G.C.E. (Advanced Level) Examination - April 2004
10 - Combined Mathematics - I
Provisional Scheme of Marking

(01) **(a)** $p(x) = x^2 - 2\lambda x + 2\lambda - 1$ (05)

The discriminant

$\Delta = 4\lambda^2 - 4(2\lambda - 1)$ (05)

$= 4\left(\lambda^2 - 2\lambda + 1\right)$

$= 4(\lambda - 1)^2 \geq 0$ (05)

$\therefore$ the roots of p(x) = 0 are real. (05) [20]

Let α and β be roots of $p(x) = 0$

Then, $\alpha + \beta = 2\lambda$ and $\alpha\beta = 2\lambda - 1$ (05)

Then the condition

$\alpha + \beta = \alpha^2 + \beta^2$ (05)

$= (\alpha + \beta)^2 - 2\alpha\beta$ ..... be comes (05)

$2\lambda = 4\lambda^2 - 2(2\lambda - 1)$

$4\lambda^2 - 6\lambda + 2 = 0$

$2\lambda^2 - 3\lambda + 1 = 0$ (05)

$(2\lambda - 1)(\lambda - 1) = 0$

$\therefore \lambda = \frac{1}{2}$ and $\lambda = 1$ are the required values.

(05) (05) [30]

(b) It is given that $P(1) = 1$, $P(2) = \frac{1}{2}$, $P(3) = \frac{1}{3}$

(05) (05) (05)

Now $Q(x) = x\,P(x) - 1$

$Q(1) = 1P(1) - 1$

$= 1 \cdot 1 - 1 = 0$ (05)

and hence (x-1) is a factor of Q(x)

$Q(2) = 2P(2) - 1 = 2 \times \frac{1}{2} - 1 = 0$ (05)

$\Rightarrow$ (x -2) is a factor of Q(x)

$Q(3) = 3P(3) - 1$

$= 3 \times \frac{1}{3} - 1 = 1 - 1 = 0$ (05)

$\Rightarrow$ (x -3) is a factor of Q(x) [30]

AS Q(x) = xP(x) - 1 are P(x) is of degree 2, Q(x) must be degree 3. (05)

$\therefore$ Q(x) = k(x - 1) (x -2) (x - 3) for some constant K (05)

We have

k(x - 1) (x - 2) (x -3) = xP(x) - 1 (05)

Substituting x = 0 on both sides

-6k = -1

$k = \frac{1}{6}$

$\therefore$ $Q(x) = \frac{1}{6}(x-1)(x-2)(x-3)$ (05) [20]

(02) **(a)** Number of ways that the six questions can be chosen

$= {}^9C_6 = \frac{9!}{6!\,3!} = 84$ (05) [05]

(i) If the first three questions are compulsory then the number of ways

$= {}^6C_3 = \frac{6!}{3!\,3!} = 20$ (10) [10]

(ii) There are two possibilities,

+ first five questions and one out of the other four. (05)
+ four out of first five questions and two out of the other four.

$\therefore$ The number of ways $= {}^4C_1 + {}^5C_4 \times {}^4C_2$

$= \frac{4!}{1!\,3!} + \frac{5!}{4!} \times \frac{4!}{2!\,2!}$ (05) (05)

$= 4 + 30$

$= 34$ (05) [20]

(b) Let $(1+7x)^{23} = A_0 + A_1x + \ldots\ldots + A_rx^r + \ldots\ldots + A_{23}x^{23}$

where $A_r = {}^{23}C_r\ 7^r$

(i) $\dfrac{A_{r+1}}{A_r} = \dfrac{{}^{23}C_{r+1}7^{r+1}}{{}^{23}C_r 7^r}$ (05)

$= \dfrac{7(23-r)}{r+1} = \dfrac{161-7r}{r+1}$ (05)

$\dfrac{A_{r+1}}{A_r} \gtrless 1 \Leftrightarrow \dfrac{161-7r}{r+1} \gtrless 1$ (05)

