G.C.E. (Advanced Level) Examination - April 2003
10 - Combined Mathematics - I
Provisional Scheme of Marking

(01) (*a*) $P(x) = (\lambda - 2)x^2 - 3(\lambda + 2)x + 6\lambda$

(i) If $\lambda = 2$, then $P(x) = -12x + 12$ and we get

$P(2) = -12 < 0$, There fore $P(x)$ is not positive (05)

for all real values of x

If $\lambda \neq 2$, then $P(x)$ is quadratio and the conditions for $p(x)$ to be positive for all real values of x are

(05) $(\lambda - 2) < 0$ and the discriminant

$\Delta = 9(\lambda + 2)^2 - 4(\lambda - 2)6\lambda < 0$ (05) (05)

i.e. $\lambda > 2$ and $\Delta = -15\lambda^2 + 84\lambda + 36$ (05)

$= -3(5\lambda + 2)(\lambda - 6) < 0$,

or $\lambda > 2$ and $(5\lambda + 2)(\lambda - 6) > 0$ this implies (05)

$\lambda > 6$ is the required condition (05)

$\therefore$ least intergral value is $\lambda = 7$ (05) **40**

(ii) T_0 have two distinct real roots, it is required that $\lambda \neq 2$ (05)

(otherwise p(x) is linear and it has only one real root)

and $\Delta > 0$ (05)

i.e $\lambda \neq 2$ and $(5\lambda + 2)(\lambda - 6) < 0$ (05)

Therefore, the rquired set of values is

$$\left\{\lambda \in \mathbb{R} : -\frac{2}{5} < \lambda < 2\right\} \cup \{\lambda \in \mathbb{R} : 2 < \lambda < 6\}$$

or $\left(-\frac{2}{5}, 2\right) \cup (2,6)$ (10) **25**

(iii) From (ii), two real roots of $p(x) = 0$ are given by

$\frac{3(\lambda+2) \pm \sqrt{\Delta}}{2(\lambda-2)}$ *when* $\lambda \in \left(-\frac{5}{2}, 2\right) \cup (2,6)$ (10)

Then difference of the real roots of $p(x) = 0$, is

$= \pm \frac{2\sqrt{\Delta}}{2(\lambda-2)} = 3$ (05)

(05)

i.e $\Delta = 9(\lambda - 2)^2$ This implies

$-15\lambda^2 + 84\lambda + 36 = 9(\lambda^2 - 4\lambda + 4)$

$\Rightarrow 8\lambda^2 - 40\lambda = 0 \Leftrightarrow \lambda(\lambda - 5) = 0$ Therfore (05)

$\lambda = 0$ or $\lambda = 5$

(05) (05) **35**

(02) (*a*) There are 5 possinilities for the sizes of teams.

Sizes of teams	Number of ways to form the teams
5, 1, 1, 1	${}^8C_5 = 56$ (05)
4, 2, 1, 1	${}^8C_4 \times {}^4C_2 = 420$ (05)
3, 3, 1, 1	${}^8C_3 \times {}^5C_3 / 2! = 280$ (05)
3, 2, 2, 1	(10) ${}^8C_3 \times {}^5C_2 \times {}^3C_2 / 2! = 840$
2, 2, 2, 2	(10) ${}^8C_2 \times {}^6C_2 \times {}^4C_2 \times {}^2C_2 / 4! = 105$

(15)

Therefore total number of ways to form the required teams is

$= 56 + 420 + 280 + 840 + 105$

$= 1701$ (10) **60**

(b) ${}^nC_{r+1} + {}^nC_r = \frac{n!}{(r+1)!(n-r-1)!} + \frac{n!}{r!(n-r)!}$ (05)

$= \frac{n!(n-r)}{(r+1)!(n-r)!} + \frac{n!(r+1)}{(r+1)!(n-r)!}$ (05)

$= \frac{n!}{(r+1)!(n-r)!}(n-r+r+1)$ (05)

$$= \frac{(n+1)!}{(r+1)!(n-r)!} \quad (05)$$

$$= {}^{n+1}C_{r+1}$$

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From the above tesult we have

${}^{2003}C_r = {}^{2004}C_{r+1} - {}^{2003}C_{r+1}$ (05)

${}^{2004}C_r = {}^{2005}C_{r+1} - {}^{2004}C_{r+1}$ (05)

.....

