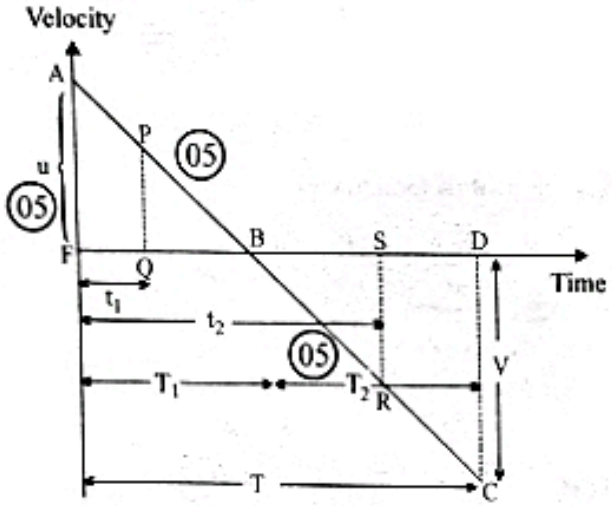
G.C.E. (Advanced Level) Examination - April 2003
10 - Combined Mathematics - II
Provisional Scheme of Marking

(01) (*a*) Velocity time graph



15

Upward motion in time T_1. At B the velocity is zero. for the downward motion the velocity is negative.
Downward motion in time T_2. v is the speed of the Particle when it falls back to O.

(i) The distance travelled during the upward motion = The distance travelled during the downward motion.

area of Δ AFB = area of Δ BCD

$$\therefore \frac{1}{2} T_1 u = \frac{1}{2} T_2 V \rightarrow (01) \text{ (05)}$$

But $\frac{u}{T_1} = \frac{V}{T_2} = g$ (05)

$$\therefore T_1 = \frac{u}{g} \text{ and } \rightarrow (02)$$

$$T_2 = \frac{v}{g} \rightarrow (03)$$

Substituting (02) and (03) in (01)

$$\frac{1}{2}\frac{u^2}{g} = \frac{1}{2}\frac{v^2}{g} \text{ (05)} \Rightarrow \therefore v^2 = u^2 \Rightarrow v = u (\because v > 0)$$

and $T_2 = \frac{u}{g} = T_1$

15

(ii) Maximum height = area of Δ AFB $= \frac{1}{2} T_1 u = \frac{1}{2} \frac{u^2}{g}$

from (02) (05)

05

(iii) Height after time t_1 = Height after time t_2

∴ Distance travelled intime $t_2 - T_1$ = Distance travelled intime $T_1 - t_1$ (05)

Area of Δ BSR = area of PQB

$$\therefore \frac{1}{2}(BS)(gBS) = \frac{1}{2}(BQ)(gBQ) \text{ (05)}$$

$$\therefore BS^2 = BQ^2 \Rightarrow BS = BQ (\because BS \neq BQ)$$

i.e. $t_2 - T_1 = T_1 - t_1$ } (05)
$t_2 - T_2 = T_1 - t_1$ } $(T_1 = T_2)$ or $t_1 + t_2 = 2T_1 = T$
$t_1 + t_2 = T_1 + T_2 = T$

15

(b) Velocity of C relative to A = Vel C, A =  (05)

Vel . C, E = 

Vel . E, A =

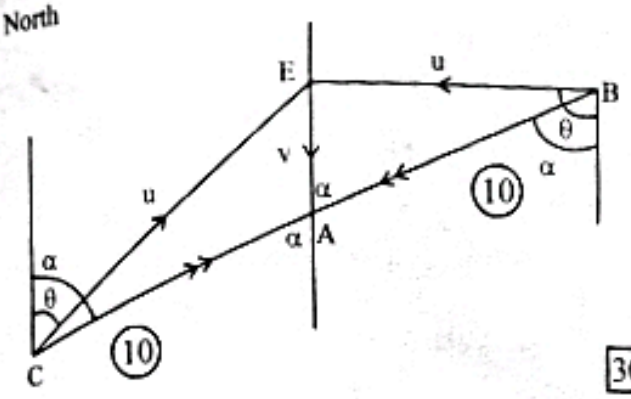
Vel . B, A =  (05)

Vel . B, E = 

Adding vectorially

Vel B, A = Vel B, E + Vel E, A

and Vel C, A = Vel C, E + Vel E, A



(10) (10)