$\Leftrightarrow 161-7r \gtrless r+1$

$\Leftrightarrow 160 \gtrless 8r$

$\Leftrightarrow 20 \gtrless r$ (05)

i.e. $r < 20 \Rightarrow A_{r+1} > A_r$

$r = 20 \Rightarrow A_{r+1} = A_r$

$r > 20 \Rightarrow A_{r+1} < A_r$ i.e. $A_{21} = A_{20}$ (05)

∴ The greatest numerical coefficient of the expansion is

$A_{20} = A_{21} = {}^{23}C_2 \cdot 7^2$

$= 253 \times 7^{21}$ (05)

The terms of the expansion corresponding to it are the twentieth and twenty first terms. (05) 40

(ii) Let T_r be the term of the expression containing x^r

Then $T_r = {}^{23}C_r 7^r x^r$ (05)

$\dfrac{T_{r+1}}{T_r} = \dfrac{A_{r+1}x^{r+1}}{A_r x^r} = \dfrac{161-7r}{r+1}x$ (05)

$T_{r+1} \gtrless T_r \Leftrightarrow 161x - 7rx \gtrless r+1$

$\Leftrightarrow \dfrac{161x-1}{1+7x} \gtrless r$

If the fourth term is the large term, then we have

$2 \le \dfrac{161x-1}{1+7x} \le 3$ (05) i.e. $2 + 14x \le 161x - 1 \le 3 + 21x$

i.e. $\dfrac{1}{4g} \le x$ (05) and $x \le \dfrac{1}{35}$ Therefore the required range is

$\dfrac{1}{49} \le x \le \dfrac{1}{35}$ (05) 25

(03) (a) When n = 1

L.H.S. $= \dfrac{1}{3}$ and R. H. S $= \dfrac{3}{4} - \dfrac{1}{4} - \dfrac{1}{6}$

$= \dfrac{18-6-4}{24}$

$= \dfrac{8}{24} = \dfrac{1}{3}$

Hence the result is true for n = 1 (05)

Now we assume that the result is true fer n = p

Where p is any positive integer.

Then $\dfrac{1}{2^2-1} + \dfrac{1}{3^2-1} + \ldots\ldots + \dfrac{1}{(p+1)^2-1}$

$= \dfrac{3}{4} - \dfrac{1}{2(p+1)} - \dfrac{1}{2(p+2)}$ (05)

Adding $\dfrac{1}{(p+2)^2-1}$ to both sides, (05)

$\dfrac{1}{2^2-1} + \dfrac{1}{3^2-1} + \ldots\ldots + \dfrac{1}{(p+1)^2-1} + \dfrac{1}{(\overline{p+1}+1)^2-1}$

$= \dfrac{3}{4} - \dfrac{1}{2(p+1)} - \dfrac{1}{2(p+2)} + \dfrac{1}{(p+2)^2-1}$

$= \dfrac{3}{4} - \dfrac{1}{2(p+1)} - \dfrac{1}{2(p+2)} + \dfrac{1}{(p+1)(p+3)}$ (05)

$= \dfrac{3}{4} - \dfrac{1}{2(p+1)} - \dfrac{1}{2(p+2)} + \dfrac{1}{2(p+1)} - \dfrac{1}{2(p+3)}$ (05)

$= \dfrac{3}{4} - \dfrac{1}{2(\overline{p+1}+1)} - \dfrac{1}{2(\overline{p+1}+2)}$ (05)

Hence if the result is true for n = p, where p is any positive integer then it is also true for n = p +1