.....

${}^{2013}C_r = {}^{2014}C_{r+1} - {}^{2013}C_{r+1}$ (05)

Now adding all of them to gether we get (05)

${}^{2003}C_r + {}^{2004}C_r + \ldots\ldots\ldots + {}^{2013}C_r = {}^{2014}C_{r+1} - {}^{2003}C_{r+1}$ 20

$$= \frac{1}{2} + \frac{1}{8}\sum_{k=1}^{n-1}(k+2) \quad (05)$$

$$= \frac{1}{2} + \frac{1}{8}\cdot\frac{(n-1)n}{2} + \frac{2(n-1)}{8}$$

$$= \frac{1}{16}\left(n^2+3n+4\right) \quad (05)$$

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The n^{th} partial sum of $\sum_{k=1}^{\infty}\frac{k!}{2^k}$ is $S_n = \sum_{k=1}^{n}\frac{k!}{2^k}$

$\therefore$ $S_n > \frac{1}{16}\left(n^2+3n+4\right)$ for each positive integer, n Since (05)

we have

$\lim_{n\to\infty}\frac{1}{16}\left(n^2+3n+4\right)=\infty$, $\lim_{n\to\infty} S_n = \infty$

$\therefore$ The series is not convergent. (05) 10

(03) *(a)*

Let p(n) : $8(n+1)! > 2^{n+1}(n+2)$

When n = 1, L.H.S. = 16 and R.H.S. = 12

$\therefore$ p(1) is true

Assume that p(k) is true i.e., (05)

$8(K+1)! > 2^{K+1}(K+2)$ (05)

Now $8(K+2)! = 8(k+1)!(k+2) > 2^{K+1}(K+2)^2$ (05)

$= 2^{K+1}(K^2+4K+4)$

$= 2^{K+1}[(2K+3)+(K+1)^2]$

$> 2^{K+1}[(2K+3)+3]$ as $(K+1)2 > 3$ for each positive integerk (05)

i.e. $8(K+2)! > 2^{K+2}(K+3)$ or

$8\left(\overline{K+1}+1\right)! > 2^{\overline{K+1}+1}\left(\overline{K+1}+2\right)$ (05)

$\therefore$ P(K + 1) is true.

Hence by the Principle of Mathematical Induction, P(n) is true for each positive integer, n. (05) 30

From (*a*) $\frac{(n+1)!}{2^{n+1}} > \frac{(n+2)}{8}$ (05) for each n = 1, 2, 3, ...

$\therefore \sum_{k=1}^{n}\frac{k!}{2^k} = \frac{1}{2} + \sum_{k=1}^{n-1}\frac{(k+1)!}{2^{k+1}} > \frac{1}{2} + \sum_{k=1}^{n-1}\frac{(k+2)}{8}$ (05)

(b) When $x < -2$, the inequality becomes.

$-(x+2)+[-(x-1)] = -x-2-x+1 = 2x-1 > 5$ (05)

i.e. $x < -3$, Hence the inequality holds when $x < -3$ (05)

When $-2 \le x < 1$ the inequality becomes.

$(x+2)+[-(x-1)] = x+2-x+1 = 3 > 5$ (05)

Which is a contradiction. (05)

When $x \ge 1$. the inequality becomes.

$(x+2)+(x-1) = 2x+1 > 5$ (05)

i.e. $x > 2$ Hence the inequality holds when $x > 2$ (05)

The set of all real values of x satisfying the inequality is,

$\{x \in \mathbb{R} : x < -3\} \cup \{x \in \mathbb{R} : x > 2\}$

(05) (05) 40

(b) Another Method

Let $y_1 = |x+2|$ and $y_2 = 5-|x-1|$ Then the in equality is equivalent to $y_1 > y_2$ (05)