30

From Δ CAE $\frac{u}{\sin\alpha} = \frac{v}{\sin(\alpha-\theta)}$ (05)

From Δ BAE $\frac{u}{\sin\alpha} = \frac{v}{\sin(\phi-\alpha)}$ (05)

$\therefore \frac{u}{\sin\alpha} = \frac{v}{\sin(\alpha-\theta)} = \frac{v}{\sin(\phi-\alpha)}$

10

Vel B, C = vel B, E + Vel E, C or = Vel B, A + Vel A, C
In either case it is represented by BC. (05)

Draw ED $\perp$ BC

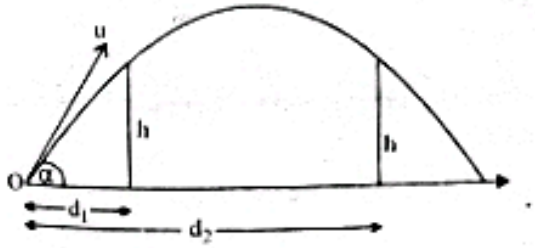
Δ BED = Δ CED

$\therefore$ BD = DC (05)

$|\text{Vel B,C}| = BC = 2DC = 2\sqrt{u^2 - v^2\sin^2\alpha}$

10

(02) (a)



Let the particle be at a horizontal distance x and vertical distance y from o, after time t.

Then $x = (u\cos\alpha_1)t_1 \rightarrow (01)$ (05)

$y = (u\sin\alpha_1)t - \frac{1}{2}gt^2 \rightarrow (02)$ (05)

Substituting (01) in (02)

$y = x\tan\alpha_1 - \frac{1}{2}g\frac{x^2}{u^2\cos^2\alpha_1}$ (05)

i.e. $x^2 - \frac{2u^2}{g}\sin\alpha_1\cos\alpha_1 + \left(\frac{2u^2}{g}\cos^2\alpha_1\right)y = 0 \rightarrow (03)$

When y = h, (03) gives

$x^2 - \frac{u^2}{g}\sin 2x + \left(\frac{2u^2}{g}\cos^2\alpha_1\right)h = 0 \rightarrow (04)$ (05)

The roots of (04) are d_1 and d_2

$\therefore d_1 + d_2 = \frac{u^2}{g}\sin 2\alpha_1$, $d_1 d_2 \frac{2hu^2}{g}\cos^2\alpha_1$ (05)

(05)

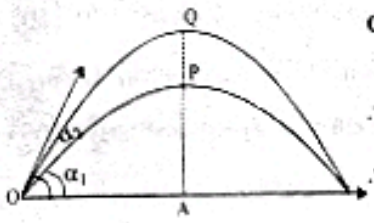
30

Maximum height when $d_1 - d_2 = d = OA$ (05)

i.e. $2OA = \frac{u^2}{g}\sin 2\alpha_1$

$OA = \frac{u^2}{2g}\sin 2\alpha_1$ (05)

10



For the particle Q

$OA = \frac{u^2}{2g}\sin 2\alpha_2$

$\therefore \frac{u^2}{2g}\sin 2\alpha_1 = \frac{u^2}{2g}\sin 2\alpha_2$ (05)

$\therefore \sin 2\alpha_1 = \sin 2\alpha_2$

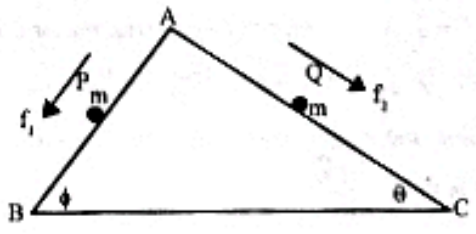
$\therefore \alpha_1 < \frac{\pi}{2}$ *and* $\alpha_2 < \frac{\pi}{2}$

$2\alpha_1 = \pi - 2\alpha_2$

$\alpha + \alpha_2 = \frac{\pi}{2}$ (05)

10

(b) Wedge fixed



Let f_1 ↙ and f_2 ↘ be the accelerations of P and Q respectively.

Apply P = mf ↙ for particle P.

