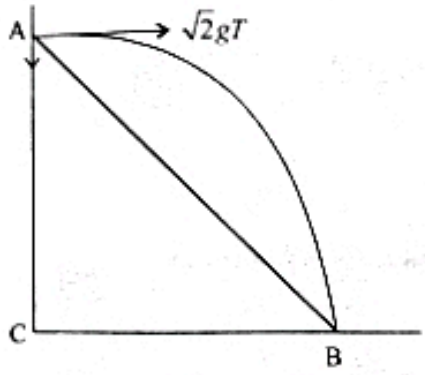
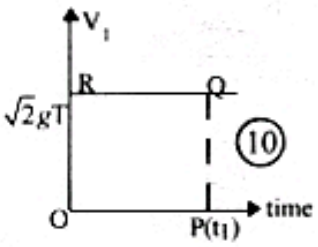
G.C.E. (Advanced Level) Examination - April 2004
Combined Mathematics - II
Provisional Scheme of Marking

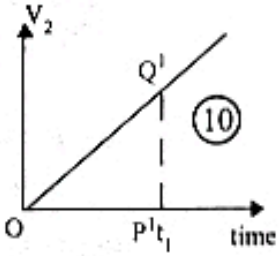
(01) (a)



Let V_1 be the horizontal component of velocity and let V_2 be the vertical component of velocity.



(10)



(10)

20

Let t_1 be the time taken to reach the ground.

BC = Area of the rectangle OPQR $= \sqrt{2}gT \cdot t_1 \rightarrow (01)$ (05)

AC = Area of the triangle $OP'Q' = \frac{1}{2}t_1 \cdot gt_1 = \frac{1}{2}gt_1^2 \rightarrow (02)$ (05)

It is given that $AB = \frac{3}{2}gT^2$

Since $AB^2 = AC2 + BC^2$, from (01) and (02) we have

$\frac{9}{4}g^2T^4 = \frac{1}{4}g^2t_1^4 + 2g^2T^2t_1^2 \Rightarrow t_1^2 + 8T^2t_1^2 - 9T^4 = 0$ (05)

$(t_1^2 + 9T^2)(t_1^2 - T^2) - 0 \Rightarrow t_1 = T(\because t_1 > 0)$ (05) (05)

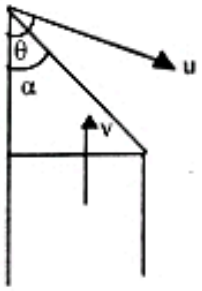
25

$h = AC = \frac{1}{2}gt_1^2$ (By (02))

$= \frac{1}{2}gT^2 (\because t_1 = T)$ (05)

05

(b)



Vel (M,E) = ↑V (05)

Vel (P,E) = ↘u (05)

Vel (P,M) = Vel(P,E)+ Vel (EM)

$= \overrightarrow{AB} + \overrightarrow{BC}$ (05)

$= \overrightarrow{AC}$

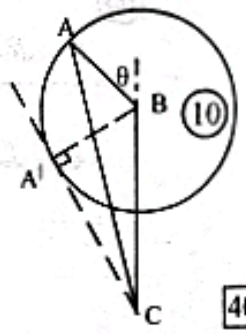
Relative to the car the pedestrian will cross the road furtherest from the car, when the derection of Vel (P,M) is along the tangent AC. (05)

i.e. When $\beta = \sin^{-1}\left(\frac{u}{v}\right)$

The pedestian will cross the road safely if $\beta > \alpha$

i.e. if $\sin\beta > \sin\alpha$ (05)

i.e. if $u > v\sin\alpha$ (05)



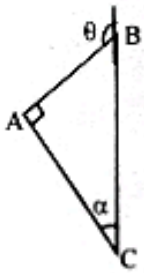
(10)

40

If $u = \sin\alpha$, then $\beta = \alpha$

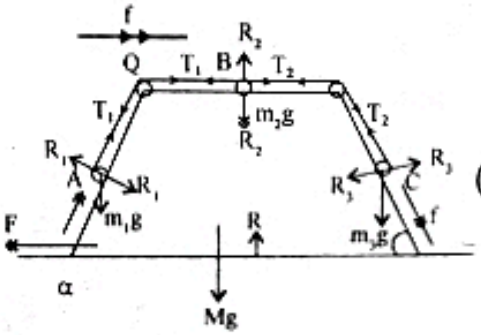
The required angle $= \pi - \theta$ (05)

$= \pi - \left(\frac{\pi}{2} + \alpha\right) = \frac{\pi}{2} - \alpha$ (05)



10

(02)



(10)

Let f ← be the acceleration of the Block relative to the plane and f (α) be the acceleration of A relative to the Block.

