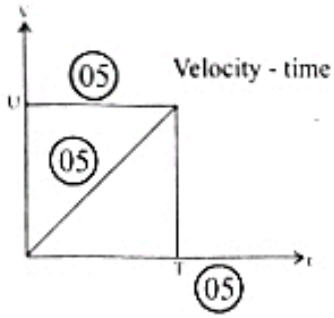
G.C.E. (Advanced Level) Examination - April 2006
Combined Mathematics - II
Provisional Scheme of Marking

(01) (a)



(05) Velocity - time graph

(05)

(05)

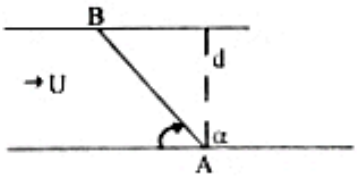
Acceleration = Gradient of the V - t graph $= \frac{u}{T}$ (05)

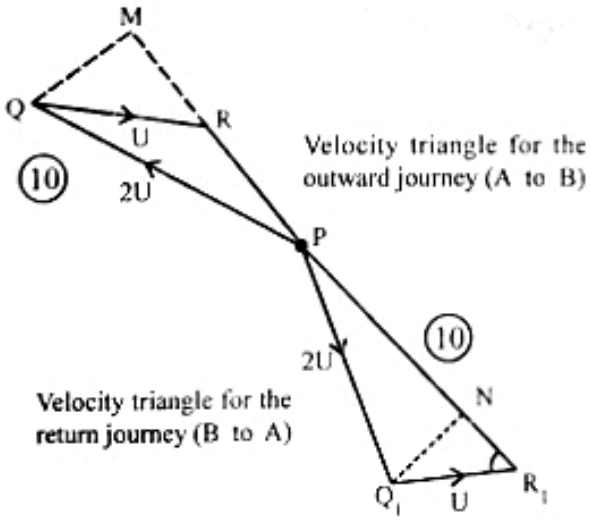
Initial distance of the man from the bus stop = Area of the shaded triangle Δ (05)

$= \frac{1}{2}uT$ (05)

30

(b)





10

Velocity triangle for the outward journey (A to B)

10

Velocity triangle for the return journey (B to A)

From the two velocity triangles,

$V + u\cos\alpha = \sqrt{4u^2 - u^2\sin^2\alpha} \rightarrow (01)$ (05)

$2V - u\cos\alpha = \sqrt{4u^2 - u^2\sin^2\alpha} \rightarrow (02)$ (05)

$(02) - (01)$ $V = 2u\cos\alpha \rightarrow (03)$ (05)

$(01) + (02)$ $3V = 2u\sqrt{4 - \sin^2\alpha} \rightarrow (04)$ (05)

Eliminating V, $4 - \sin^2\alpha = 9\cos^2\alpha$ (05)

$\sin^2\alpha = \frac{5}{8} \rightarrow (05)$

Since α is acute, $\sin\alpha = \sqrt{\frac{5}{8}}$ (05)

$\cos\alpha = \sqrt{\frac{3}{8}} \rightarrow (06)$ (05)

Substituting (06) in (03), $V = 2u\sqrt{\frac{3}{8}}$ (05)

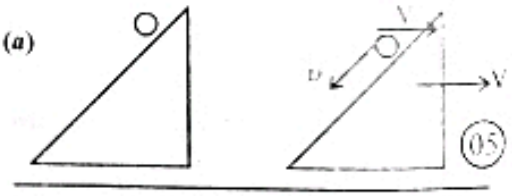
$= u\sqrt{\frac{3}{2}}$

Total time taken $= \frac{3d\ \mathrm{cosec}\,\alpha}{2V}$ (05)

$= \frac{3d}{2} \frac{\sqrt{\frac{8}{5}}}{u\sqrt{\frac{3}{2}}} = \frac{6d}{u\sqrt{15}}$ (05)

70

(02) **(a)**



(05)

Since the horizontal force on the system = 0, horizontal linear momentum, in the direction of motion, remains constant = initial value = 0 (05)