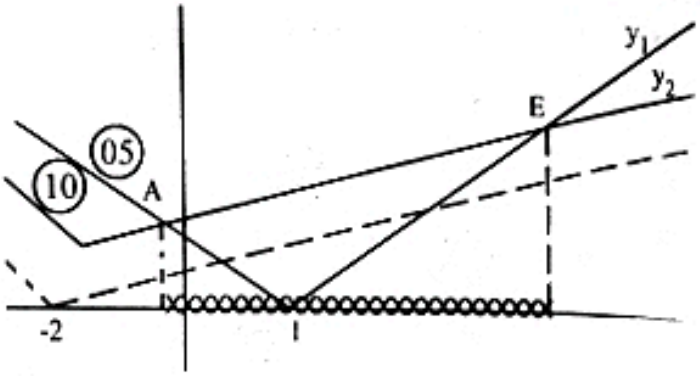
∴ By the principle of Mathematical Induction the result holds. (05) 35

$\lim_{n\to\infty} \sum_{r=1}^{n} \dfrac{1}{r(r+2)} = \lim_{n\to\infty} \sum_{r=1}^{n} \dfrac{1}{(r+1)^2-1}$ (05)

$= \lim_{n\to\infty} \left\{\dfrac{3}{4} - \dfrac{1}{2(n+1)} - \dfrac{1}{2(n+2)}\right\} = \dfrac{3}{4}$ (05)

$\therefore$ The series $\sum_{r=1}^{\infty} \frac{1}{r(r+2)}$ is convergent and series

Sum $= \frac{3}{4}$ (05) [15]



(b) Method I

$$|x-1| = \begin{cases} x-1 & x \geq 1 \\ -(x-1) & x < 1 \end{cases}$$ (05)

$$\left|\frac{1}{2}x+1\right| = \begin{cases} \frac{1}{2}x+1 & x \geq -2 \\ -\left(\frac{1}{2}x+1\right) & x < -2 \end{cases}$$ (05)

Case (i) $x < -2$

Then the inequality becomes.

$-(x-1)+\frac{x}{2}+1<1$ i.e. $x>2$ (05) This is impossible (05)

Case (ii) $-2 \leq x < 1$

Then the ineguality becomes

(05) $-(x-1)-\left(\frac{x}{2}+1\right)<1$ *i.e.* $x > -\frac{2}{3}$ (05)

Hence, all $x \in \left(\frac{-2}{3}, 1\right)$ are solutions.

Case (iii) $x \geq 1$

Then the ineguality becomes

(05) $(x-1)-\left(\frac{x}{2}+1\right)<1$ *i.e.* $x<6$ (05)

Hence all $x \in [1,6]$ are solution.

combining all three cases the set of all solutions of

$$|x-1|-\left|\frac{x}{2}+1\right|<1$$

is $\left\{x : -\frac{2}{3} < x < 6\right\} = \left(\frac{-2}{3}, 6\right)$ (05) [45]

Method II

Let $y_1 = |x-1|$ and $y_2 = \left|\frac{1}{2}x+1\right|+1$ Then the inequality becomes $y_1 < y_2$ (05)

The shaded interval is the required solution set.

It is sufficient to find the x coordinate of point A and B.

A is the point of intersection of the lines.

$y = -(x-1)$ and $y = \left(\frac{1}{2}x+1\right)+1$ (05)

$\therefore -x+1 = \frac{1}{2}x+2 \Rightarrow \frac{3}{2}x = -1 \rightarrow x = -\frac{2}{3}$ (05)

B is the point of intersection of the lines

$y = x-1$ and $y = \left(\frac{1}{2}x+1\right)+1 = \frac{1}{2}x+2$ (05)

$\therefore x-1 = \frac{1}{2}x+2 \Rightarrow \frac{1}{2}x = 3 \Rightarrow x = 6$ (05)

$\therefore$ Required solution set $= \left\{x : \frac{-2}{3} < x < 6\right\} = \left(\frac{-2}{3}, 6\right)$ (05)

[45]

The greatest integral value in the solution set is 5. (05) [05]

(04) $2z^2 = \frac{1}{2}\left(1+\sqrt{3}\right)^2 = -1+i3$ (05)