Then the acceleration of B relative to the Block = f →

and the acceleration of C relative to the Block = f (α) ↘

Let T_1 and T_2 be the rension in the portion AB and BC respectively.

Apply P = mf

→ for the system:

$m_1(f\cos\alpha - F) + m_2(f - F) + m_3(f\cos\alpha - F) + MF = 0 \rightarrow (01)$ (15)

↗ for A:

$T_1 - m_1 g\sin\alpha = m_1(f - F\cos\alpha) \rightarrow (02)$ (10)

→ for B:

$T_1 - T_2 = m_2(f - F) \rightarrow (03)$

(05) (05)

↘ for C: (05) (05)

$m_3 g\sin\alpha - T_2 = m_3(f - F\cos\alpha) \rightarrow (04)$ 55

$m_2 = 0$ In this case (03) gives us

$T_1 - T_2 = 0$

i.e. $T_1 = T_2$ (05) 05

$m_2 = 0$ and $M = 0$ In this case (01) implies that

$m_1(f\cos\alpha - F) + m_3(f\cos\alpha - F) = 0$ (05)

$\Rightarrow F = f\cos\alpha$ (05)

Since $T_1 = T_2$, (02) + (04) gives us

$(m_3 - m_1)g\sin\alpha = (m_1 + m_3)(f - F\cos\alpha)$

$= (m_1 + m_3)(1 - \cos^2\alpha)\dfrac{F}{\cos\alpha}$

$= (m_1 + m_3)\dfrac{\sin^2\alpha}{\cos\alpha}F$

$\therefore F = \dfrac{(m_3 - m_1)g\cos\alpha}{(m_1 + m_3)\sin\beta}$ (10)

Apply P = mf

→ for the Block: $R_1\sin\alpha - R_3\sin\alpha = 0$ (05)

$\Rightarrow R_1 = R_3$ (05)

↖ for A:

$R_1 - m_1 g\cos\alpha = m_1 F\sin\alpha$ (05)

$\Rightarrow R_1 = m_1 g\cos\alpha + \dfrac{m_1(m_3 - m_1)g\cos\alpha}{(m_1 + m_3)}$

$= \dfrac{m_1 g\cos\alpha}{(m_1 + m_3)}[m_1 + m_3 + m_3 - m_1]$ (05)

$= \dfrac{2m_1 m_3}{m_1 + m_3}g\cos\alpha$ 40

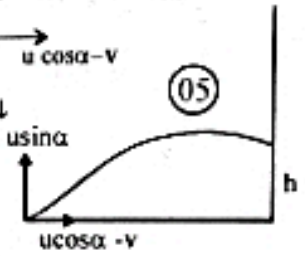
(03) (a) Consider the motion of the particle relative to the vertical screen.

Let t be the time taken to hit the screen.

Vel (Particle, Target Plane) = u sinα ↑, u cosα − v →

Acc (Particle, Target Plane) = g↓

Apply $S = ut + \frac{1}{2}gt^2$



→ $d = (u\cos\alpha - v)t \rightarrow (01)$ (05)

↑ $h = (u\sin\alpha)t - \frac{1}{2}gt^2 \rightarrow (02)$ (05)

Since d > o, (01) implies that ucosα > v (05) 20

$(01) \Rightarrow t = \dfrac{d}{u\cos\alpha - v}$ (05)

$Now(02) \Rightarrow h = u\sin\alpha\cdot\dfrac{d}{(u\cos\alpha - v)} - \dfrac{1}{2}g\dfrac{d^2}{(u\cos\alpha - v)^2}$