At the instant considered (just before impulsive action)

Velocity of m, M = $v \swarrow$ and velocity of M, table $T = \overline{V}$ (05)

Velocity of m, T has componenets : $V - v\cos\alpha \rightarrow$ (05)

$\downarrow V \sin\alpha$ (05)

Hosizontal linear momentum, $MV + m(V - v\cos\alpha) = 0$

$$V = \frac{mv\cos\alpha}{M+m}$$ (05)

Just after impulsive action, M and m have the same common velocity, $V_1 \rightarrow$ (05)

Horizontal linear momentum of the system remains constant at zero,

Since there is no external impulse in this direction. (05)

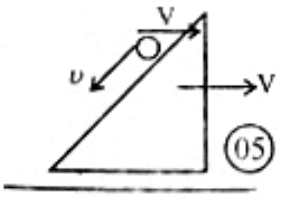
$(M + m) V_1 = 0 \Rightarrow$ Velocity of the wedge, $V_1 = 0$ (05) (05)

Impulse on the system (M and m) from the table

$= 0 - (-m v \sin\alpha)$ (05)

Impulse on the table $= m v \sin\alpha$ (05) 60

Alternative method : 2(*a*)



(05)

Acceleration of m, $M = v \swarrow$ and M, table $T = \dot{V} \rightarrow$ (05)

Horizontal acceleration of m, T is $\dot{V} - \dot{v}\cos\alpha \rightarrow$ (05)

Equation of motion for the system, in the horizontal direction of motion is

$$M\dot{V} + m(\dot{V} - \dot{v}\cos\alpha) = 0$$ (05)

integration w.r.t. time $(M+m) V - v\cos\alpha$ = Constant

$= 0$ (05)

Since the system is initially at rest (05)

Just after impulsive action, M and m have the same common velocity, $V_1 \rightarrow$ (05)

Horizontal linear momentum of the system remains constant at zero.

Since there is no external impulse in this direction (05)

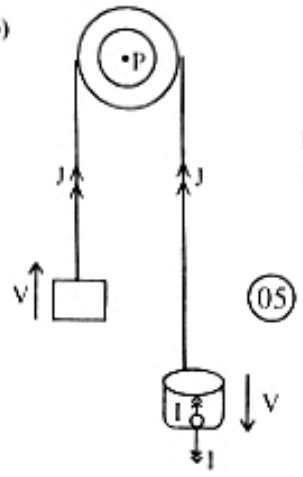
$(M + m) V_1 = 0 \Rightarrow$ Velocity of the Wedge, $V_1 = 0$ (05) (05)

Impulse on the system (M and m), from the table

$= 0 - (-m v \sin\alpha)$ (05)

Impulse on the table $= mv \sin\alpha$ (05) 60

(b)



(05)

Impulse of collision = I

Impulse of in the string = J

Just after the first impulsive action velocity of the bucket $= V \downarrow$

Velocity of the ball, relative to bucket = $eu \uparrow$ (05)

Velocity of ball, relative to P = $(eu - v) \uparrow$

For the ball $\uparrow$, $I = m(eu - V) - (-mu) \rightarrow (01)$ (05)

For the bucket $\downarrow$, $I - J = MV \rightarrow (02)$ (05)

For the counterpoise $\uparrow$, $J = MV \rightarrow (03)$ (05)

$(02) + (03) - (01) \Rightarrow 2MV - mu(1 + e) + mV = 0$ (05)

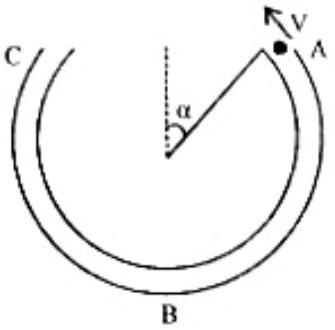
$$\Rightarrow V = \frac{mu(1+e)}{2M+m}$$

