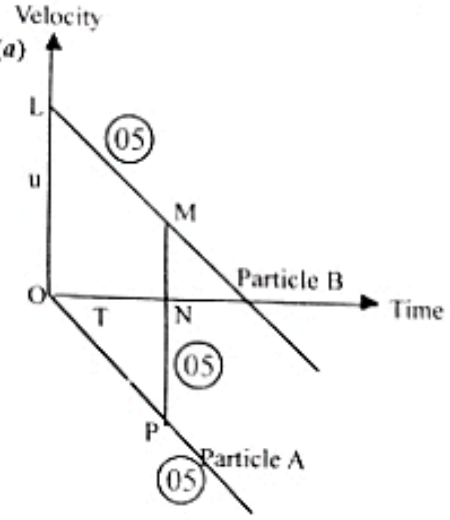
G.C.E. (Advanced Level) Examination - April 2005
Combined Mathematics - II
Provisional Scheme of Marking

(01) (a)



05 05 05 — 15

Let the two particles be at the same height at time T.

The distance travelled by particle B = the area of trapezium ONML (05)

The distance travelled by particle A = the area of triangle OPN (05)

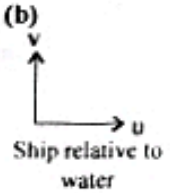
The sum of the distance travelled by Particles A and B = area of the parallelogram OPML (05)

$= h$ (05)

i.e. $UT = h$

$T = \frac{h}{u}$ (05) — 25

(b)



Ship relative to water

Velocity of the torpedo relative to the ship = velocity of the terpedo relative to water + velocity of water relative to the ship. (10)

(05)

Velocity of the torpedo relative to the ship =

$\overrightarrow{W \sin\theta} + W\cos\theta\uparrow + \overleftarrow{u} + \downarrow v$ (05)

$= \overrightarrow{W\sin\theta - u} + (W\cos\theta - v)\uparrow$ (05)

Torpedo strikes the ship if $W\sin\theta - u = 0$ (05)

$\sin\theta = \frac{u}{w}$

$\theta < \frac{\pi}{2}$ (05) $\quad \sin\theta < 1$ (05) $\quad \therefore u < W$ (05) — 45

time taken $T = \frac{d}{W\cos\theta - V}$ (05)

$\therefore \sin\theta = \frac{u}{w}$

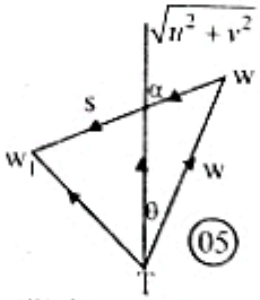
$\cos\theta = \frac{\sqrt{w^2 - u^2}}{w}$ (05)

$T = \frac{d}{\sqrt{w^2 - u^2} - v}$ (05) — 15

OR

Velocity of the torpedo relative to the ship

Velocity of the torpedo relative to water velocity of water relative to the ship (10)



W_1 is not admissible

$\overline{TW}$ - Torpedo relative to water

$\overline{WS}$ - Water relative to ship (05)

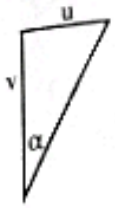
$\overline{TS}$ - Torpedo relative to ship

$|\overline{TW}| = TW = W$

$|\overline{WS}| = WS = \sqrt{u^2 + v^2}$

For Δ Tw S $\quad \frac{W}{\sin\alpha} = \frac{\sqrt{u^2 + v^2}}{\sin\theta} \rightarrow (01)$ (05)

$\tan\alpha = \frac{u}{v}$ $\qquad \sin\alpha = \frac{u}{\sqrt{u^2+v^2}}$ (05)



$(01) \Rightarrow W\sin\theta = \sqrt{u^2+v^2}\sin\alpha = u$ (05)

$\sin\theta = \frac{u}{w}$

$\theta < \frac{\pi}{2}$ (05) $\qquad \therefore u < W$ (05)

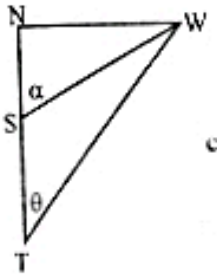
40

Time taken $T = \frac{d}{TS}$ (05)