Let $A = (x_A, y_A)$ and $E = (x_E, y_E)$ Note that

$B = (-2,0)$, $C = (-4,0)$

$D = (1,5)$, and $F = (0,4)$

considering the right angled isosceles triangles ABC and DEF we get

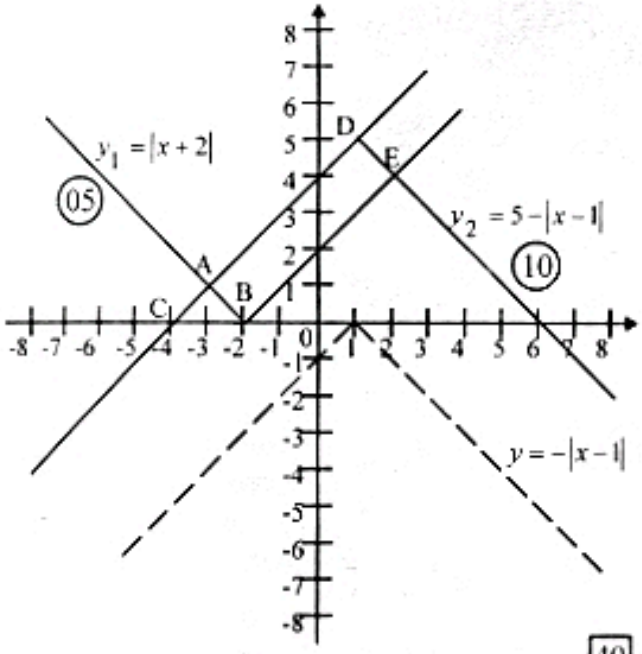
$$x_A = \frac{-2+(-4)}{2} = -3 \quad (05) \quad \text{and} \quad 1 = \frac{0+x_E}{2}$$

respectively, i.e. $x_A = -3$ and $x_E = 2$ (05)

The required solution set is

$\{x|x<-3\} \cup \{x|x>2\}$

(05) (05)



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(04) $\omega = \sqrt{3}+i = 2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) = 2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$ (05) (05)

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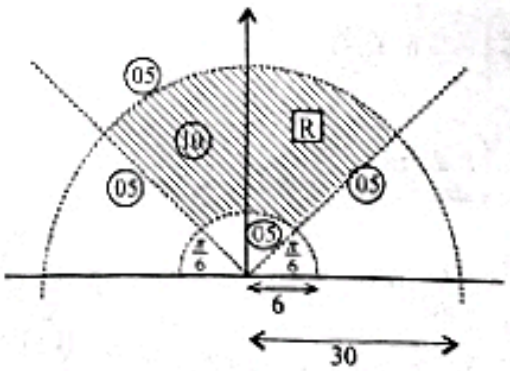
$$\omega^2 = \omega\omega$$
$$= 2^2\left[\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right]\left[\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right]$$
$$= 4\left\{\left[\cos^2\frac{\pi}{6}-\sin^2\frac{\pi}{6}\right]+i\,2\sin\frac{\pi}{6}\cos\frac{\pi}{6}\right\} \quad (05)$$
$$= 4\left(\cos\frac{2\pi}{6}+i\sin\frac{2\pi}{6}\right) = 4\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right) \quad (05)$$

$$\omega^3 = \omega^2\omega = 8\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$$
$$= 8\left(\cos\frac{\pi}{3}\cos\frac{\pi}{6}-\sin\frac{\pi}{3}\sin\frac{\pi}{6}\right)+i\left(\begin{matrix}\sin\frac{\pi}{3}\cos\frac{\pi}{6}\\+\cos\frac{\pi}{3}\sin\frac{\pi}{6}\end{matrix}\right)$$
$$= 8\left[\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right] \quad (10)$$

$$\omega^4 = \omega^3\omega$$
$$= 16\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$$
$$= 16\left(\cos\frac{\pi}{2}\cos\frac{\pi}{6}-\sin\frac{\pi}{2}\sin\frac{\pi}{6}\right)+i\left(\begin{matrix}\sin\frac{\pi}{2}\cos\frac{\pi}{6}\\+\cos\frac{\pi}{2}\sin\frac{\pi}{6}\end{matrix}\right)$$
$$= 16\left[\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right] \quad (10)$$

$$\omega^5 = \omega^4\cdot\omega$$
$$= 32\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$$
$$= 32\left[\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right] \quad (10)$$

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$|\omega| = 2 < 6$

$|\omega^2| = 4 < 6$

and $|\omega^5| = 32 > 30$

imply that ω, ω^2 and ω^5 are not in the region R

(10)