$\Rightarrow gd^2 - 2u\sin\alpha(u\cos\alpha - v)d + 2h(u\cos\alpha - v)^2 = 0$ (05)

10

If $d > \dfrac{2u}{g}\sin\alpha(u\cos\alpha - v)$, then

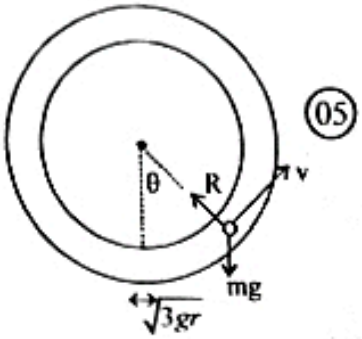
$h = \dfrac{d[gd - 2u\sin\alpha(u\cos\alpha - v)]}{(u\cos\alpha - v)^2} < 0$ (05)

This is impossible. (05) 10

The forces acting on the particle are R↖ and mg↓ R↖ is perpendicular to the direction of motion. (05)

Thus there is no work done by R↖ and the force mg↓ is a conservative force. (05)

∴ The law of conservation of energy can be applied for the motion of the particle.



(05) 15

Conservation of energy :

$$\frac{1}{2}mv^2 - mgr\cos\theta = \frac{1}{2}m(3gr) - mgr \quad (10)$$

$$\Rightarrow v^2 = gr(1+2\cos\theta)$$

↖ $\underline{P} = m\underline{f}$ $\quad R - mg\cos\theta = m\frac{v^2}{r}$

$$= mg(1+2\cos\theta)$$

$$\therefore R = mg(1+3\cos\theta) \quad (05)$$

The reaction of the tube changes when R = 0

i.e. When $1 + 3\cos\theta = 0$

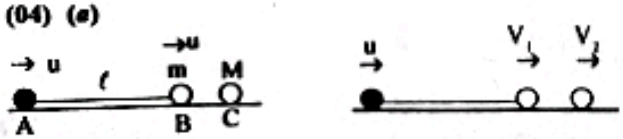
$$\theta = \cos^{-1}\left(\frac{1}{3}\right) \quad (05) \qquad 20$$

When $\theta = \cos^{-1}\left(-\frac{1}{3}\right)$, $V^2 = gr\left[1+2\left(\frac{-1}{3}\right)\right]$ (05)

$$= \frac{gr}{3}$$

The required speed is $\sqrt{\frac{gr}{3}}$ (05) 10

(04) (a)



Conservation of momentum for m and M

$$\rightarrow mv_1 + Mv_2 = mu \rightarrow (01) \quad (10)$$

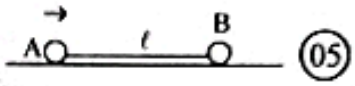
Newton's law of restitution.

$$v_2 - v_1 = eu \rightarrow (02) \quad (10)$$

$$(01) - (02)\times M : (m+M)v_1 (m - eM)u$$

$$\therefore v_1 = \frac{(m-eM)}{(m+M)} \quad (05) \qquad 25$$

Consider the motion of A relative to B ofter the impulse:



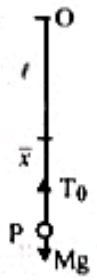
(05)

Vel (A, B) = $u - v_1$

The time required = $\frac{l}{(u-v_1)}$ (05) $= \frac{(m+M)\ell}{[(m+M)-(m-eM)]u}$

$$= \frac{(m+M)\ell}{M(1+e)u} \quad (05) \qquad 15$$

(b)



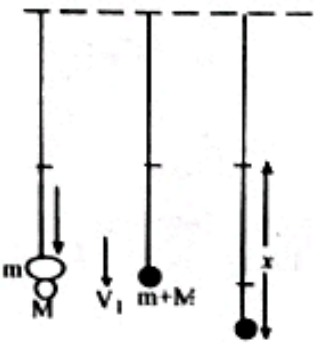
Tention of the string.