$= 2\left[-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right] = 2\left[\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right]$ (05)

implies that the modulus of $2z^2$ is 2 and the argument (05)

(principal) is $\frac{2\pi}{3}$ (05) [20]

$\frac{3}{z^2} = \frac{6}{-1+i\sqrt{3}} \times \frac{-1-\sqrt{3}i}{-1-\sqrt{3}i} = \frac{3}{2}\left(-1-i\sqrt{3}\right)$ (05)

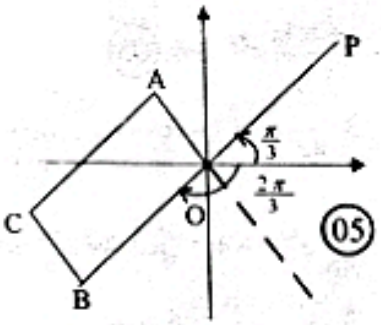
$= 3\cos\left(\frac{-2\pi}{3}\right)+i\sin\left(\frac{-2\pi}{3}\right)$ (05)

implies that the modulus of $\frac{3}{z^2}$ is 3 and the argument (principal) is $\left(\frac{-2\pi}{3}\right)$ (05) (05) [20]

Let P represents z, Then

$\hat{POB} = \left(\pi - \frac{\pi}{3}\right) + \left(\pi - \frac{2\pi}{3}\right) = 2\pi - \pi = \pi$ (05) (05)

Therefore the points lie on the same line. (05) [15]



(05)

The complex number represented by C is

$= 2z^2 + \frac{3}{z^2} = -1 + \sqrt{3}i - \frac{3}{2}\left(1 + \sqrt{3}i\right)$ (05) (05)

$= -\frac{5}{2} - \frac{\sqrt{3}i}{2}$ (05) [20]

Length of $OC = \sqrt{\frac{25}{4} + \frac{3}{4}} = \frac{\sqrt{28}}{2} = \sqrt{7}$ (05) (05) [10]

Length of $AB = \left|2z^2 - \frac{3}{z^2}\right| = \left|-1 + \sqrt{3}i + \frac{3}{2} + \frac{3\sqrt{3}}{2}\right|$ (05)

$= \left|\frac{1}{2} + \frac{5\sqrt{3}}{2}i\right|$ (05)

$= \sqrt{\frac{1}{2} + \frac{75}{4}}$

$= \sqrt{19}$ (05) [15]

(05) (a) $y = e^{-x}(\cos 2x + \sin 2x)$

$\frac{dy}{dx} = e^{-x}(-2\sin 2x + 2\cos 2x) + (\cos 2x + \sin 2x)\left(-e^{-x}\right)$ (05)

$= e^{-x}(-3\sin 2x + \cos 2x)$

$\frac{dy}{dx} + y = -3e^{-x}\sin 2x + e^{-x}\cos 2x + e^{-x}\cos 2x + e^{-x}\sin 2x$

$= 2e^{-x}\sin 2x + 2e^{-x}\cos 2x$ (05)

$= 2e^{-x}(\cos 2x - \sin 2x)$ (05) [15]

$\therefore \frac{d^2y}{dx^2} + \frac{dy}{dx}$

$= 2e^{-x}(-2\sin 2x - 2\cos 2x) + 2(\cos 2x - \sin 2x)\left(-e^{-x}\right)$ (05) (05)

$= -4e^{-x}\sin 2x - 4e^{-x}\cos 2x - 2e^{-x}\cos 2x + 2e^{-x}\sin 2x$

$= -4e^{-x}(\sin 2x + \cos 2x) - 2e^{-x}(\cos 2x - \sin 2x)$ (05) (05)

$= -4y - \left(\frac{dy}{dx} + y\right)$ (05)

$\therefore \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$ $\therefore p = 2, q = 5$ (05) [25]