Considering motion relative to the bucket which moves with constant velocity, the time between the first and second impact $= \frac{2(\text{initial relative velocity})}{g}$ (05)

$$= \frac{2eu}{g}$$ (05)

40

(03) (a)



By the principle of conservation of energy.

$$\frac{1}{2}mv^2 + mga\cos\alpha = \frac{1}{2}mu^2 - mga \quad (15)$$

The speed v of the particle, as it leaves the tube at A, is given by $v^2 = u^2 - 2ga(1+\cos\alpha) \rightarrow (01)$ (05)

Direction of velocity at A makes an angle α with the horizontal (05)

For free motion from A to C, under gravity.

$$0 = (v\sin\alpha)t - \frac{1}{2}gt^2 \quad (05)$$

$$\rightarrow \text{time of flight } t = \frac{2v\sin\alpha}{g} \quad (05)$$

Hosizontal range, $2a\sin\alpha = (v\cos\alpha)\frac{2v\sin\alpha}{g}$

$$\rightarrow v^2 = ga\sec\alpha \rightarrow (02) \quad (05)$$

Equating (01) and (02) $u^2 - 2ga(1+\cos\alpha) = ga\sec\alpha$ (05)

$$u^2 = ga\left[2(1+\cos\alpha)+\sec\alpha\right]$$

Greatest height , h, of the particle above the level of AC is given by

$$O = V^2\sin^2\alpha - 2gh \rightarrow (03) \quad (05)$$

from (02) and (03) $h = (ga\sec\alpha)\frac{\sin^2\alpha}{2g}$ (05)

$$= \frac{a}{2}(\sec\alpha - \cos\alpha) \quad (05)$$

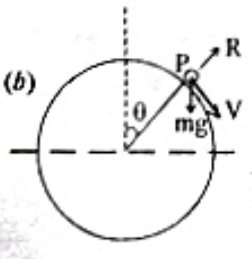
Greatest height, above the level of O, is

$$H = h + a\cos\alpha \quad (05)$$

$$= \frac{a}{2}(\sec\alpha + \cos\alpha)$$

65

(b)



By the principle of conservation of energy,

$$\frac{1}{2}m(a\dot{\theta})^2 + mga\cos\theta = 0 + mga\cos\alpha \quad (15)$$

$$\Rightarrow a\dot{\theta}^2 = 2g(\cos\alpha - \cos\theta) \quad (10)$$

Equation of motion : ✓ $mg\cos\theta - R = ma\dot{\theta}^2$

$$R = mg\cos\theta - 2mg(\cos\alpha - \cos\theta)$$

$$R = mg(3\cos\theta - 2\cos\alpha) \quad (05)$$

Particle will leave the surface when R = 0

$$\Rightarrow \theta = \cos^{-1}\left(\frac{2\cos\alpha}{3}\right) \quad (05)$$

35

(04) (a) On the level road :

When moving with uniform speed, tractive force = resistance R

$$Power = P \times \frac{160,000}{60\times 60} = 3,000,000\text{W} \quad (05)$$

$$R = 67,500\text{ N} \quad (05)$$

On the inclined road :

Tractive force at speed 60kmh^{-1} is

$$P_1 = 3,000,000 \times \frac{60\times 60}{60,000} \quad (05)$$

$$R = 180,000\text{ N} \quad (05)$$

Equation of motion is P1 - Mgsin α - R = Ma (05)

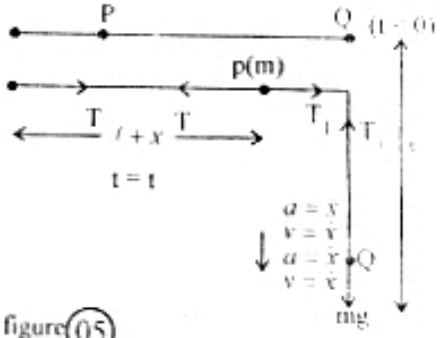
Acceleration, $a = \frac{P_1 - R}{M} - g\sin\alpha$ (05)

$$= \frac{112,500}{450,000} - (9.8)\times\frac{1}{70} = 0.25 - 0.14 = 0.11\text{ms}^{-2} \quad (05)$$

(05)

40

(b)



For the figure (05)

By the principle of conservation of energy.