$$T_0 = \frac{2Mg\bar{x}}{\ell} \quad (05)$$

↑ $T_0 = Mg$

$$\frac{2Mg\bar{x}}{\ell} = Mg$$

$$\therefore \bar{x} \quad (05) \qquad 10$$



Apply $V^2 = u^2 + 2fs$

↓for the ring

$$v^2 = 2g\cdot\frac{3\ell}{2}$$

$$v = \sqrt{3g\ell} \quad (05)$$

Apply $\underline{I} = \Delta(m\underline{v})$ for the ring at P :

$$\downarrow mv = (m+M)v_1 \quad (10)$$

$$\therefore v_1 = \frac{m}{(M+m)}\sqrt{3g\ell} \quad (05)$$

20

Apply $\underline{P} = m\underline{f}$ for the composite body.

$$(m+M)g - \frac{2Mg}{\ell}x = (m+M)\ddot{x} \rightarrow (01)$$ (10)

Let $y = x - \bar{y}$, where $\bar{y}$ is given by $2Mg \cdot \frac{\bar{y}}{\ell} = (M+m)g$

$$i.e. \quad y = x - \left(\frac{M+m}{2M}\right)\ell$$ (05)

$$Then \quad \ddot{y} = \ddot{x}$$ (05)

$(01) \Rightarrow$

$$\frac{2Mg}{\ell}\left\{\frac{(M+m)}{2M}\ell - x\right\} = (M+m)\ddot{x}$$

$$\Rightarrow \frac{-2Mg}{\ell}y = (M+m)\ddot{y}$$ (05)

$$\Rightarrow \ddot{y} + \frac{2Mg}{(M+m)\ell}y = 0$$

∴ The composite body performs single harmonic motion with frequency.

$$\sqrt{\frac{2Mg}{(M+m)\ell}}$$ (05) [30]

(05) (a)

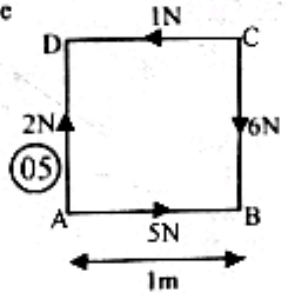
Resolving vertically ↑ $y = (2 - 6)N = -4$ N (05)

Resolving horizontally → $x = (5 - 1)N = 4N$ (05)

∴ Magnitude of the resultant force

$$R = \sqrt{4^2 + 4^2}\,N$$

$$= 4\sqrt{2}\ N$$ (05)



Suppose the resultant in passing through E such that AE = x and making an angle α with the downward vertical.

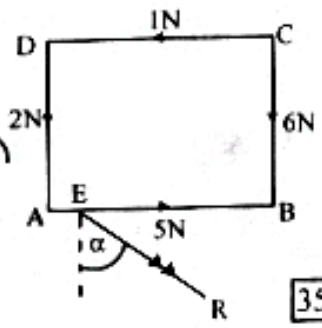
$\tan\alpha = 1$

$\Rightarrow \alpha = \frac{\pi}{4}$ (05)

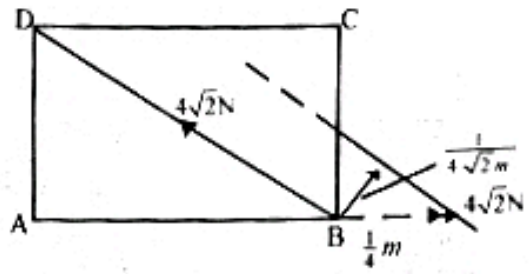
Taking the moment about E

$6(1 - x) + 2x - 1 = 0$ (05)

$x = \frac{5}{4}$ (05)



[35]

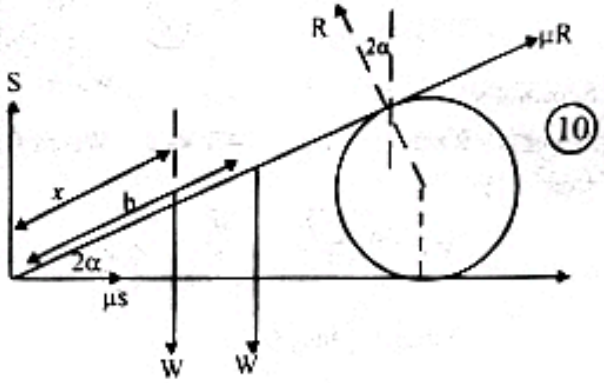


Forces are equal and opposite

∴ System reduces to a couple of magnitude. (05)