$mg \sin \phi = mf_1$

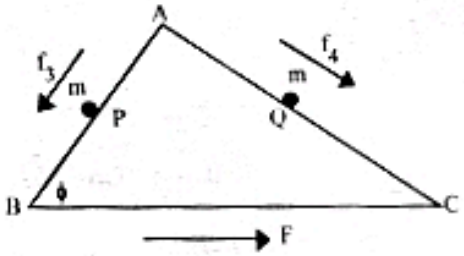
$\therefore f_1 = g \sin(\phi)$ (05)

Apply p = mf ↘ for particle Q

$mg \sin(\theta) = mf_2$

$\therefore f_2 = g \sin \theta$ (05) [10]

Wedge in motion



Let F be the acceleration of the wedge relative to the plane and f_3 ↙ and f_4 ↘ be the a acceleration of P and Q respectively relative to the wedge.

Apply P = mf→ for the system

$O = MF + m(F - f_3 \cos\phi) + m(F + f_4 \cos\theta) \rightarrow (01)$ (10)

Apply p = mf ↙ for P $mg\sin(\phi) = m(f_3 - F\cos\phi) \rightarrow (02)$ (05)

Apply p = mf ↘ for Q $mg\sin\theta = m(f_4 + F\cos\theta) \rightarrow (03)$ (05)

(01) + (02) x cos ϕ - (03) x cosθ gives

$mg \sin\phi \cos\phi - mg\sin\theta \cos\theta = MF + m(F - F\cos^2\phi) + m(F - F\cos^2\theta)$

$$F = \frac{mg \sin 2\phi - \sin 2\theta}{2\left[M + m\sin^2\theta + \sin^2\phi\right]}$$ (10) [30]

(05)

When $\theta = \phi$, F = 0 : wedge moves with uniform velocity.

wedge is an inertial frame. ∴ The acceleration of P and Q are g sinϕ and g sinθ respectively. as in the case when the wedge is fixed. (05) [10]

(03) (*a*)



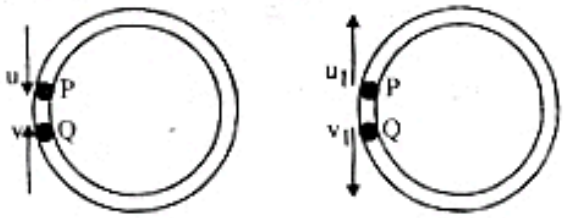
(05)

Speed of P relative to Q is u + v, which is not changes as the spheres and the grove are smooth.

(05)

The spheres collide after P travels a distance $2\pi a$ relative to Q. i.e. after time $\frac{2\pi a}{u+v}$ (05) [15]

First collision.



before after

Conservation of momentum : mu - mv = $mv_1 - mu_1$

$u_1 - v_1 = v - u \rightarrow (01)$ (05)

Newton's law of restitution :

$u_1 + v_1 = e(u + v) \rightarrow (02)$ (05)

(05)

(i) (02) - (01) gives $2v_1 = e(u+v) + u - v > 0$ ($\therefore u > v$)

Q moves in the opposite direction after the collision. [15]

(ii) The two spheres move in opposite derections, if further, $u_1 > 0$ (05)

(01) + (02) gives $2u_1 = e(u + v) + v - u$

$u_1 > 0$, if $e(u + v) > u - v$

i.e. if $e > \frac{u-v}{u+v}$ (05) [10]

Speed of P relative to Q after the first collision is $u_1 + v_1$ (05)

After the first collision, the spheres collide again after a time $\frac{2\pi a}{u_1 + v_1}$

∴ They collide again at time t where t

$$= \frac{2\pi a}{u+v} + \frac{2\pi a}{u_1+v_1}$$

$$= \frac{2\pi a}{u+v} + \frac{2\pi a}{e(u+v)} \quad [from(02)]$$

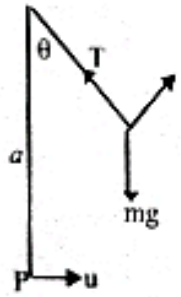
$$= \frac{2\pi a(1+e)}{e(u+v)} \quad (05)$$

[10]

(b) Let u be the velocity of P, after the impuls apply

$I = \Delta mv$

$I = mu$ *i.e.* $u = \frac{I}{m} \rightarrow (01)$ (05)