$|\omega^3| = 8$ and $\arg(z) = \frac{\pi}{2}$ (05)

and $|\omega^4| = 16$ and $\arg(z) = \frac{2\pi}{3}$ (05)

imply that ω^3 and ω^4 are in the region R. [20]

(05) (a)

$y = e^{\cos x} \Rightarrow \frac{dy}{dx} = e^{\cos x}(-\sin x)$ (05)

$\therefore \left(\frac{dy}{dx}\right)_{x=0} = 0$ (05)

$\left(\frac{d^2y}{dx^2}\right) = e^{\cos x}(-\cos x) + (-\sin x)e^{\cos x}(-\sin x)$ (05)

$= -e^{\cos x}\cos x + \sin^2 x\, e^{\cos x}$

$\therefore \left(\frac{d^2y}{dx^2}\right)_{x=0} = -e$ (05)

$\left(\frac{dy}{dx}\right) = -y\sin x \Rightarrow \frac{d^2y}{dx^2} = -\left(\frac{dy}{dx}\right)\sin x - y\cos x$

$\Rightarrow \left(\frac{d^3y}{dx^3}\right) = -\frac{d^2y}{dx^2}\sin x - \left(\frac{dy}{dx}\right)\cos x - \left(\frac{dy}{dx}\right)\cos x + y\sin x$

$\therefore \left(\frac{d^3y}{dx^3}\right)_{x=0} = 0$ (05)

$\frac{d^3y}{dx^3} = -\left(\frac{d^2y}{dx^2}\right)\sin x - 2\left(\frac{dy}{dx}\right)\cos x - \frac{dy}{dx}$ (05)

$$\frac{d^4y}{dx^4} = \begin{pmatrix} -\left(\frac{d^3y}{dx^3}\right)\sin x - \left(\frac{d^2y}{dx^2}\right)\cos x - 2\left(\frac{d^2y}{dx^2}\right)\cos x \\ +2\left(\frac{dy}{dx}\right)\sin x - \left(\frac{d^2y}{dx^2}\right) \end{pmatrix}$$ (05)

$\Rightarrow \therefore \left(\frac{d^4y}{dx^4}\right)_{x=0} = 4e$ (05)

$$\left(\frac{d^5y}{dx^5}\right) = \begin{pmatrix} -\left(\frac{d^4y}{dx^4}\right)\sin x - \left(\frac{d^3y}{dx^3}\right)\cos x - 3\left(\frac{d^3y}{dx^3}\right)\cos x \\ +3\left(\frac{d^2y}{dx^2}\right)\sin x + 2\left(\frac{d^2y}{dx^2}\right)\sin + 2\left(\frac{dy}{dx}\right)\cos x \\ -\left(\frac{d^3y}{dx^3}\right) \end{pmatrix}$$ (05)

$\therefore \left(\frac{d^5y}{dx^5}\right)_{x=0} = 0$ (05) [50]

(b) $y = \frac{2x}{1+x^2} \Rightarrow \frac{dy}{dx} = \frac{(1+x^2)2 - 2x(2x)}{(1+x^2)^2}$ (05)

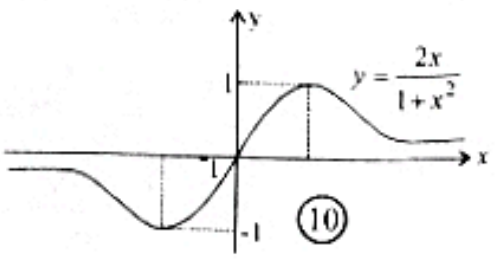
$= \frac{2(1-x^2)}{(1+x^2)^2} = \frac{2(1+x)(1-x)}{(1+x^2)^2}$ (05)

$\therefore \frac{dy}{dx} = 0 \Rightarrow x = \pm 1$ (05)

Note that $x = 1 \Leftrightarrow y = 1$ and $x = -1 \Leftrightarrow y = -1$

Interval	$\frac{dy}{dx}$	f	
$x < -1$	(-)	decreasing	(05)
$-1 < x < 1$	(+)	increasing	(05)
$1 < x$	(-)	decreasing	(05)