$TS = TN - SN$

$= TW\cos\theta - SW\cos\alpha$

$= W\cos\theta - \sqrt{u^2+v^2}\cos\alpha$ (05)



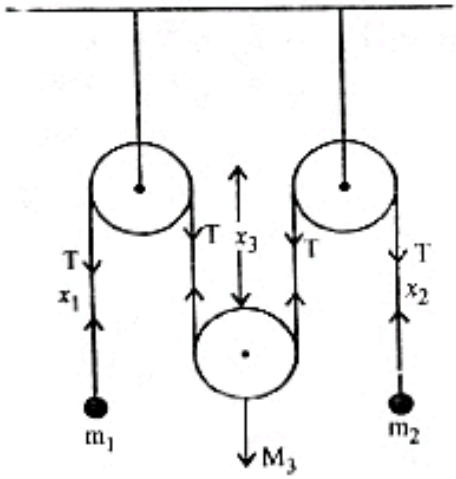
But,

$\cos\theta = \frac{\sqrt{w^2+u^2}}{w}$ and $\cos\alpha = \frac{v}{\sqrt{v^2+u^2}}$

$\therefore TS = \sqrt{w^2+u^2} - v$

$\therefore T = \frac{d}{\sqrt{w^2+u^2}-v}$ (05) 15

(02) (a)



$x_1 + x_2 + 2x_3 + d_1 + d_2 = \ell$ (05)

Where d_1 is the lenght of the portions of the string in contact with the pulleys,

d_2 is the difference in lengthes of the vertical portions of the string around the pulley C and ℓ is the total length of the string.

$\ddot{x}_1 + \ddot{x}_2 + 2\ddot{x}_3 = 0$ (05)

Let $f_1\downarrow$ and $\downarrow f_2$ be the accelerations of m_1 and m_2 respectively.

Then the acceleration of

$M_3 = \ddot{x}_3 = -\frac{1}{2}(\ddot{x}_1 + \ddot{x}_2) = -\frac{1}{2}(f_1 + f_2)$ (05)

Applying $P = mf$ to $m_1\downarrow$ $m_1g - T = m_1f_1 \rightarrow (01)'$ (05)

Applying $P = mf$ to $m_2\downarrow$ $m_2g - T = m_2f_2 \rightarrow (02)'$

Applying $P = mf$ to $M_3\downarrow$ $M_3g - 2T = M_3f_3$

$= -\frac{M_3}{2}(f_1 + f_2) \rightarrow (03)'$

$(01)' \Rightarrow g - \frac{T}{m_1} = f_1$

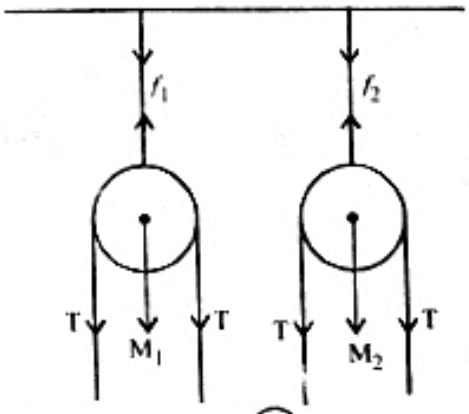
$(02)' \Rightarrow g - \frac{T}{m_2} = f_2 \rightarrow (02)$ (05)

$(03)' \Rightarrow 2g - \frac{4T}{M_3} = -(f_1 + f_2) \rightarrow (03)$ (05)

$(01) + (02) + (03) \Rightarrow 4g - T\left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{4}{M_1}\right) = 0$ (05)

$T = \frac{4m_1m_2M_3g}{4m_1m_2 + M_3(m_1+m_2)}$

35



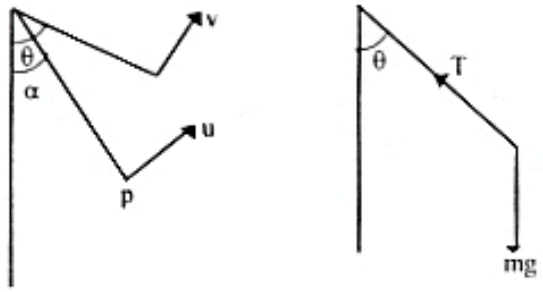
Force exerted on the ceiling (05)

$= f_1 + f_2 = 2T + M_1g + 2T + M_2g$ (05)

$= 4T + (M_1 + M_2)g$

$= \frac{\left[16m_1m_2M_3 + 4m_1m_2(M_1+M_2) + M_3(m_1+m_2)(M_1+M_2)\right]}{m_1m_2 + M_3(m_1+m_2)}g$ (05) 15

(b)



Conservation of energy.