When the particle Q is at a depth x,

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{x}^2 - mgx + \frac{1}{2}\frac{(mg)}{\ell}x^2 = \text{const} = 0 \quad (20)$$

$$\dot{x}^2 = gx - x^2\frac{g}{2\ell} = \frac{g}{2\ell}\left[2\ell x - x^2\right] \quad (05)$$

$$2\dot{x}^2 = \frac{g}{2\ell}\left[\ell^2 - (\ell - x)^2\right] \quad (05)$$

$$= \omega^2\left[\ell^2 - (\ell - x)^2\right], \text{ Where } \omega^2 = \frac{g}{2\ell}$$

(05)

SHM with centre at $x = \ell$ (the edge of the table)

and amplitude = ℓ (05)

Time taken by P from th initial (extreme) Position $x = 0$ to the centre $x = \ell$ is

$$\frac{\pi}{2}\frac{1}{\omega} = \frac{\pi}{2}\sqrt{\frac{2\ell}{g}} \quad (05)$$

$$= \pi\sqrt{\frac{\ell}{2g}}$$

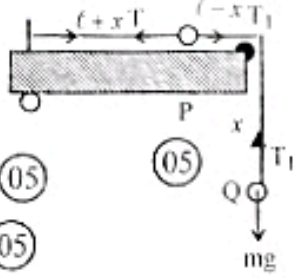
The speed of P at this instant is $= \omega\ell$

$$= \ell\sqrt{\frac{g}{2\ell}} \quad (05)$$

$$= \sqrt{\frac{g\ell}{2}} \quad (05)$$

60

Alternative Method 4(b)



Equations of motion

For P, →T1 - T = $m\ddot{x}$ →(01) (05)

Where $T = \left(\frac{mg}{\ell}\right)x$ →(02) (05)

For Q ↓ mg - T = $m\ddot{x}$ →(03) (05)

using (01), (02), (03)

$$mg - \frac{mg}{\ell}x = 2m\ddot{x} \quad (05)$$

Writing $X = x - \ell$, $\ddot{x} + \omega^2 X = 0$ →(04) (05)

Particle P Performs SHM, before it reaches the edge, centre being at $x = 0$ or $x = \ell$ (at the edge of the table) (05)

$(04) \Rightarrow \dot{x}^2 = \omega^2\left(a^2 - x^2\right)$ where a denotes the amplitude (05)

Initially, $x = 0$ or $x = -\ell$ and $\dot{x} = 0 \Rightarrow a = \ell$ (05)

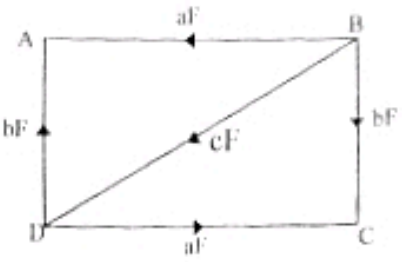
Time taken by P from the initial (extreme) Position $x = 0$ to the centre $x = \ell$ is

$$\frac{\pi}{2}\frac{1}{\omega} = \frac{\pi}{2}\sqrt{\frac{2\ell}{g}} = \pi\sqrt{\frac{\ell}{2g}} \quad (05) \quad (05)$$

The speed of P at this instant is $= \omega\ell = \ell\sqrt{\frac{g}{2\ell}} = \sqrt{\frac{g\ell}{2}}$ (05)

60

(05) (a)



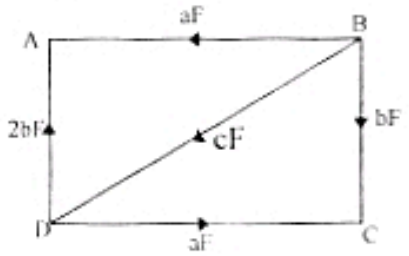
The two couples formed by forces along two // sides balance, (05)