T does no work

Conservation of energy:

$$\frac{1}{2}mv^2 - mga\cos\theta = \frac{1}{2}mu^2 - mga$$

(05) (05)

$$v^2 = u^2 - 2ga + 2ga\cos\theta$$

$$= \frac{I^2}{m^2} - 2ga + 2ga\cos\theta \rightarrow (02)$$

$$[from(01)]$$

Apply P = mf ↖

$$T - mg\cos\theta = \frac{mv^2}{a} \rightarrow (03) \quad (05)$$

$$= \frac{I^2}{ma} - 2mg + 2mg\cos\theta \quad [from(02)]$$

i.e. $T = \frac{I^2}{ma} - 2mg + 3mg\cos\theta \rightarrow (04)$ (05) [25]

(i) If the particle describes a complete circle then $v^2 > 0$ and $T \geq 0$, at $\theta = \pi$ (minimum at $\theta = \pi$)

From (02) at $\theta = \pi$, $v^2 = \frac{I^2}{m^2} - 4ag > 0 \Rightarrow I > m\sqrt{4ag}$ (05)

From (04) at $\theta = \pi$, $T = \frac{I^2}{ma} - 5mg \geq 0 \Rightarrow I \geq m\sqrt{5ag}$

$\left(> m\sqrt{4ag}\right)$ (05)

[10]

(ii) If the particle leaves circular motion when OP makes an acute angle α with the upward vertical.

Then $v^2 > 0$, at $\theta = \frac{\pi}{2}$, and $T < 0$ at $\theta = \pi$

From (02), at $\theta = \frac{\pi}{2}$, $v^2 = \frac{I^2}{m^2} - 2ga > 0$,

From (04), at $\theta = \pi$, $T = \frac{I^2}{ma} - 5mg < 0$,

i.e. $m\sqrt{2ga} < I < m\sqrt{5ga}$ (05)

$T = 0$ and $v^2 > 0$, when $\theta = \pi - \alpha$:

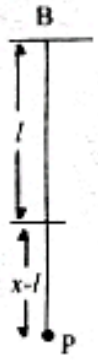
From (04), $\frac{I^2}{ma} - 2mg - 3mg\cos\alpha = 0 \rightarrow (05)$

Also at $\theta = \pi - \alpha$, from (02)

$v^2 = \frac{I^2}{m^2} - 2ga - 2ga\cos\alpha = ga\cos\alpha\left[from(05)\right] > 0$ (05)

From (05), $\cos\alpha = \frac{I^2}{3m^2ga} - \frac{2}{3}$ (05) [15]

(04) (a)



Let x be the maximum length

(i) conservation of energy gives

(05) $\frac{1}{2}4mg\frac{(x-\ell)^2}{\ell} = mgx$ (05)

$2(x-\ell)^2 = x\ell$

$2x^2 - 5\ell x + 2\ell^2 = 0$

$(2x-\ell)(x-2\ell) = 0$ (05)

$x = 2\ell$ as $\frac{\ell}{2}$ is inadmissible.

∴ Maximum length is 2ℓ (05)

[20]

(ii) Let v be the velocity of p when the string is just stretched.

conservation of energy gives $\frac{1}{2}mv^2 - mg\ell$ (05)

$v = \sqrt{2g\ell}$

[05]



conservation of energy

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}4mg\frac{(x-\ell)^2}{\ell} = mgx \quad (05)$$ (05)

Differentiate with respect to time

$$\dot{x}\ddot{x} + 4g - \frac{(x-\ell)}{\ell}x = gx \quad (05)$$

$$i.e. \ddot{x} + \frac{4g}{\ell}\left(x - \frac{5\ell}{4}\right) = 0 \rightarrow (02)$$

Write $y = x - \frac{5\ell}{4}$

$(02) \Rightarrow \ddot{y} + \frac{4g}{\ell}y = 0 \rightarrow (03)$ (05) $x \geq l$,

$\therefore y \geq \frac{-l}{4}$

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Substituting $y = A\cos\omega t + B\sin\omega t \rightarrow (04)$ (05)

in (03) $\omega^2 + \frac{4g}{\ell} - 0$

$\omega = 2\sqrt{\frac{g}{\ell}} \rightarrow (05)$ (05)

Take $t = 0$ *at* $y = -\frac{\ell}{4}$

From (04), $A = \frac{-\ell}{4}$ *and* (05)

$y = -\omega A\sin\omega t + \omega B\cos\omega t \rightarrow (06)$

At $t = 0$, $y = x = \sqrt{2g\ell}$

From (06), $\sqrt{2g\ell} = \omega \quad B = 2\sqrt{\frac{g}{\ell}}B$ [*from* (05)]