$\therefore$ (-1, -1) is a local minimum and (1, 1) is a local maximum. (05) (05)



(10) [50]

(05)

(06) (*a*) Let $3\sqrt{x} = y$, Then $x = y^3$ and $dx = 3y^2dy$

$$\therefore \int_1^8 \frac{dx}{1+x^{\frac{1}{3}}} = \int_1^2 \frac{3y^2 dy}{1+y} \quad (05)$$

$$= 3\int_1^2 \left[(y-1) + \frac{1}{1+y}\right] dy \quad (05)$$

$$= 3\left[\frac{y^2}{2} - y + \ell n|1+y|\right]_1^2 \quad (05)\ (05)$$

$$= 3\left[\ell n 3 - \frac{1}{2} + 1 - \ell n 2\right]$$

$$= 3\left[\frac{1}{2} + \ell n\frac{3}{2}\right] \quad (05) \qquad \boxed{30}$$

(*b*) $$\int_0^1 x^2 e^{2x+3} dx = e^3 \int_0^1 x^2 e^{2x} dx$$

$$= e^3 \int_0^1 x^2 \frac{d}{dx}\left(\frac{e^{2x}}{2}\right) dx$$

$$= e^3 \left[\left(x^2 \frac{e^{2x}}{2}\right)_0^1 - \int_0^1 \frac{e^{2x}}{2} 2x dx\right] \quad (05) \quad (05)$$

$$= e^3 \left[\frac{e^2}{2} - \int_0^1 x \frac{d}{dx}\left(\frac{e^{2x}}{2}\right) dx\right] \quad (05)$$

$$= e^3 \left[\frac{e^2}{2} - \left(x\frac{e^{2x}}{2}\right)_0^1 - \int_0^1 \frac{e^{2x}}{2} dx\right] \quad (05) \quad (05)$$

$$= e^3 \left[\frac{e^2}{2} - \frac{e^2}{2} + \left(\frac{e^{2x}}{4}\right)_0^1\right] \quad (05) \quad (05)$$

$$= \frac{e^3(e^2-1)}{4} \quad (05) \qquad \boxed{40}$$

(*c*) $$\frac{1}{x(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3}$$

$\Rightarrow A = \frac{1}{3}$, $B = -\frac{1}{3}$ *and* $C = 0$ (10)

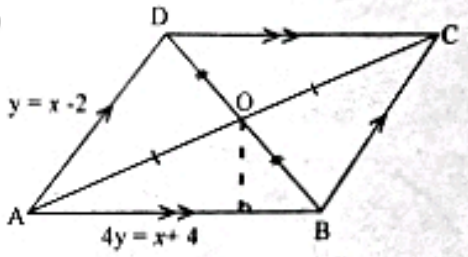
$$\therefore \int \frac{dx}{x(x^2+3)} = \int \frac{1}{3x} dx - \int \frac{x}{3(x^2+3)} dx \quad (05)$$

$$= \frac{1}{3}\left[\underset{(05)}{(\ell n|x|)} - \underset{(05)}{\frac{1}{2}\ell n|x^2+3|}\right] + D \quad (05)$$

$$= \frac{1}{3}\ell n\left(\frac{|x|}{\sqrt{x^2+3}}\right) + D$$

Where D is an arbitrary constant **30**

(07)



(i) By solving $y = x - 2$ and $4y = x + 4$ we get the coordinates of A ≡ (4, 2) Hence C ≡ (-4, -2) (05) (05)

Equation of BC : $y - x = -2 + 4 = 2 \rightarrow y = x + 2$ (10)

$4y - x = -8 + 4 = -4 \rightarrow 4y = x - 4$ (10)

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Equation of AC $\frac{y-0}{x-0} = \frac{-2-2}{-4-4} = \frac{1}{2} \Rightarrow 2y = x$ (05)

Using the equations of AB & BC, we can write the equation of BD in the form

$4y - x - 4 + \lambda(y - x - 2) = 0$, where $\lambda \in \mathbb{R}$ (10)

This goes through $0 \Rightarrow -4 - 2\lambda = 0 \Rightarrow \lambda = -2$ (05)

Equation of BD : $4y - x - 4 - 2(y - x - 2) = 0 \Rightarrow$

$2y + x = 0$ (05)