(05)

because they are of equal magnitude abF, and are in opposite senses. (05)

The system reduces to a force cF along BD, in the direction from B to D. (05)

(05)



When the force along DA is increased to 2bF, while other forces remain the same as before, the new system is equivalent to forces cF along BD and bF along DA. (05)

Since these two forces are proportional to the length BD, DA respectively, (05)

and acts through D, their resultant is a single force aF along CD. (05) (05) (05)

50

Alternative Method

For the system → $x = cF\left(-\frac{a}{c}\right) = -aF$ (05)

↑ $y = cF\left(-\frac{b}{c}\right) = -bF$ (05)

Magnitude of resultant $= \sqrt{X^2 + y^2} = cF$ (05)

Direction is that from B to D. (05)

Line of action is BD. (05)

For the new system $\rightarrow x_1 = -aF$

$\uparrow y_1 = bF + y = 0$ (05)

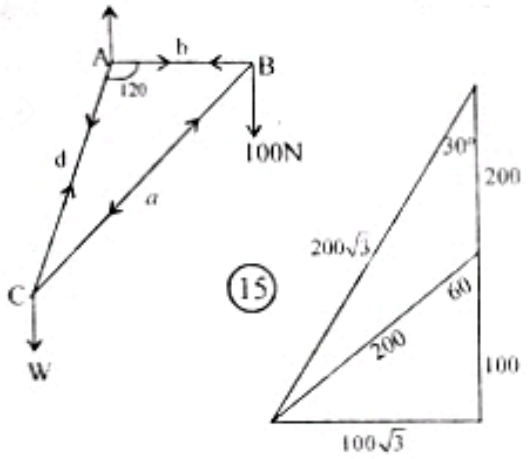
Magnitude of resultant = aF (05)

Direction is that from B to A (or C to D) (05)

Moment of the system about D being zero,

the resultant acts through D (05)

Line of action is CD. (05) [50]



(15)

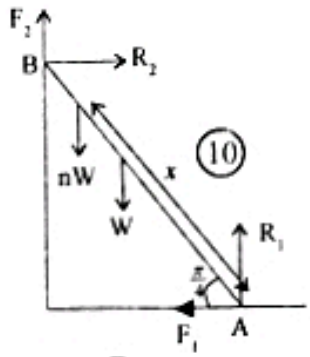
AB (bc) - Tension (05) $100\sqrt{3}$ N (05)

BC (ca) - Thrust (05) 200 N (05)

AC (cd) - Tension (05) $200\sqrt{3}$ N (05)

W(ad) 200 N (05) [50]

(06)



(10)

$\rightarrow \quad F_1 = R_2 \quad \rightarrow (01)$ (10)

$\uparrow \quad F_2 + R_1 = (n+1)W \rightarrow (02)$ (10)

B $\curvearrowleft \quad f_1 \cdot \frac{2a}{\sqrt{2}} + W\frac{a}{\sqrt{2}} + nW\frac{(2a-x)}{\sqrt{2}} = R_1 \cdot \frac{2a}{\sqrt{2}}$ (15)

$2F_1 a + Wa + nW(2a - x) = 2R_1 a \rightarrow (03)$ (15)

For limiting equalibrium, $F_1 = \mu R_1 \rightarrow (04)$ (05)

$F_2 = \mu R_2 \rightarrow (05)$ (05)

From (01), (02), (04) and (05)

$$(1+\mu^2)(R_1) = (n+1)W$$

$$R_1 = \frac{(n+1)W}{1+\mu^2} \quad (05)$$

From (03), (04) and (05)

$$(\mu - 1)2a\frac{(n+1)W}{1+\mu^2} + Wa(1+2n) = nWx \quad (15)$$

$$x = \frac{a\left[(1+2n)(1+\mu^2) + 2(n+1)(\mu-1)\right]}{n(1+\mu^2)}$$

$$= \frac{a\left[\mu^2(1+2n) + 2(n+1)\mu - 1\right]}{n(1+\mu^2)} \quad (05)$$

[75]