$B = \frac{\ell}{\sqrt{2}}$ (05)

$\therefore y = \frac{-\ell}{4}\cos\omega t + \frac{\ell}{\sqrt{2}}\sin\omega t$ (05)

25

(iii) i.e.

$$y = \frac{3\ell}{4}\left[\frac{2\sqrt{2}}{3}\sin\omega t - \frac{1}{3}\cos\omega t\right] = \frac{3\ell}{4}\sin(\omega t - \alpha) \rightarrow (07)$$ (05)

Where $\sin\alpha = \frac{1}{3}$

Maximum value of $y = \frac{3\ell}{4}$ (05)

$\therefore$ Maximum length = Maximum x

= Maximum $y + \frac{5\ell}{4}$

= 2ℓ (05)

15

(iv) From (07) $\dot{y} = \frac{3\ell\omega}{4}\cos(\omega t - \alpha) \rightarrow (08)$ (05)

Greatest speed is achieved in the S.H.M. Phase.

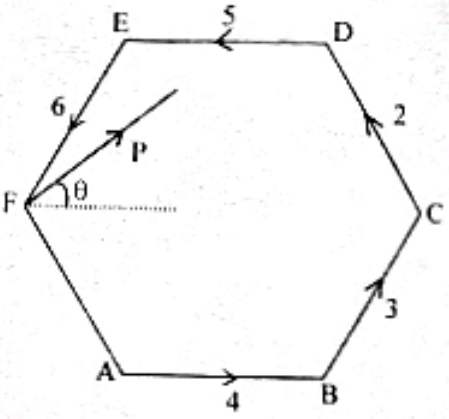
(05)

$\therefore$ Greatest speed *maximum* $\dot{y} = \frac{3\ell\omega}{4}$ [*from*(08)]

$= \frac{3}{2}\sqrt{g\ell}$ [*from*(05)] (05)

15

(05) (a)



(i) Resolving horizontally;

$P\cos\theta = -4\frac{-3}{2} + \frac{2}{2} + 5 + \frac{6}{2} = \frac{7}{2}$ N (05)

Resolving vertically;

$P\sin\theta = -3\frac{\sqrt{3}}{2} - 2\frac{\sqrt{3}}{2} + 6\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ N (05)

Therefore P = $\sqrt{13}$N (05) and $\theta = \tan^{-1}\left(\frac{\sqrt{3}}{7}\right)$ (05)

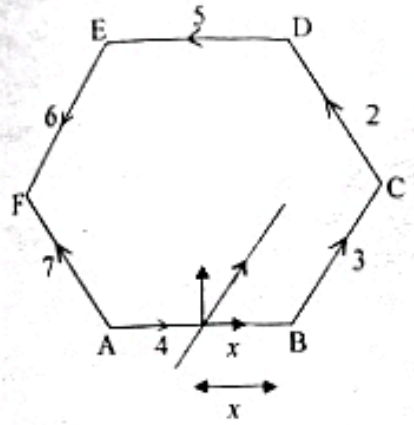
Taking the moment about F anticlockwise.

$G = 4\times 2\frac{\sqrt{3}}{2} + \frac{3}{2}\times 2\frac{\sqrt{3}}{2} + 3\frac{\sqrt{3}}{2}\times 3 + 2\frac{\sqrt{3}}{2}\times 4 + 2\frac{\sqrt{3}}{2}\times 5$ (10)

$= 19\sqrt{3}$ Nm (05)

35

(ii)



Resolving horizontally;

$$\rightarrow x = 4 + 3\times\frac{1}{2} - 2\times\frac{1}{2} - 5 - 6\times\frac{1}{2} - 7\times\frac{1}{2}$$

$$\therefore = 7\text{N} \quad (05)$$

Resolving vertically;

$$\uparrow y = 7\times\frac{\sqrt{3}}{2} - 6\times\frac{\sqrt{3}}{2} + 3\times\frac{\sqrt{3}}{2} + 2\times\frac{\sqrt{3}}{2}$$

$$= 3\sqrt{3}\text{N} \quad (05)$$

Therefore, the single force $R = 2\sqrt{19}$ N

Taking the moments about the point of intersection of single force and AB anticlockwise;

$$\frac{3\sqrt{3}}{2}x + \frac{2}{2}\times\frac{2\sqrt{3}}{2} + \frac{2\sqrt{3}}{2}\times(x+1) + 5\times 2\sqrt{3} + \frac{6\sqrt{3}}{2}\times(2-x)$$