(05) $\frac{1}{2}mv^2 - mga\cos\theta = \frac{1}{2}mu^2 - mga\cos\alpha$ (05)

$v^2 = u^2 - 2ga\cos\alpha + 2ga\cos\theta \rightarrow (01)$ [10]

Applying p = mf in the radial direction.

(05) $T - mg\cos\theta = \frac{mv^2}{a}$ (05)

$\frac{mu^2}{a} - 2mg\cos\alpha + 2mg\cos\theta$

$\frac{aT}{m} = u^2 - 2ga\cos\alpha + 3ga\cos\theta \rightarrow (02)$

As θ increases both v^2 and $\frac{aT}{m}$ decrease

When, $\alpha \le \theta < \frac{\pi}{2}$ $\quad v^2 < \frac{aT}{m}$

$\theta = \frac{\pi}{2}$ $\quad v^2 = \frac{aT}{m}$

$\theta > \frac{\pi}{2}$ $\quad v^2 > \frac{aT}{m}$

The particle describes a circular are and then begins to move freely under gravity at θ = β provided $v^2 > 0$ and T = 0 at θ =β (β ≠ π) (05)

This is possible provided that $\pi > \beta > \frac{\pi}{2}$

From (02),

T = 0 when $\cos\theta = \frac{1}{3}\left(2\cos\alpha - \frac{u^2}{ga}\right)$

$\theta = \cos^{-1}\left[\frac{1}{3}\left(2\cos\alpha - \frac{u^2}{ga}\right)\right]$ (05)

[20]

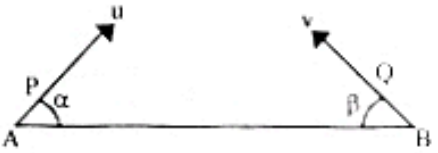
$\pi > \beta > \frac{\pi}{2} \Rightarrow 0 > \cos\theta > -1$ (05)

$i.e.\ \ 0 > \frac{1}{3}\left(2\cos\alpha - \frac{u^2}{ga}\right) > -1$ (05)

$ga(3 + 2\cos\alpha) > u^2 > 2ga\cos\alpha$ (05)

(05) [20]

(03)



Let S_p and S_a be the vertical distances travel by P & Q respectively at time t.

$S_P = u\sin\alpha t - \frac{1}{2}gt^2$

$S_Q = v\sin\beta t - \frac{1}{2}gt^2$

If P & Q collide, then $S_P = S_Q$ at t = T (say)

$u\sin\alpha T - \frac{1}{2}gT^2 = v\sin\beta T - \frac{1}{2}gT^2$ (05)

$(u\sin\alpha - v\sin\beta)T = 0 \Rightarrow u\sin\alpha = v\sin\beta$ as $T \neq 0$ (05)

Vertical component of velocity of P at time t,

u sin α - gt

Vertical component of velocity of Q at time t,

V sin β- gt

∴ usin α = vsinβ , usinα - gt = vsinβ - gt (05)

until P & Q collide [15]

If p moves horizontally then its vertical component is zero. (05)

Since vertical components are equal, vertical component of velocity of Q is also zero.

There fore Q moves horizontally. (05) [10]

Since the vertical component of velocity is zero at T,

$u\sin\alpha - gT = 0$

$$T = \frac{u\sin\alpha}{g}$$ (05)

The horizontal distance travel by P in time

$$T \quad \frac{u\cos\alpha \cdot u\sin\alpha}{g} = \frac{u^2\sin\alpha\cos\alpha}{g}$$ (05)

sum of the horizontal distances travel by P and Q at is

$$\frac{u^2\sin 2\alpha}{g}$$ (05)

$$= \frac{2u^2\sin\alpha\cos\alpha}{g}$$

$\therefore$ horizontal distances travelled by Q at time T is

$$= \frac{2u^2\sin\alpha\cos\alpha}{g} - \frac{u^2\sin\alpha\cos\alpha}{g}$$

$$= \frac{u^2\sin\alpha\cos\alpha}{g}$$ (05)