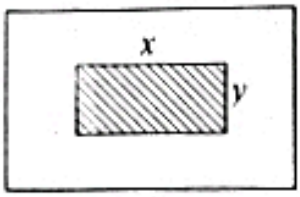
$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$ (05) Note that y(0) = 1

$\left(\frac{dy}{dx}\right)_{x=0} = 1, \left(\frac{d^2y}{dx^2}\right)_{x=0} = -2\left(\frac{dy}{dx}\right)_{x=0} - 5y(0) = -7$ (05)

$\left(\frac{d^3y}{dx^3}\right)_{x=0} = -2\left(\frac{d^2y}{dx^2}\right)_{x=0} - 5\left(\frac{dy}{dx}\right)_{x=0} = 14 - 5 = 9$ (05)

[15]

(b)



Let the sides of rectangular printed region be x cm and y cm (05)

Then the sides of the poster are $(x+12)$cm and $(y+16)$cm

Area of the poster $A=(x+12)(y+16)$. It is given that $xy = 972$ (05)

$$\therefore A(x)=(x+12)\left(\frac{972}{x}+16\right) \quad (05)$$

$$A'(x)=(x+12)\left(\frac{-972}{x^2}\right)+\left(\frac{972}{x}+16\right) \quad (05)$$

$$=\frac{-12\times 972}{x^2}+16=0$$

$$=\frac{16\left(x^2-27^2\right)}{x^2}=0$$

$$\Rightarrow x=\pm 27 \quad (05)$$

As x has to be positive we take $x = 27$

Now $A'(x)=\dfrac{16(x-27)(x+27)}{x^2}$

$\Rightarrow$ $A'(x) < 0$ for $0 < x < 27$ (05)
$A'(x) > 0$ for $x > 27$

$\therefore$ $A(x)$ has a local minimum at $x = 27$ (05)

$\therefore$ Poster has the least area when $x = 27$, and the corresponding $y=\dfrac{972}{27}=36$

$\therefore$ Dimension of the poster are 39 cm x 57cm (05) [45]

(06) (a) $I=\displaystyle\int_{11}^{23}\frac{1}{(x+1)\sqrt{2x+3}}dx$

Let $t=\sqrt{2x+3}$ (05)

Then $t^2 = 2x + 3$

and $2tdt = 2dx$ Now $x+1=\dfrac{t^2-3}{2}+1=\dfrac{t^2-1}{2}$ (05)

$x = 11 \Rightarrow t = 5$ and $x = 23 \Rightarrow t = 7$ (05)

$$I=\int_5^7\frac{2dt}{t^2-1}=\int_5^7\frac{2dt}{(t-1)(t+1)} \quad (05)$$

(05) $=2\displaystyle\int_5^7\left[\frac{\frac{1}{2}}{t-1}+\frac{-\frac{1}{2}}{t+1}\right]dt=\int_5^7\frac{1}{t-1}dt-\int_5^7\frac{1}{t+1}dt$

$=\left[\ln(t-1)\right]_5^7-\left[\ln(t+1)\right]_5^7$ (05)

$=\ln\frac{6}{4}+\ln\frac{8}{6}$

$=\ln\frac{9}{8}$ (05) [35]

(b) Let us find one antiderivative of $e^{3x}\cos 4x$

$J=\int e^{3x}\cos 4x\,dx=\int e^{3x}\frac{d}{dx}\left(\frac{\sin 4x}{4}\right)dx$ (05)

$=e^{3x}\frac{\sin 4x}{4}-\int\frac{\sin 4x}{4}e^{3x}\cdot 3dx$ (05)

$=e^{3x}\frac{\sin 4x}{4}-\frac{3}{4}\int e^{3x}\frac{d}{dx}\left(\frac{-\cos 4x}{4}\right)dx$ (05)

$=e^{3x}\frac{\sin 4x}{4}+\frac{3}{16}\left[e^{3x}\cos 4x-\int\cos 4x\,e^{3x}\cdot 3dx\right]$ (05)