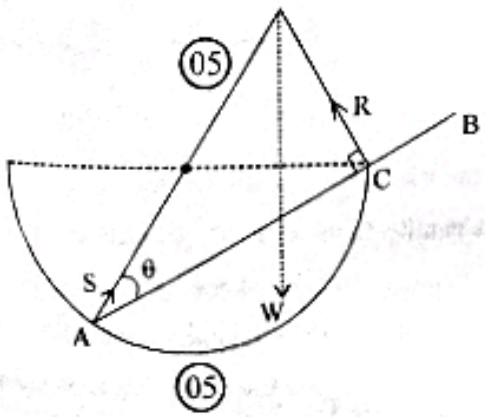
$$+3\times 2\sqrt{3} - \frac{7\sqrt{3}}{2}\times(2-x) = 0 \quad (10)$$

$$\Rightarrow 3x + 17 = 0$$

$$\Rightarrow x = -5\frac{2}{3}$$

Therefore, the point of intersection of its line of action with AB is $7\frac{2}{3}$ meters from A to the right. (05) **25**

(b)



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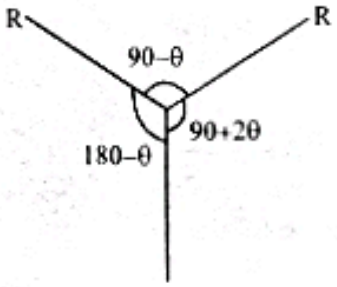
Taking moments about A anticlockwise:

$W\ell\cos = R \cdot 2a\cos$

$$W\ell\cos\theta = R.2a\cos\theta \quad (05)$$

$$\Rightarrow R = \frac{w\ell}{2a} \rightarrow (01)$$

05



applying sine law; (05)

$$\frac{R}{\sin(90+2\theta)} = \frac{W}{\sin(90-\theta)}$$

05

This implies $R = \frac{W\cos 2\theta}{\cos\theta} \rightarrow (02)$ (05)

using (01) and (02) we get

$$\frac{W\ell}{2a} = \frac{W\cos 2\theta}{\cos\theta} \Rightarrow 4a\cos^2\theta - \ell\cos\theta - 2a = 0 \quad (05)$$

$$\Rightarrow \cos\theta = \frac{\left(\ell + \sqrt{\ell^2 + 32a^2}\right)}{8a} \quad (05)$$

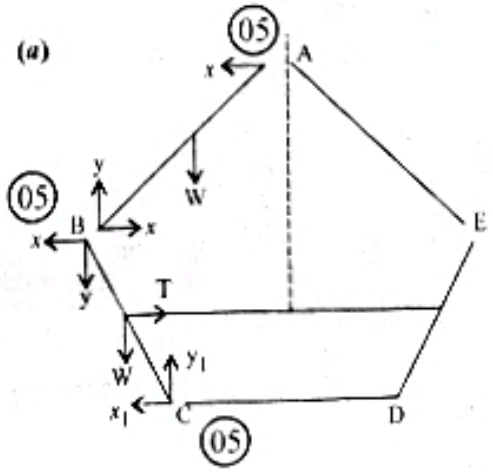
Therefore $CB = 2\ell - 2a\cos\theta$

$$= 2\frac{\left[\ell - \sqrt{\ell^2 + 32a^2}\right]}{8} \quad (05)$$

$$= \frac{1}{4}\left[7\ell - \sqrt{\ell^2 + 32a^2}\right]$$

20

(06) (a)



15

Consider the equilibrium of the rod AB;

Resolving vertically $Y = W$ (05)

Taking the moments about $B \times \sin\frac{\pi}{5} = \frac{1}{2} W \cos\frac{\pi}{5} \rightarrow (01)$ (05)

If T denotes the tension of the light rod, by taking the moments about C for the rod BC, We get

$$\frac{1}{2} T \sin\frac{2\pi}{5} = \frac{1}{2} W \cos\frac{2\pi}{5} + W \cos\frac{2\pi}{5} + X \sin\frac{2\pi}{5} \quad (10)$$