But the horizontal distance travelled by Q at T is

$$v\cos\beta T = \left(v\cos\beta \cdot \sin\alpha\right)/g$$ (05)

$$\therefore v\cos\beta\frac{u\sin\alpha}{g} = \frac{u^2\sin\alpha\cos\alpha}{g}$$ (05)

$$v\cos\beta = u\cos\alpha$$ 30

(ii) $\xrightarrow[m]{u\cos\alpha}$ $\xleftarrow[\lambda m]{u\cos\alpha}$ $\xrightarrow[(\lambda+1)m]{W}$

conservation of momentum

$mu\cos\alpha - \lambda mu\cos\alpha = (\lambda + 1)\,mW$

$$W = \frac{(1-\lambda)u\cos\alpha}{1+\lambda}$$ (05) 05

The vertical distance travelled by either P or Q

$$= u\sin\alpha\cdot\frac{u\sin\alpha}{g} - \frac{1}{2}g\left(\frac{u\sin\alpha}{g}\right)^2$$ (05)

$$= \frac{1}{2}\frac{u^2\sin^2\alpha}{g}$$

Let T_1 be the time taken by the composite particle to fall to the ground.

Then $\frac{1}{2}\frac{u^2\sin^2\alpha}{g} = 0 + \frac{1}{2}gT_1^2 \Rightarrow T_1^2 = \frac{u^2\sin^2\alpha}{g^2}$

$$\Rightarrow T_1 = \frac{u\sin\alpha}{g}$$ (05)

$\therefore$ The horizontal distance travel by the composite particle is

$$\left(\frac{1-\lambda}{1+\lambda}\right)u\cos\alpha T_1 = \left(\frac{1+\lambda}{1+\lambda}\right)\frac{u^2\sin\alpha\cos\alpha}{g}$$ (05)

$\therefore$ The composite Particle will fall to the ground at a distance

$$\frac{u^2\sin\alpha\cos\alpha}{g} + \left(\frac{1-\lambda}{1+\lambda}\right)\frac{u^2\sin\alpha\cos\alpha}{g}$$ from A (05)

i.e. At a distance $\frac{2u^2\sin\alpha\cos\alpha}{(1+\lambda)g} = \frac{u^2\sin 2\alpha}{(1+\lambda)g}$ from A. (05)

25

If the mass of Q is negligible compared to that of P then $\lambda = 0$ and the composite particle falls to the ground at a distance $\frac{u^2\sin 2\alpha}{g}$ from A, i.e. at B. (05)

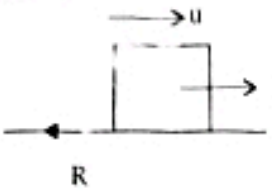
If the mass of P is negligible copared to that of Q then $\lambda \to \infty$ and the composite particle falls to the ground at zero distance from A.

i.e. at A (or form symmetry) (05) 10

If the masses of P and Q are equal then $\lambda = 1$ and the composite particle falls to the ground at a distance $\frac{u^2\sin 2\alpha}{2g}$ from A, i.e at the midpoint of AB

(or the horizontal component of the velocity of the cimposite particle is zero. $\therefore$ It falls vertically under gravity) (05) 05

(04)(a) For horizontal motion, equation of motion is P - R = 0 and the power equation (with tractive force P and Speed u) is P. u = 1000 H (05)



$\Rightarrow R = \frac{1000H}{u}$ (05)

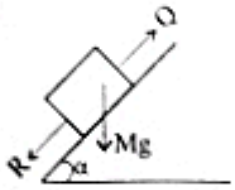
For motion on inclined road equation of motion is

Q - R - Mg sin α = 0, and the power equation

(with tractive force Q and speed v) is Q v = 1000H (05)

Eliminating Q and R, $\frac{1000H}{v} - \frac{1000H}{u} = Mg\sin\alpha$ (05)

$\Rightarrow H = \frac{uvMg\sin\alpha}{1000(u-v)}$ (05)



25

(b) For equilibrium of particle P, Mg = $T_0 = \frac{\lambda c}{\ell}$ (05)