$=e^{3x}\frac{\sin 4x}{4}+\frac{3}{16}e^{3x}\cos 4x-\frac{6}{19}\int e^{3x}\cos 4x\,dx$

$16J = 4e^{3x}\sin 4x + 3\cos 4x - 9J$ (05)

$\Rightarrow J=\frac{e^{3x}}{25}\left[4\sin 4x+3\cos 4x\right]$ (05)

There fore $\frac{e^{3x}}{25}\left[4\sin 4x+3\cos 4x\right]$ is an antiderivative of $e^{3x}\cos 4x$

$\therefore \int e^{3x}\cos 4x\,dx=\frac{e^{3x}\left[4\sin 4x+3\cos 4x\right]}{25}+C$ (05)

Where C is an arbitrary constant. [35]

(c) $\int\sin^4 2x\,dx=\int\left[\frac{1}{2}(1-\cos 4x)\right]^2dx$ (05)

$=\frac{1}{4}\int(1-2\cos 4x+\cos^2 4x)dx$

$=\frac{1}{4}\int\left(1-2\cos 4x+\frac{1}{2}(1+\cos 8x)\right)dx$ (05)

$$\frac{1}{8}\int(3-4\cos 4x+\cos 8x)dx$$

$$=\frac{3x}{8}-\frac{\sin 2x}{4\times 8}+\frac{\sin 8x}{8\times 8}+C$$

(05) (05) (05)

$$=\frac{3x}{8}-\frac{\sin 2x}{8}+\frac{1}{64}\sin 8x+C$$

(05) 30

Where C is in arbitrary constant.

(07) Let us first note that u and v cannot be vertical.

Let m be the gradient of u and v. Then the (05) equations of u and v can be written as

y = m (x - 5) and y = m(x + 5) respectively

(05) (05)

$$\left.\begin{aligned} y-mx&=-5m\\ 3y+4x&=25\end{aligned}\right\}\Rightarrow x=\frac{25+15m}{4+3m}\ and\ y=\frac{5m}{4+3m}$$

Which gives $P=\left(\frac{25+15m}{4+3m},\frac{5m}{4+3m}\right)$ (05) (05)

Similarly

$$\left.\begin{aligned} y-mx&=5m\\ 3y+4x&=25\end{aligned}\right\}\Rightarrow x=\frac{25-15m}{4+3m}\ and\ y=\frac{45m}{4+3m}$$

Which gives $Q=\left(\frac{25-15m}{4+3m},\frac{45m}{4+3m}\right)$ (05) (05)

$$PQ^2=\left(\frac{25+15m}{4+3m}-\frac{25-15m}{4+3m}\right)^2+\left(\frac{5m}{4+3m}-\frac{45m}{4+3m}\right)^2$$

$$=\left(\frac{50m}{4+3m}\right)^2=5^2\ (05)$$

$$\frac{10m}{4+3m}=\pm 1\Rightarrow m=\frac{4}{7}\ (05)\ or\ \frac{-4}{13}\ (05)$$

Therefore m has two possible values and hence there exists two possible pairs of (u,v)

50

The equations of one pair are $y=\frac{4}{7}(x-5)$ (05) and

$y=\frac{4}{7}(x+5)$ (05)

i.e. $7y-4x=-20$ and $7y-4x=20$

The equations of the second pair are $y=-\frac{4}{13}(x-5)$ (05) and

$y=-\frac{4}{13}(x+5)$ (05)

i.e $13y+4x=20$ and $13y+4x=-20$ 20

The equation of one diagonal is y = 0 (05)