$= 4\sqrt{2} \cdot \frac{1}{4\sqrt{2}}\,Nm = 1Nm$ (05) [10]

(b)



(10)

Given that $\mu = \tan\lambda$ Resolving vertically ↑

$S + R\cos 2\alpha + \mu R \sin 2\alpha = 2W$ (10)

or $S + R(\cos 2\alpha + \tan\lambda \sin 2\alpha) = 2W \rightarrow (01)$

Resolving horizontally

$R \sin 2\alpha = \mu S + \mu R \cos 2\alpha$ (10)

or $\tan\lambda S + R(\tan\lambda \cos 2\alpha - \sin 2\alpha) = 0 \rightarrow (02)$

Substituting s from (02) in (01)

$R(\sin 2\alpha - \tan\lambda \cos 2\alpha) + R(\cos 2\alpha + \tan\lambda \sin 2\alpha)$

$\tan\lambda = 2W\tan\lambda$

Simplifying

$$R = \frac{2W\tan\lambda}{\sin 2\alpha \sec^2\lambda} \rightarrow (03)$$ (05)

Taking the moment about the foot,

$R.r\cot\alpha = W\,b\cos 2\alpha + Wx\cos 2\alpha$ (10)

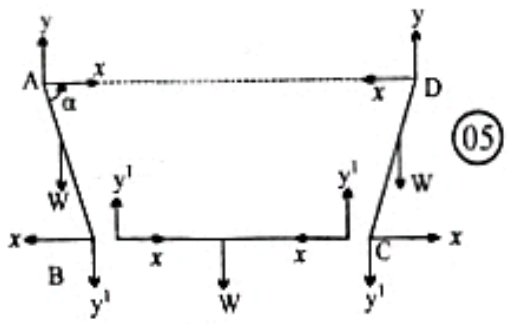
Substituting R from (03).

$$\frac{2W\tan\lambda}{\sin 2\alpha \sec^2\lambda} = \frac{W\cos 2\alpha(b+x)}{r\cot\alpha}$$ (05)

Simplifying this we get

$(b + x)\sin^2\alpha \cos 2\alpha = r \sin\lambda \cos\lambda$ (05) [55]

06) (a)



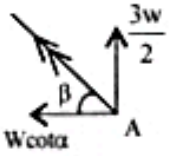
05

Resolving (AB) vertically ↑ $y = y^1 + W \rightarrow (01)$ (05)

For (AB) taking the moment about A)

$W.a\cos\alpha + x.2a\sin\alpha + y.2a\cos\alpha = 0 \rightarrow (02)$ (10)

Where 2a is the length of the rod.



Resolving (BC) vertically $Y^1 = \frac{W}{2}$ (05)

Substituting in (02)

$W\cos\alpha + 2x\sin\alpha + W\cos\alpha = 0 \Rightarrow x = -W\cot\alpha$ (05)

from (01)

$$Y = \frac{3w}{2}\tan\beta = \frac{3W/2}{W\cot\alpha}$$ (05) (05)