If $\mu = \frac{1}{2}$

$$x = \frac{a\left[\frac{1}{4}(1+2n)(n+1) - 1\right]}{n\left(1+\frac{1}{4}\right)} \quad (05)$$

$$= \frac{a}{5n}[1 + 2n + 4n] \quad (05)$$

$$= \frac{a}{5n}(6n+1)$$

Cat can reach the top of the laddeer before the ladder slips. if

$$\frac{a}{5n}(6n+1) > 2a \Rightarrow 6n + 1 > 10n \quad (05)$$

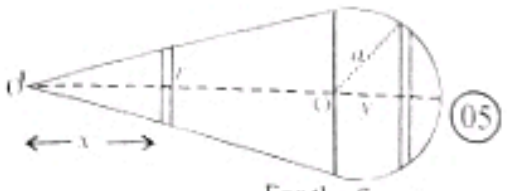
$$\Rightarrow 1 > 4n$$

$$\Rightarrow n < \frac{1}{4} \quad (05)$$

If $n = \frac{1}{4}$, cat reaches top of the ladder. (05)

[25]

(07)



(05)

For the figure

For solid hemisphere, centre of gravity is at $(\bar{y}, 0)$, Where

$$\bar{y}\int_0^a \rho\pi\left(a^2 - y^2\right)dy = \int_0^a \rho\pi\left(a^2 - y^2\right)ydy$$ (05) (05)

$$\Rightarrow \bar{y} = \frac{\rho\pi\left[a^2\frac{a^2}{2} - \frac{a^4}{4}\right]}{\rho\pi\left[a^2 \cdot a - \frac{a^3}{3}\right]}$$ (05) (05)

$$= \frac{\frac{a}{4}}{\frac{2}{3}} = \frac{3}{8}a$$ (05)

Centre of Gravity of hemisphere is at $\left(-\frac{3a}{8}, 0\right)$ (05)

35

For the solid cone, CG is at $(\bar{x}, 0)$, W.r.t. origin O^1

Where $\bar{x}\int_0^h \rho\pi r^2\,dx = \int_0^h \rho\pi r^2 x dx$ (05)

Now $\frac{r}{x} = \frac{a}{h} \Rightarrow r = \frac{a}{h}x$ (05)

$$\bar{x} = \frac{\rho\pi\int_0^h \left(\frac{a}{h}\right)^2 x^2 x dx}{\rho\pi\int_0^h \left(\frac{a}{h}\right)^2 x^2 dx}$$ (05)

$$= \frac{\left[\frac{h^4}{4}\right]}{\left[\frac{h^3}{3}\right]}$$

$= \frac{3}{4}h$ measured from vertex O^1. (05)

Distance of G_1 from O is $\frac{1}{4}h$

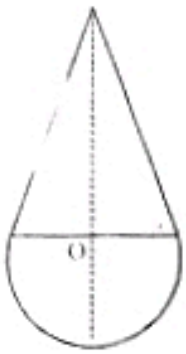
i.e. $G_1 = \left(\frac{1}{4}h, 0\right)$ W. r. t. 0 (05)

25

Composite body has centre of gravity at $G^1(y,0)$, Where

$$\bar{y} = \frac{\frac{1}{4}h \cdot \frac{1}{3}\pi a^2 h - \frac{3}{8}a \cdot \frac{2}{3}\pi a^3}{\frac{1}{3}\pi a^2 h + \frac{2}{3}\pi a^3}$$ (10)

$$= \left|\frac{\left(h^2 - 3a^2\right)}{4(h+2a)}\right|$$ (05)



The body will topple over, if $y > 0$ (05)

$\Rightarrow h^2 > 3a^2$; i.e. $h > \sqrt{3}\,a$ (05)

The body will return to equilibrium position if $y < 0$ (05)

$\Rightarrow h^2 < 3a^2$; i.e. $h < \sqrt{3}\,a$ (05)

If $h = \sqrt{3}\,a$, body will remain in the displaced position (05)