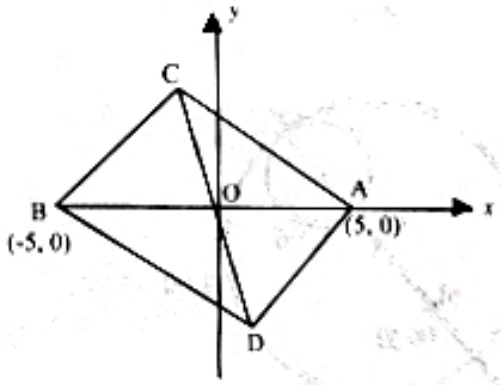
The centre of the parallelogram is (0, 0)

Let the equation of the other diagonal be

$7y-4x+20+\lambda(13y+4x-20)=0$ (05)

Since the diagonal passes through the origin We have $\lambda=-1$

$\therefore$ the equation of the second diagonal is $3y+4x=0$ (05)



15

$$\left.\begin{aligned} 7y-4x&=20\\ 3y+4x&=0\end{aligned}\right\}\Rightarrow y=2\ and\ x=\frac{-3}{2}$$

$$\therefore C=\left(\frac{-3}{2},2\right)\ (05)$$

The area of ABCD $2\times ABC\Delta=2\times\frac{1}{2}\times 2\times 10$ (05)

$= 20 square\ units$ (05) 15

(08) $S_1 \equiv x^2+y^2-2=0$

$S_2 \equiv x^2+y^2+3x+3y-8=0$

$S_1=0$ center $c_1=(0,0)$

Radius $r_1 = \sqrt{0+0+2} = \sqrt{2}$ (05)

$S_2 = 0$; Centre $C_2 = \left[\frac{-3}{2}, \frac{-3}{2}\right]$ (05)

Radius $r_2 = \sqrt{\frac{9}{4}+\frac{4}{4}} = \frac{5\sqrt{2}}{2}$ (05)

$\therefore C_1C_2 = d = \sqrt{\frac{9}{4}+\frac{9}{4}} = \frac{3\sqrt{2}}{2}$ (05) and $r_2 - r_1 = \frac{5\sqrt{2}}{2} - \sqrt{2}$

$= \frac{3\sqrt{2}}{2}$ (05)

Since $r_2 - r_1 = d$, $S_1 = 0$ and $S_2 = 0$ touch internally. (05) [30]

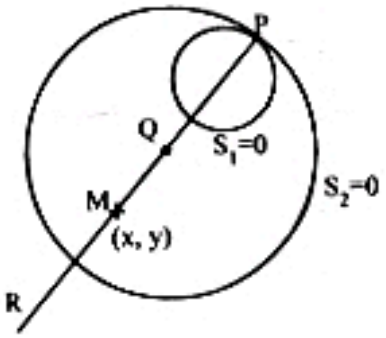
Let the point contact be $(x_0\ y_0)$. Then $(x_0\ y_0)$ lies on both $S_1 = 0$ and $S_2 = 0$

$\therefore\ 3x_0 + 3y_0 - 6 = 0$ or $x_0 + y_0 = 2$ (05)

Also it lies on the line which passes through C_1 and C_2

$y = x \Rightarrow y_0 = x_0$ (05)

$\therefore\ x_0 = y_0 = 1$ and $P = (1, 1)$ (05) [15]



Let the mid point of Q and R be $M = (x, y)$ Parametric form of straight line passing throng h(1,1) is $x = 1 + t\sin\theta$ and $y = 1 + t\cos\theta$

Let t_Q and t_R be the values of t corresponding to Q and respectively (05)

When Q is on $S_1 = 0$,

$2 = x^2 + y^2 - (1 + t\sin\theta)^2 + (1 + t\cos\theta)^2 = 2 + 2t(\cos\theta + \sin\theta) + t^2$ (05)

$\Rightarrow\ t[t + 2(\cos\theta + \sin\theta)] = 0$

Thus $t_Q = -2(\cos\theta + \sin\theta)$ as $t_Q \neq 0$ (05)

When R is on $s_2 = 0$,

$8 = x^2 + y^2 + 3x + 3y = 2 + 2t(\cos\theta + \sin\theta) + t^2 + 6 + 3t(\cos\theta + \sin\theta)$ (05)