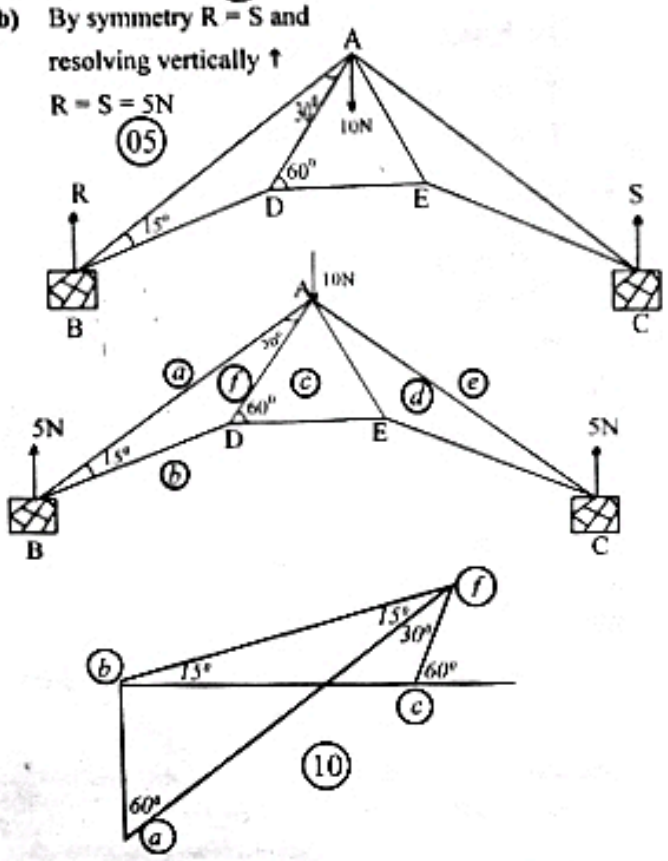
$$\tan\alpha = \frac{2}{3}\tan\beta$$

40

(b) By symmetry R = S and resolving vertically ↑

R = S = 5N

05



10

Concidering (a) (b) (f) $\Delta, \frac{5}{\sin 15^0} = \frac{(b)(f)}{\sin 60^0}$

$$(b)(f) = \frac{5\sqrt{3}}{2}\operatorname{cosec}15^0$$

$$= \frac{5}{\sin 15^0} = \frac{(a)(f)}{\sin 105^0} = \frac{(a)(f)}{\cos 15^0}$$

$$\Rightarrow (a)(f) = 5\cot 15^0$$

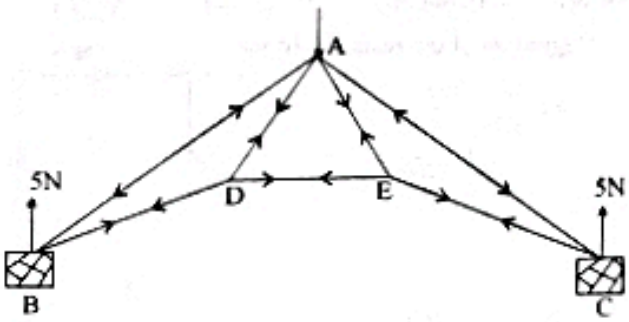
Concidering (a) (b) (f) $\Delta, \frac{(c)(f)}{\sin 15^0} = \frac{(b)(f)}{\sin 120^0}$

$$= \frac{\frac{5\sqrt{3}}{2}\operatorname{cosec}15^0}{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow (c)(f) = 5$$

$$\frac{(b)(c)}{\sin 45^0} = \frac{5}{\sin 15^0} \Rightarrow (b)(c) = \frac{5}{\sqrt{2}}\operatorname{cosec}15^0$$

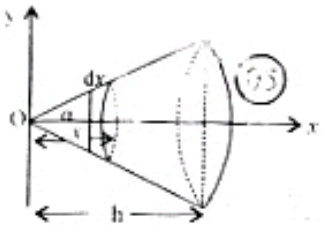
∴ The rod CE has a tension of magnitude $\frac{5\sqrt{3}}{2}$ $\operatorname{cosec}15^0$N (05)



Rod	Magnitude	Tension/Thrust
AB, AC	5 cot 15° N	Thrust (10)
BD, CE	$\frac{5\sqrt{3}}{2}\operatorname{cosec}15^0 N$	Tension (05)
AD, AE	5N	Tension (10)
DE	$\frac{5}{\sqrt{2}}\operatorname{cosec}15N$	Tension (10)

60

(07)



(15)

By symmetry centre of gravity is on the X - axis. (05)

Let $\bar{x}$ be the centre of gravity of the cone from O.