$\Rightarrow \lambda = \frac{mgl}{c}$ (05) 10

Velocity of Q, just before impact is $u = \sqrt{2gc}$ (05)

conservation of linear momentum ⇨ 2mv = mu (05)

initial velocity of composite body, $v = \frac{u}{2} = \sqrt{\frac{gc}{2}}$ (05)

for motion of composite body, $2mg - T = 2m\ddot{x}$ (05)

Where the tension $T = \frac{\lambda x}{\ell}$ (05)

Eliminating T and substituting for $\frac{\lambda}{\ell}$: $2mg - \left(\frac{mg}{c}\right)x = 2m\ddot{x}$ (05)

$\ddot{x} + \frac{g}{2c}(x - 2c) = 0$ (05)

$\ddot{x} + \omega^2(x - 2c) = 0$, *where* $\omega^2 = \frac{g}{2c}$ 35

Given : $x = 2C + a\cos\omega t + b\sin\omega t$

$\dot{x} = -a\omega\sin\omega t + b\omega\cos\omega t$ (05)

When t = 0, x = C ⇨ a = -c (05)

When $t = 0, \dot{x} = v = \sqrt{\frac{gc}{2}} \Rightarrow \sqrt{\frac{gc}{2}} = b\sqrt{\frac{g}{2c}} = b = c$ (05)

15

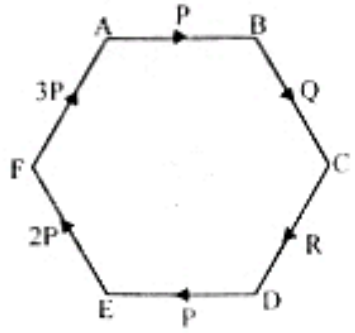
$\dot{x} = c\left[2 - \cos\omega t + \sin\omega t\right]$, $\dot{x} = c\omega(\sin\omega t + \cos\omega t)$

$\dot{x} = 0$ for the first time when $\sin\omega t + \cos\omega t = 0$ (05)

$\omega t = \frac{3\pi}{4} \Rightarrow t = \frac{3\pi}{4}\sqrt{\frac{2c}{g}}$ (05) 10

Then, the extension $x = 2c + \frac{2c}{\sqrt{2}} = c(2 + \sqrt{2})$ (05) 05

(05)



$\rightarrow x = p + \frac{1}{2}(Q - R) - P + \frac{1}{2}P(3 - 2)$

$= \frac{1}{2}(P + Q - R)$ (05)

Resolving perpendicular to AB, $y = \frac{\sqrt{3}}{2}(5P - Q - R)$ (05)

(1) couple x = 0, y = 0 ⇨ Q - R = -P

Q + R = 5P (05)

Q = 2P, R = 3P (05) (05) 25

Moment (taken about O) $= a\sqrt{3}\cdot 2\cdot(P + 2P + 3P)$

$= 12aP\sqrt{3}Nm$ (05)

in the sense of ABC (05) 10

(ii) Single force along AD

Taking moments

about A : $a\sqrt{3}(Q + 2P) + 2a\sqrt{3}(R + P) = 0$

$\Rightarrow Q + 2R = -4P$ (05)

about $D: a\sqrt{3}(Q+2P)+2a\sqrt{3}(3P+P)=0$

$\Rightarrow Q+10P=0$ (05)

Solving $Q = -10P$ (05)

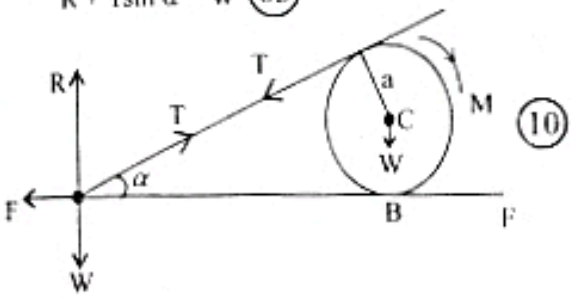
$R = 3P$ (05) [20]

(b) for particle; in limiting equilibrium,

$T\cos\alpha = F$; (05)

$F = \mu R$ (05)

$R + T\sin\alpha = W$ (05)



(10)