40

(08) (a)

If two events x and y are such that $x \cup y = s$, then the events are said to be exhaustive. (05)

In this case $p(x \cup y) = 1$

If two events x and y are such that $x \cap y = \phi$, then the two events are said to be mutually exclusive. (05)

In this case $p(x \cap y) = 0$

If either of the events x and y can occur without being affected by the other, then the two events are independent Equivalently, $p(x \cap y) = p(x) \times p(y)$ (05)

Since A and B are exhaustive and mutually exclusive events in S, each event is the compliment of the other, and P(A) = 0.4 we get P(B) = 1- P(A) = 1 - 0.4 = 0.6 (05)

20

(i) Since A and C are independent events in S,

$p(A) = \frac{2}{5}$, $P(C) = \frac{1}{2}$ we get

$$P(A \cap C) = P(A) \times P(C)$$

$$= \frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$$ (05)

(ii) Using $P(A \cup C) = P(A) + P(C) - P(A \cap C)$ (05)

We have $P(A \cup C) = \frac{2}{5} + \frac{1}{2} - \frac{1}{5} = \frac{7}{10}$ (05)

Since $A \cup C$ and $\overline{A \cup C} = \overline{A} \cap \overline{C}$ are exhaustive and mutually exclusive events (05)

$$P(\overline{A} \cap \overline{C}) = 1 - P(A \cup C)$$

$$= 1 - \frac{7}{10} = \frac{3}{10}$$ (05)

(iii) Since $P(A) = \frac{2}{5}$ and $P(C) = \frac{1}{2}$, We get $P(\overline{A}) = \frac{3}{5}$

and $P(\overline{C}) = \frac{1}{2}$ Thus We have

$$P(\overline{A} \cap \overline{C}) = \frac{3}{10} = P(\overline{A}) \times P(\overline{C})$$ (05)

Therefore $\overline{A}$ and $\overline{C}$ are independent (05)

Since B and D are mutually exclusive,

$$P(B \cup D) = P(B) + P(D) = \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$$

Since $B \cup D$ and $P(\overline{B \cup D}) = \overline{B} \cap \overline{D}$ are exhaustive and mutually exclusive.

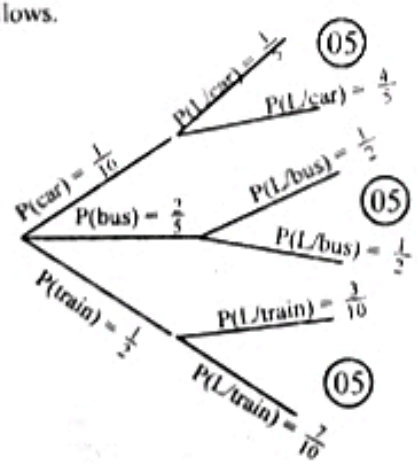
events $P(\overline{B} \cap \overline{D}) = 1 - P(B \cup D) = 1 - \frac{4}{5} = \frac{1}{5} \neq 0$ (05)

Thus $\overline{B}$ and $\overline{D}$ are not mutually exclusive. (05)

45

(b) Let L be the event that the government servant gets late for work.

Then, the tree diagram inrespect of the problem is as follows.



By Bayes' theorem

$$P\left(\frac{Train}{L}\right) = \frac{P\left(\frac{L}{Train}\right)P(Train)}{P(L)}$$

$$= \frac{P\left(\frac{L}{Train}\right)P(Train)}{P\left(\frac{L}{Car}\right)\cdot P(Car) + P\left(\frac{L}{Bus}\right)P(Bus) + P(\quad)}$$

$$= \frac{\frac{3}{10} \times \frac{1}{2}}{\left(\frac{1}{5}\right)\left(\frac{1}{10}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{5}\right) + \left(\frac{3}{10}\right)\left(\frac{1}{2}\right)}$$

$$= \frac{\frac{3}{20}}{\frac{1}{50} + \frac{1}{5} + \frac{3}{20}}$$ (05)

$$= \frac{\left(\frac{3}{20}\right)}{\frac{2 + 20 + 15}{100}}$$

$$= \frac{3}{20} \cdot \frac{100}{37} = \frac{15}{37}$$ (05)