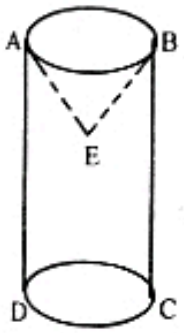
(05)

$\frac{1}{3}\pi h^3 \tan^2\alpha\ \rho\bar{x} = h\int_0^h \pi x^2 \tan^2\alpha\ \rho x dx$, Where ρ is the density of the cone. (05)

Now $\frac{1}{3}h^3\bar{x} = \left[\frac{x^4}{4}\right]_0^h = \frac{h^4}{4}$ and hence $\bar{x} = \frac{3}{4}h$ (10)

$\therefore$ The centre of gravity of the cone from the base is $\frac{1}{4}h$

30



By symmetry centre of gravity of S is on the axis. (05)

Let the centre of gravity of S be $\bar{x}$ from AB.

Then,

$$\pi R^2\left(H-\frac{1}{3}h\right)\bar{x} = \pi R^2 H\cdot\frac{H}{2} - \frac{1}{3}\pi R^2 h\cdot\frac{1}{4}h \quad (20)$$

$$\left(H-\frac{1}{3}h\right)\bar{x} = \frac{H^2}{2} - \frac{h^2}{12} = \frac{1}{12}\left(6H^2 - h^2\right)$$

$$\bar{x} = \frac{6H^2 - h^2}{4(3H-h)} \quad (10)$$

If the centre of gravity of S at E,

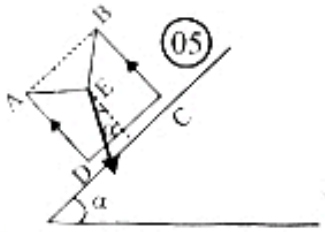
$$\frac{6H^2 - h^2}{4(3H-h)} = h \quad (05)$$

$\Rightarrow$ $3h^2 - 12Hh + 6H^2 = 0$ or $h^2 - 4Hh + 2H^2 = 0$ (05)

$$\therefore h = \frac{4 \pm \sqrt{16H^2 - 8H^2}}{2} = \left(2 \pm \sqrt{2}\right)H \quad (05)$$

Since $h < H$, $h = \left(2-\sqrt{2}\right)H$ (05)

55



(05)

The resulting body will not topple if OD > OF

i.e. $R > (H - h)\tan\alpha$ (10)

or $R > \left(\sqrt{2}-1\right)H\tan\alpha$

15

(08) (a)

P[item selected at random is faulty] = P_1

P[defect is detected / item is faulty] = P_2

P[defect is detected in an item selected at random]

=P[item selected is faulty] P[defect is detected / item is faulty]

+ P [item selected is not faulty] P[defect is detected / item is not faulty]

$= P_1P_2 + (1-P_1) = P_1P_2$

(05) (05) (05)

15

$\therefore$ P[defect is not detected in an item selected at random]

$= 1 - P_1P_2$ (05)

P[no defects are detected among the three items]

$= (1 - P_1P_2)^3$ (10)

15

(05)

$\therefore$ P[the defect are detected in two items]

= 3P[no defects are detected in one item and defects are defected in two items] (05)

$= 3(1 - P_1P_2)(P_1P_2)^2$ (05)

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(i) P[the defect are detected at least in two items]

= P[the defects are detected in two items out of the three items] + P[the defects are detected in all three items] (10)

$= 3(1 - P_1P_2)(P_1P_2)^2 + (P_1P_2)^3$ (05)

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(b) P[x Predicts weather correctly] = α

P[y Predicts weather correctly] = β

P[x Predicts fair weather and y predicts bad weather] (10)

= P[x Predicts weather correctly and y predicts weather incorrectly] + P[x Predicts weather incorrectly and y Predicts weather correctly] (10)

= P[x Predicts weather correctly] P[y predicts weather incorrectly] + P[x Predicts weather incorrectly] P[y Predicts weather correctly]

= $\alpha(1-\beta)+(1-\alpha)\beta = \alpha+\beta-2\alpha\beta$ (05)

= P[x Predicts weather correctly / x Predicts fair weather and Predicts bad weather] (05)

$$= \frac{\text{P[}x\text{ Predicts weather correctly]}}{\text{P [}x\text{ Predicts fair weather and }y\text{ Predicts bad weather]}} \quad (05)$$

$$= \frac{\alpha(1-\beta)}{\alpha+\beta-2\alpha\beta} \quad (05) \qquad \boxed{40}$$