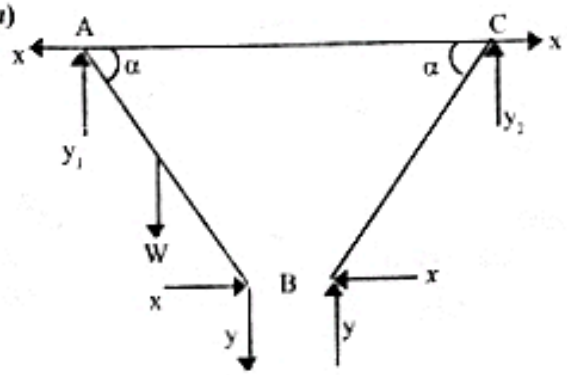
Eliminating R, $T(\cos\alpha+\mu\sin\alpha)=\mu W$

$\Rightarrow T = \dfrac{\mu W}{\cos\alpha+\mu\sin\alpha}$ (05) [30]

Taking moments about B, $G = T(a + a\cos\alpha)$ (10)

$\Rightarrow G = \dfrac{\mu W a(1+\cos\alpha)}{\cos\alpha+\mu\sin\alpha}$ (05) [15]

(06)(a)



For AB taking moments about A

$X.2a\sin\alpha - y.2a\cos\alpha = W.a\cos\alpha$ (05)

For CB, taking moments about C,

$X.2a\sin\alpha + Y.2a\cos\alpha = 2W.a\cos\alpha$ (05)

adding, $X(4a\sin\alpha) = 3Wa\cos\alpha$ (05)

$\Rightarrow X = \dfrac{3}{4}W\cot\alpha$;

Subtracting and simplifying, $Y = \dfrac{1}{4}W$ (05)

Horizontal reactions at A and C are each of magnitude

$X = \dfrac{3}{4}W\cot\alpha$; (05)

Vertical reaction at A is $Y_1 = Y + W = \dfrac{5}{4}W$ (05)

Vertical reaction at C is $Y_2 = 2W - Y = \dfrac{7}{4}W$ (05)

If the lines of action of the total reactions at A and C are perpendicular to each other,

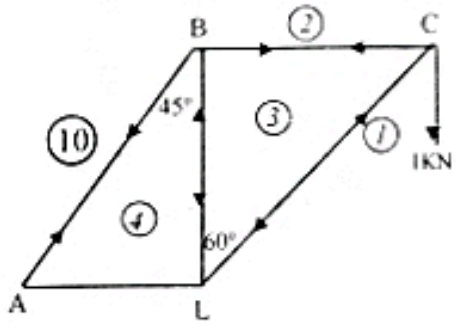
Then $\dfrac{Y_2}{X} = \tan\theta = \dfrac{X}{Y_1}$ (05)

$Y_1Y_2 = X^2 \Rightarrow \dfrac{5}{4}W\cdot\dfrac{7}{4}W = \left(\dfrac{3}{4}W\cot\alpha\right)^2$ (05)

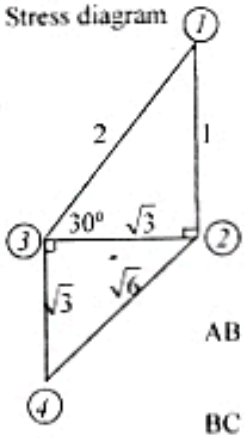
$\tan^2\alpha = \dfrac{9}{35}$ (05)

$\tan\alpha = \dfrac{3}{\sqrt{35}}$ [50]

(b)



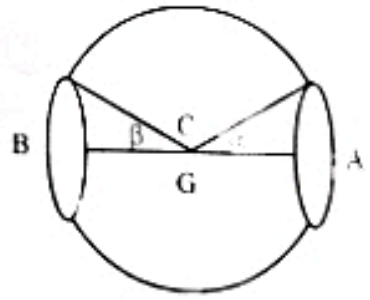
Stress diagram



AB	(05) Tension	(05) $1000\sqrt{6}N$
BC	(05) Tension	(05) $1000\sqrt{3}N$
CD	(05) Thrust	(05) $2000N$
BD	(05) Thrust	(05) $1000\sqrt{3}N$

[50]

(07)



Mass of the zone $M = \sigma \int_{\alpha}^{\pi-\beta} 2\pi a \sin\theta (a d\theta)$ (05)

