + P(NM)P(PM)$ (05)

$= \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} = \frac{1}{4}$ (05)

(iii) P(All three buy the same kind of packets)

$= P\{WF \cap NF \cap PF\}$

$= P(WF)P(NF) + P(PF)$ (05)

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{4}$

$= \frac{1}{16}$ (05)

(iv) P(Wimal, Nimal and Piyal buy different kinds of packets)

$= P\begin{Bmatrix}(WV \cap NM \cap PF) \cup (WV \cap NF \cap PM) \\ \cup (WF \cap NM \cap PV)\end{Bmatrix}$

$= \begin{pmatrix}P(WV)P(NM)P(PF) + P(WV)P(NF) \\ P(PM) + P(WF)P(NM)P(PV)\end{pmatrix}$ (05)

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$ (05)

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(c) Let A = Student knows the answer and B = student select the correct answer.

$P(A) = 0.7$, $P(B/A) = 1$ and $P(B/A^1) = 0.2$

Where A^1 is the complementary event of A

(i) $P(B) = P(A)P(B/A) + P(A^1)P(B/A^1)$ (05)

$= 0.7 \times 1 + 0.3 \times 0.2$ (05)

$= 0.76$ (05)

(ii) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B/A)}{P(B)}$

$= \frac{0.7 \times 1}{0.76}$ (05)

$= \frac{35}{38}$ (05)

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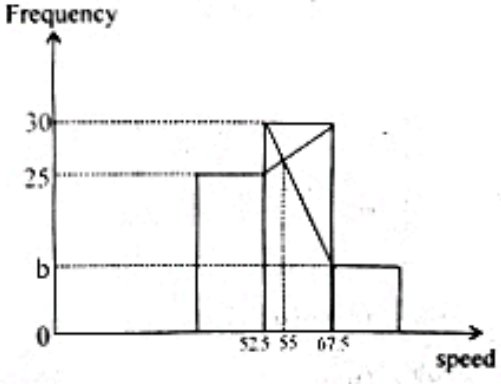
(09)

(a)

Mid-value of the class interval	15	30	45	60	75	90
Frequency	10	a	25	30	b	10

Mode = 55 ⇨ class boundaries of the modal class is 52.5 - 67.5 (05)

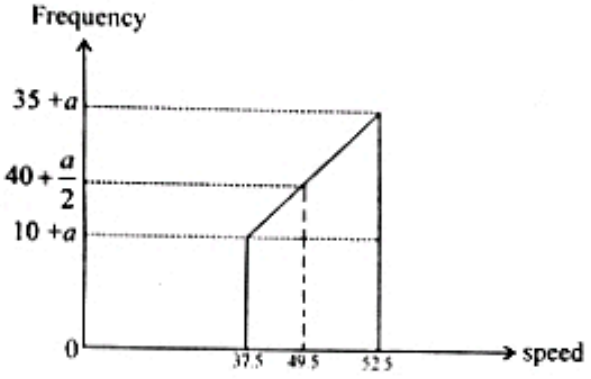
$\frac{30-25}{30-b} = \frac{55-52.5}{67.5-55} = \frac{2.5}{12.5} = \frac{1}{5} \Rightarrow b = 5$ (05) (05)



Median = 49.5 ⇔ class boundaries of the median class is 37.5 - 52.5 (05)

$$\frac{(35+a)-(10+a)}{\left(40+\frac{a}{2}\right)-(10+a)}=\frac{52.5-37.5}{49.5-37.5}$$ (05)

$$\frac{25}{30-\frac{a}{2}}=\frac{5}{4}\Rightarrow a=20$$ (05)



Mid-value of the class interval(x)	Frequency (f)	$d=\frac{x-60}{15}$	fd	fd^2
15	10	-3	-30	90
30	20	-2	-40	80
45	25	-1	-25	25
60	30	0	0	0
75	5	1	5	5
90	10	2	20	40
Total	100		-70	240
			(05)	(05)

Mean $=\frac{1}{100}\sum_{i=1}^{6}f_ix_i=\frac{15}{100}\sum_{i=1}^{6}f_id_i+60$ (05)

$=-10.5+60=49.5$ (05)

Variance $=15^2\left\{\frac{1}{100}\sum_{i=1}^{6}f_id_i^2-\left(\frac{\sum_{i=1}^{6}f_id_i}{100}\right)^2\right\}$

$=225(2.4-0.49)$ (05)

$=429.75$ (05)

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(b) For the first set of numbers $n_1 = 12$, $\mu_x = 4$ and $\sigma_x = 2$

$\therefore \sum_{i=1}^{12}x_i=n_i\mu_x=12\times4=48$ *and* $\sum_{i=1}^{12}x_i^2$ (05)

$=n_i\left(\sigma_x^2+\mu_x^2\right)$

$=12(4+16)$

$=240$ (05)

For the second set of numbers $n_2 = 20$, $\mu_y = 5$ and $\sigma_y = 3$

$\therefore \sum_{i=1}^{20}y_i=n_2\mu_y=20\times5=100$ *and* (05)

$\sum_{i=1}^{20}y_i^2=n_2\left(\sigma_y^2+\mu_y^2\right)$

$=20(9+25)$

$=680$ (05)

The mean of the combined set

$=\frac{\sum_{i=1}^{12}x_i+\sum_{i=1}^{20}y_i}{n_1+n_2}=\frac{48+100}{32}=4.625$ (05)

$\sum_{i=1}^{12}x_i^2+\sum_{i=1}^{20}y_i^2=240+680=920$ (05)

The variance of the combined set

$=\frac{\sum_{i=1}^{12}x_i^2+\sum_{i=1}^{20}y_i^2}{n_1+n_2}-(4.625)^2$

$=\frac{920}{32}-(4.625)^2$

$=7.359$ (05)

$\therefore$ The standard deviation of the combined set

$=\sqrt{7.359}=2.71$ (05)

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