(09) (a)

$$\text{Mean} = \frac{35\times6000+30\times10000+25\times15000+10\times20000}{35+30+25+10} \quad (05)$$

$$= \frac{210000+300000+375000+200000}{100}$$

$$= \frac{1085000}{100} = 10850 \text{ Rupees} \quad (05) \qquad \boxed{10}$$

Median

Since the number of items is even the median is the arithmetic mean of the two middle items when arranged in ascending order. (05)

$$\therefore \text{Median} = \frac{10000+10000}{2} = 10000 \text{ Rupees} \quad (05) \qquad \boxed{10}$$

Mode

Mode is the most frequently occurring item. (05)

$\therefore$ mode = 6000 Repees (05) $\boxed{10}$

Only the mean will be altered (05) $\boxed{05}$

Mean will be altered because the total of the wages will be increased due to the increase of wages of the four workers. (05) $\boxed{05}$

Median occurs in the second category of wages.

For median to be altered it should be shifted to the third category of wages, for this to happened we need the increase of wages of at least 15 workers in the second category of wages. (05) $\boxed{05}$

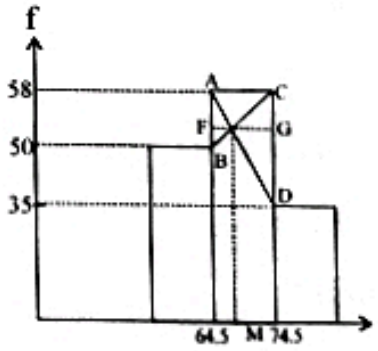
Mode occurs in the first category of wages. The minimum difference between the frequency of the modal category and the frequency of any other category is 5. Therefore, the mode cannot be altered by increasing the wages of only 4 workers. (05) $\boxed{05}$

(b) Modal class is 65 - 74 (64.5 - 74.5)

$$\frac{M-64.5}{74.5-M} = \frac{58-50}{58-35} = \frac{8}{23} \quad (05)$$

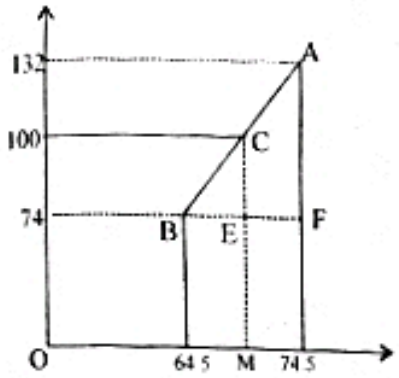
$$\Rightarrow 31M = 23\times64.5+8\times74.5$$

$$M = \frac{1483.5+596}{31} = 67.08 \quad (05)$$



$\boxed{10}$

Median class is 65 - 74 (64.5 - 74.5) (05) $\boxed{05}$



$$\frac{M-64.5}{74.5-64.5} = \frac{100-74}{132-74} = \frac{13}{29} \quad (05)$$

$$\Rightarrow M = 64.5 + \frac{130}{29} = 64.5 + 4.48$$

$$= 68.98 \quad (05)$$

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Mid-value of the class interval x	Frequency f	$d=\frac{x-69.5}{10}$	fd	fd^2
49.5	24	-2	-48	96
59.5	50	-1	-50	50
69.5	58	0	0	0
79.5	35	1	35	35
89.5	21	2	42	84
99.5	12	3	36	108
Total	200		15	373

(10)

Mean $= 69.5 + 10\bar{d} = 69.5 + .75 = 70.25$ (05)

$$\text{Variance} = \frac{1}{200}\sum_{i=1}^{6} f_i\left(x_i-\bar{x}\right)^2$$

$$= \frac{10^2}{200}\sum_{i=1}^{6} f_i\left(d_i-\bar{d}\right)^2$$

$$= \frac{1}{2}\left\{\sum_{i=1}^{6} f_i d_i^2 - 200\bar{d}^2\right\}$$

$$= \frac{1}{2}(373-1.125)$$

$$= \frac{260.5}{2} = 185.93$$

Therefore the standard deviation $= \sqrt{185.93}$ (05)

$= 13.64$

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