35

By solving the equations of AB and BC, We can find the coordinate of B :

$4y = x + 4$ and $y = x + 2 \Rightarrow B \equiv \left[\frac{-4}{3}, \frac{2}{3}\right]$ (05)

Area of Parallelogram = 4 x *area* Δ ABO

$$= 4 \times \frac{1}{2} AB \times OE \quad (05)$$

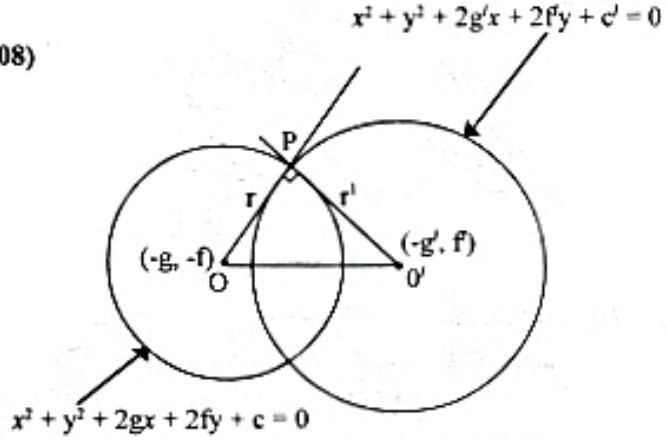
$$= 4 \times \frac{1}{2} \times \sqrt{\left(4+\frac{4}{3}\right)^2 + \left(2-\frac{2}{3}\right)^2} \times \frac{4}{\sqrt{17}} \quad (10)$$

$$= 4 \times \frac{1}{2} \times \sqrt{\left(\frac{16}{3}\right)^2 + \left(\frac{4}{3}\right)^2} \times \frac{4}{\sqrt{17}} \quad (10)$$

$$= \frac{32}{3} \text{ units} \quad (05)$$

[35]

(08)



If the two circles intersect orthogonally.

$(PO)^2 + (PO')^2 = (OO')^2$ then, (05)

$r^2 + r'^2 = (-g + g')^2 + (-f + f')^2$ (05)

Where $r = \sqrt{g^2 + f^2 - c}$ and $r' = \sqrt{g'^2 + f'^2 - c'}$

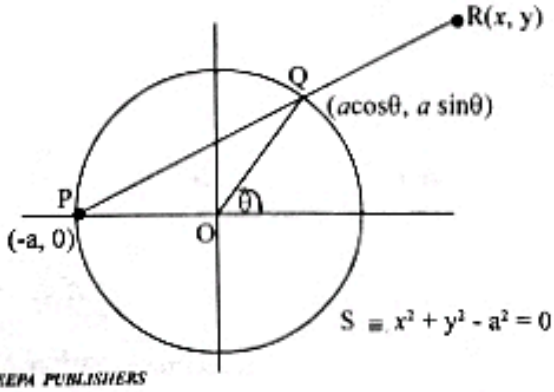
$\therefore (g^2 + f^2 - c) + (g'^2 + f'^2 - c')$ (05)

$= g^2 + g'^2 - 2gg' + f^2 + f'^2 - 2ff'$ (05)

$\Rightarrow \quad -c - c' = -2gg' - 2ff'$

$\Rightarrow \quad 2gg' + 2ff' = c + c'$ (05)

[25]



PQ : QR = 1 : 1 and R = (x, y)

$\Rightarrow \frac{(-a+x)}{2} = a\cos\theta$ *and* $\frac{0+y}{2} = a\sin\theta$ (05) (05)

$\Rightarrow \quad x = 2a\cos\theta + a \qquad y = 2a\sin\theta$

$\Rightarrow \quad R = (2a\cos\theta + a\ 2a\sin\theta)$ (05) (05)

[20]

Then $\cos\theta = \frac{(x-a)}{2a}$ and $\sin\theta = \frac{y}{2a}$ Now,

$\cos^2\theta + \sin^2\theta = 1$

$\Rightarrow \left(\frac{(x-a)}{2a}\right)^2 + \left(\frac{y}{2a}\right)^2 = 1 \Rightarrow (x-a)^2 + y^2 = 4a^2$ (05) (05) (05)

Therefore R lies on the circle S' with centre $(a, 0)$ and radius $2a$.