(05)

$= \sigma\left(2\pi a^2\right)\left[-\cos\theta\right]_{\alpha}^{\pi-\beta}$ (05)

$= 2\pi a^2 \sigma(\cos\alpha + \cos\beta)$ (05) [20]

By, symmetru C.M. G lies on AB (05)

at distance $\bar{x} = OG$, from O, Where

$M\bar{x} = \sigma \int_{\alpha}^{\pi-\beta} 2\pi a \sin\theta (a d\theta)(a\cos\theta)$ (10)

(05)

$= \sigma\left(\pi a^3\right)\left[\sin^2\theta\right]_{\alpha}^{\pi-\beta}$ (05)

$= \pi a^3 \sigma\left[\sin^2\beta - \sin^2\alpha\right]$ (05)

$= \pi a^3 \sigma\left[\cos^2\alpha - \cos^2\beta\right]$ (05)

$\bar{x} = \frac{a}{2}(\cos\alpha - \cos\beta)$ (05)

$GA = OA - OG = a\cos\alpha - \frac{a}{2}(\cos\alpha - \cos\beta)$ (05)

$= \frac{a}{2}(\cos\alpha - \cos\beta)$ (05)

G is the mid-point of AB. (05) [55]

For neutral equilibrium, the C.G. of the composite body has to be coincident with O. (05)

$M\bar{x} - (a\cos\beta)\sigma\pi a^2 \sin^2\beta = 0$ (10)

$\pi a^3 \sigma\left(\sin^2\beta - \sin^2\alpha\right) - \sigma\left(\pi a^3\right)\cos\beta\sin\beta = 0$ (05)

$\sin^2\alpha = \sin^2\beta(1 - \cos\beta)$ (05)

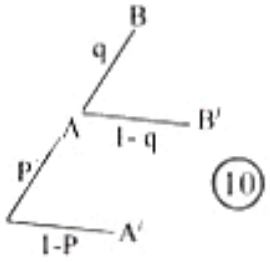
$\sin\alpha = (\sin\beta)\sqrt{(1-\cos\beta)}$ [25]

(08) Let

A = { making an offence when he parks his vehicle} and

B = {he is fined when he makes a parking offence}

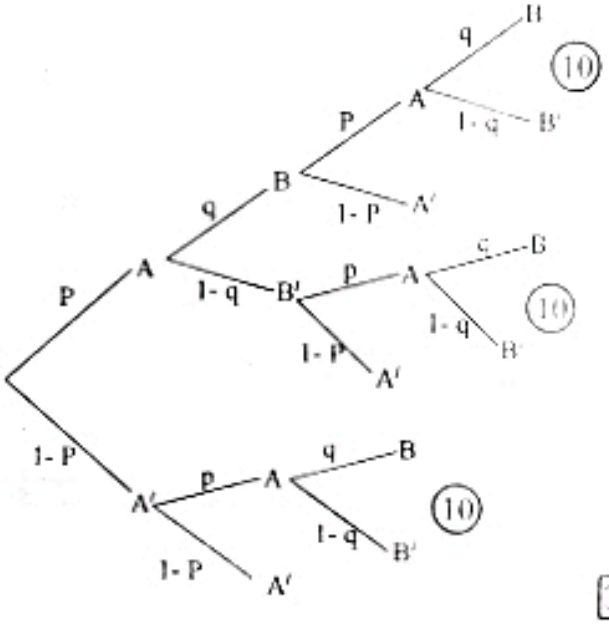
S = {AB, AB', A'} (05) [05]



(10) [10]

P[AB] = pq, $P\left[AB'\right]$ = p(1-q) and $P\left[A'\right]$ = 1 - P

(05) (05) [10]



(10) (10) (10) [30]

(ii) The probability that he is fined in both occasions

=P[ABAB] = pqpq = $p^2 q^2$ [10]

(05) (05)

(iii) The probability that he is fined only once and he makes offences on both occasions.