$\Rightarrow t[t + 2(\cos\theta + \sin\theta)] = 0$

Thus $t_R = -5(\cos\theta + \sin\theta)$ as $t_R \neq 0$ (05)

$\therefore\ t_M = \frac{t_Q + t_R}{2} = \frac{-7}{2}(\cos\theta + \sin\theta)$ (05) (05)

Then $X = 1 + t_M\sin\theta = 1 - \frac{7}{2}(\cos\theta + \sin\theta)\sin\theta$ and

$Y = 1 + t_M\cos\theta = 1 - \frac{7}{2}(\cos\theta + \sin\theta)\cos\theta$ (05)

$\therefore X + Y + \frac{3}{2} = -7\sin\theta\cos\theta$ (05)

Now

$(x-1)^2 + (y-1)^2 = t_M^2 = \frac{49}{4}(1 + 2\sin\theta\cos\theta)$

$= \frac{49}{4} + \frac{49}{2}\left(-\frac{1}{7}\right)\left(X + Y + \frac{3}{2}\right)$ (05)

By simplifying we get $x^2 + y^2 + \frac{3}{2}x + \frac{3}{2}y - 5 = 0$ (05)

Therefore M lies on the circle

$x^2 + y^2 + \frac{3}{2}x + \frac{3}{2}y - 5 = 0$ [55]

(09) (a) $\sin\theta\tan\theta - 2(1 - \cos\theta)$

$= \frac{\sin^2\theta - 2\cos\theta(1 - \cos\theta)}{\cos\theta}$ (05)

$= \frac{\cos^2\theta - 2\cos\theta + 1}{\cos\theta}$ (05)

$= \frac{(\cos\theta - 1)^2}{\cos\theta} > 0$ (05) $\therefore\ \therefore 0 < \theta < \frac{\pi}{2}$ [15]

(b) $\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$ (05)

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$ (05) [10]

$$\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$

$$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6} \quad (05)$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{4} \quad (05)$$

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$$\frac{1-\cos 2x}{\sin 2x} = \frac{2\sin^2 x}{2\sin x \cos x} \quad (05)$$

$$= \tan x \quad (05)$$

10

$$\tan\frac{\pi}{24} = \frac{1-\cos\frac{\pi}{12}}{\sin\frac{\pi}{12}} = \frac{1-\frac{\sqrt{6}+\sqrt{2}}{4}}{\frac{\sqrt{6}-\sqrt{2}}{4}} \quad (05)$$

(05)

$$= \frac{\left(4-\sqrt{6}-\sqrt{2}\right)\left(\sqrt{6}+\sqrt{2}\right)}{4} \quad (05)$$

$$= \sqrt{6}-\sqrt{3}+\sqrt{2}-2 \quad (05)$$

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(c) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ (05)

05

Let $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{\lambda}$ (05)

$a = \lambda \sin A, \quad b = \lambda \sin B, \quad c = \lambda \sin C$

$$\frac{a^2-b^2}{c} = \frac{\lambda^2\sin^2 A - \lambda^2 \sin^2 B}{\lambda^2 \sin^2 C} \quad (05)$$

$$= \frac{\sin^2 A - \sin^2 B}{\sin^2 C}$$

$$= \frac{(\sin A - \sin B)(\sin A + \sin B)}{\sin^2(A+B)} \quad (05)$$

$$= \frac{2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \cdot 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{\sin^2(A+B)} \quad (05)$$

$$= \frac{2\sin\frac{A+B}{2}\cos\frac{A+B}{2} \cdot 2\sin\frac{A-B}{2}\cos\frac{A-B}{2}}{\sin^2(A+B)}$$

$$= \frac{\sin(A+B)\sin(A-B)}{\sin^2(A+B)} \quad (05)$$

$$= \frac{\sin(A-B)}{\sin(A+B)} \quad (05)$$

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