[15]

Equation of $S' \equiv x^2 + y^2 - 2ax - 3a^2 = 0$ (05)

[05]

Let $S'' = x^2 + y^2 + 2gx + 2fy + C = 0$, Since S and S'' circles intersect orthogonally (05)

$2g(0) + 2f(0) = C - a^2 \quad \Rightarrow C = a^2$ (05)

since S and S'' circles intersect orthogonally,

$2g(-a) + 2f(0) = C - 3a^2$

$\Rightarrow \quad -2ga = C - 3a^2 = 2a^2$

$\Rightarrow g = a$ (05)

Therefore $S'' \equiv x^2 + y^2 + 2ax + 2fy + a^2 = 0$

Now S'' touches the y axis implies that when x = 0, y values are coincident.

i.e. $y^2 + 2fy + a^2 = 0$ has coincident roots then

(05) $4f^2 - 4a^2 = 0 \quad \Rightarrow f^2 = a^2 \quad \Rightarrow f = \pm a$ (05)

Therefore, there are two such circles S''.

[25]

Their equations are

$S'' \equiv x^2 + y^2 + 2ax + 2ay + a^2 = 0$ (05) and

$S'' \equiv x^2 + y^2 + 2ax - 2ay + a^2 = 0$ (05)

[10]

(09) (a)

(i) $x = \sin\theta - \cos\theta$

$\Rightarrow x^2 = \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$

$= 1 - \sin 2\theta$ (05)

$\Rightarrow \sin 2\theta = 1 - x^2$ (05) 10

(ii) $y = \tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ (05)

$= \dfrac{1}{\frac{1}{2}\sin 2\theta} = \dfrac{2}{\sin 2\theta} \Rightarrow \sin 2\theta = \dfrac{2}{y}$ (05) (05) 15

$\therefore 1 - x^2 = \dfrac{2}{y} \Rightarrow y(1 - x^2) - 2 = 0$ (05) 05

(b) $\sin 2x + \sin 4x + \sin 6x$

$= \sin 4x + \sin 2x + \sin 6x$ (05)

$= \sin 4x + 2\sin 4x\cos 2x$

$= (1 + 2\cos 2x)\sin 4x$ (05) 10

$\sin x(\sin 2x + \sin 4x + \sin 6x)$

$= \sin x\,(1 + 2\cos 2x)\sin 4x$ (05)

$= \sin 4x\,(\sin x + 2\sin x\cos 2x)$ (05)

$= \sin 4x\,(\sin x + \sin 3x - \sin x)$

$= \sin 3x \sin 4x$ (05) 15

Let $x = \dfrac{\pi}{12}$ (05). Then $\sin\dfrac{\pi}{12}\left[\dfrac{1}{2} + \dfrac{\sqrt{3}}{2} + 1\right] = \dfrac{\sqrt{3}}{2\sqrt{2}}$ (05)

$\left(3 + \sqrt{3}\right)\sin\dfrac{\pi}{12} = \dfrac{\sqrt{3}}{\sqrt{2}}$

$\sin\dfrac{\pi}{12} = \dfrac{\sqrt{3}}{\sqrt{2}} \times \dfrac{1}{3 + \sqrt{3}} = \dfrac{1}{\sqrt{2}\left(1 + \sqrt{3}\right)} = \dfrac{1}{\sqrt{6} + \sqrt{2}} = \dfrac{\sqrt{6} - \sqrt{2}}{4}$ (05)

15

(c) Sine Rule for a triangle is

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ (05) 05

Let $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c} = k$ (05)

Then $a = b + \lambda c \Rightarrow \sin A = \sin B + \lambda\sin C$ (05)

$\Rightarrow \lambda\sin C = \sin A - \sin B = \sin(B + C) - \sin B$ (05)

$\therefore A + B + C = \pi$

$\Rightarrow 2\lambda\sin\dfrac{C}{2}\cos\dfrac{C}{2} = 2\sin\dfrac{C}{2}\left(\dfrac{2B + C}{2}\right)$ (05)

(05)

$\lambda\cos\dfrac{C}{2} = \cos\left(B + \dfrac{C}{2}\right)$ 25