= P[ABAB'] + P AB' AB] (05)

= pqp (1-q) + P(1 - q)pq

= $2p^2 q(1 - q)$ (05)

The required probability $= \frac{2p^2 q(1-q)}{p^2}$ (05)

$= 2q(1-q)$ (05) [20]

(iv) The probability that he is fined only once and he makes a parking offence in only one occasion

$$= P(ABA') + P(A'AB)$$
$$= pq(1-p) + (1-p)pq$$
$$= 2pq(1-p) \quad (05)$$

The required probability $= \dfrac{2pq(1-p)}{2p(1-p)}$ (05)

$= q$ (05) [15]

(09) (a)

(i) Mean $\bar{x} = \dfrac{1}{n}\sum_{i=1}^{n} x_i$ (05) and Variance $= \dfrac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ (05) [10]

$$\bar{z} = \frac{x_1 + \ldots + x_n + y_1 + \ldots + y_m}{n+m} \quad (05)$$

But $\bar{x} = \dfrac{x_1 + \ldots + x_n}{n}$ and $\bar{y} = \dfrac{y_1 + \ldots + y_m}{m}$ (05)

$$\therefore \bar{z} = \frac{n\bar{x} + m\bar{y}}{n+m}$$ [10]

(ii) $\sum_{i=1}^{n}(x_i - \bar{z})^2$

$$= \sum_{i=1}^{n}\{(x_i - \bar{x}) + (\bar{x} + \bar{z})\}^2$$
$$= \sum_{i=1}^{n}(x_i - \bar{x})^2 + 2(\bar{x} - \bar{z})\sum_{i=1}^{n}(\bar{x}_i - \bar{x}) + \sum_{i=1}^{n}(\bar{x} - \bar{z})^2 \quad (05)$$
$$= \sum_{i=1}^{n}(x_i - \bar{x})^2 + \sum_{i=1}^{n}(\bar{x} - \bar{z})^2 \because \sum_{i=1}^{n}(\bar{x}_i - \bar{x}) = 0 \quad (05)$$
$$= n\sigma_1^2 + nd_1^2, \quad \text{where} \quad d_1 = \bar{x} - \bar{z} \quad (05)$$ [15]

(iii) $\sigma^2 = \dfrac{1}{n+m}\left\{(x_1 - \bar{z})^2 + \ldots\ldots + (x_n - \bar{z})^2 + (y_1 - \bar{z})^2 + \ldots\ldots + (y_m - \bar{z})^2\right\}$ (05)

$$= \frac{1}{n+m}\left\{\sum_{i=1}^{n}(x_i - \bar{z})^2 + \sum_{i=1}^{m}(y_i - \bar{z})^2\right\} \quad (05)$$
$$= \frac{1}{n+m}\left\{n(\sigma_1^2 + d_1^2) + m(\sigma_2^2 + d_2^2)\right\} \quad (05)$$ [15]

(b) (i)

Mid value	$x' = \dfrac{x-47}{5}$	f	fx^1	fx^2
32	-3	5	-15	45
37	-2	10	-20	40
42	-1	15	-15	15
47	0	30	0	0
52	1	5	5	5
57	2	5	10	20
Total			-35	125
			(05)	(05)

$$\bar{x} = 5\bar{x}' + 47 = 5 \times \frac{-35}{70} + 47 = 44.5 \quad (05)$$

$$\sigma^2 = 25\left[\frac{125}{70} - \left(\frac{-35}{70}\right)^2\right] = 25\left[\frac{25}{14} - \frac{1}{4}\right] \quad (05)$$
$$= \frac{25 \times 43}{28} = 38.39$$ [20]

(ii) $38 = \dfrac{30\bar{x} + 70 \times 44.5}{100}$ (05)

$$\bar{x} = \frac{3800 - 3115}{30} = \frac{685}{30} = 22.83 \quad (05)$$ [10]

$$12^2 = \frac{1}{100}\left\{30\left[\sigma_1^2 + (22.83 - 38)^2 + 70\left(38.39 + (44.5 - 38)^2\right)\right]\right\}$$ (10)

$$14400 = 30\left(\sigma_1^2 + (-15.17)^2\right) + 70\left(38.39 + 6.5^2\right)$$
$$14400 = 30\left(\sigma_1^2 + 230.13\right) + 70(38.39 + 42.25)$$
$$14400 = 30\sigma_1^2 + 6903.9 + 5644.8$$
$$30\sigma_1^2 = 14400 - 12548.7 = 1851.3$$
$$\sigma_2^2 = 61.71 \quad (10)$$ [20]