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(09) In the usual notation,

$$\text{mean } \mu = \frac{\sum_{i=1}^{n} x_i}{n}$$ (05)

and Variance $\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}$ (05)

(i) Mean of the given set of values

$\{x_i + \alpha : i = 1,2,\ldots,n\}$

$$= \frac{\sum_{i=1}^{n} (x_i + \alpha)}{n} = \frac{\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \alpha}{n}$$

$$= \frac{\sum_{i=1}^{n} x_i}{n} + \frac{1}{n}\sum_{i=1}^{n} \alpha = (\mu + \alpha)$$ (05)

Square of the standard deviation of the given set of Values

$$= \frac{\sum_{i=1}^{n}(x_i + \alpha - \mu - \alpha)^2}{n}$$

$$= \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}$$

$$= \sigma^2$$

Thus the standard deviation of the given set of Values = σ

(05)

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(ii) Mean of the given set of Values

$$= \frac{\sum_{i=1}^{n}\beta x_i}{n} = \frac{\beta\sum_{i=1}^{n} x_i}{n}$$

$$= \beta\frac{\sum_{i=1}^{n} x_i}{n} = \beta\mu$$ (05)

Square of the standard deviation of the given set of Values

$$= \frac{\sum_{i=1}^{n}(\beta x_i - \beta\mu)^2}{n} = \frac{\sum_{i=1}^{n}\beta^2(x_i - \mu)^2}{n}$$ (05)

$$= \frac{\beta^2\sum_{i=1}^{n}(x_i - \mu)^2}{n} = \beta^2\sigma^2$$ (05)

Thus the standard deviation of the given set of values = βσ

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Mean of the set of Values $\{2x_i + 3 : i = 1, 2, \ldots\ldots n\}$

$$\mu = \frac{\sum_{i=1}^{n}(2x_i + 3)}{n}$$

$$= \frac{\sum_{i=1}^{n}2x_i + \sum_{i=1}^{n}3}{n}$$

$$= 2\mu + 3$$ (05)

Square of the standard deviation of the given set of values.

$$= \frac{\sum_{i=1}^{n}\{(2x_i + 3) - (2\mu + 3)\}^2}{n}$$ (05)

$$= \frac{1}{n}\sum_{i=1}^{n}\{2(x_i - \mu)\}^2 = \frac{4}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$ (05)

Thus the standard deviation of the given set of values = 2σ

(05)

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(b) The given twelve number are

3, 6, 9, 12, 4, 6, 8, 10, 12, 14, x, y

Since the Mean of the set of numbers = 8

$$8 = \frac{84 + x + y}{12}$$

i.e. $x + y = 12$ (05)

Since the Mode = 6, at least one of the x and y should be equal to 6.

Thus x = y = 6 (05)

When the numbers are written in the ascending order they take the form

3, 4, 6, 6, 6, 6, 8, 9, 10, 12, 12,14 (05)

Thus we get the Median $= \frac{6+8}{2} = 7$ (05)

When three additional numbers 8 - k, 8, 8 + k are included the Mean of the new set of numbers. remains at the value 8. Using the result. (05)

Variance $= \frac{\sum x^2}{15} - 8^2$, We have (05)

$15.12 = 3^2 + 4^2 + 4.6^2 + 8^2 + 9^2 + 10^2 + 2.12^2 + 14^2 + 8^2 + (8-k)^2 + (8 + k)^2 - 15.8^2$ (05)

i.e. $180 = 9 + 16 + 144 + 64 + 81 + 100 + 288 + 196 + 64 + 64 + 2k^2 - 15.8^2$

$= 250 + 584 + 2k^2 - 11.8^2$

$= 834 + 2k^2 - 704$

$= 130 + 2k^2$ (05)

i.e. $2k^2 = 50$

$k = \pm 5$ (05)

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