On substituting for x in terms of W from (01), we find that

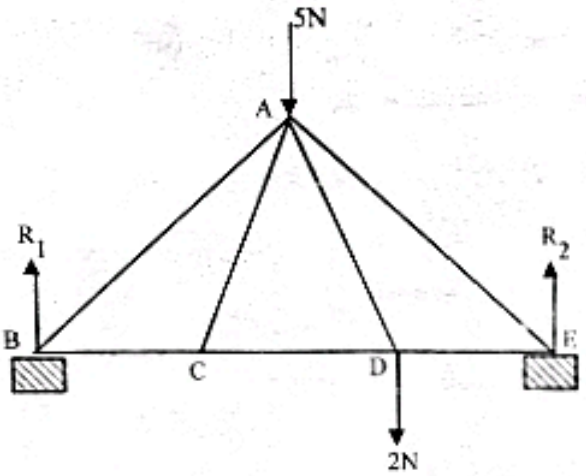
$$\frac{1}{2} T \sin\frac{2\pi}{5} = \frac{1}{2} W \cos\frac{2\pi}{5} + W \cos\frac{2\pi}{5} + \frac{1}{2} W \cot\frac{\pi}{5} \sin\frac{2\pi}{5} \quad (05)$$

$$\Rightarrow T = W\left(\cot\frac{2\pi}{5} + 2\cot\frac{2\pi}{5} + \cot\frac{\pi}{5}\right)$$

$$\Rightarrow T = W\left(\cot\frac{\pi}{5} + 3\cot\frac{2\pi}{5}\right)$$

25

(b)



Resolving vertically;

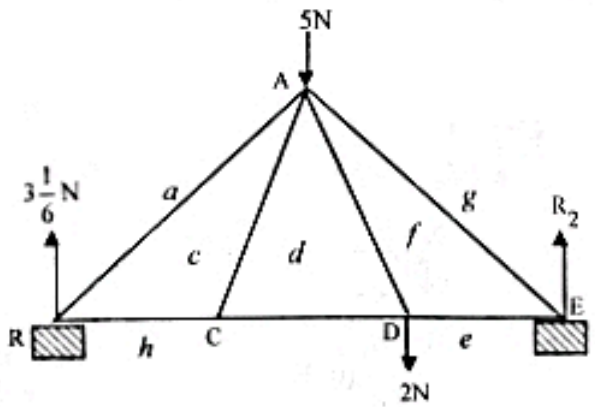
$R_1 + R_2 = 7N$ (05)

Let $BC = CD = DE = AC = AD = 2a$

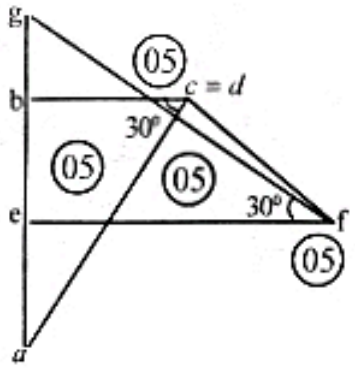
Taking the moments about B; (05)

$5 \times 3a - R_2 \times 6a = -2 \times 4a \Rightarrow R_2 = 3\frac{5}{6} N$ (05)

Therefore, $R_1 = 3\frac{1}{6} N$ (05)



The stress diagram.



40

Considering the triangle abc, we get

$$\sin 30^0 = \frac{19/6}{d(a,c)} \Rightarrow d(a,c) = 6\frac{1}{3}$$ (05)

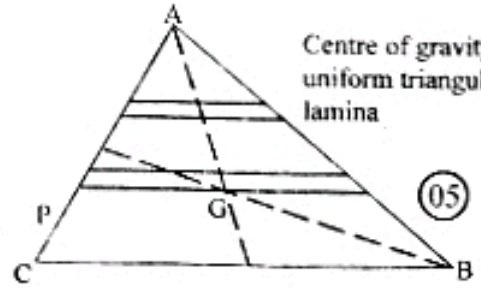
Considering the triangle efg, we get

$$\sin 30^0 = \frac{23/6}{d(f,g)} \Rightarrow d(f,g) = 7\frac{2}{3}$$ (05)

Rod	Stress	Magnitude	
AB	Thrust	$6\frac{1}{3}$ N	(05)
AE	Thrust	$7\frac{2}{3}$ N	(05)

20

(07) Centre of gravity of a uniform triangular lamina



consider the triangle as being made from a Series of strips with sides parallel to the side BC. AS each strip is approximately a rectangle its centre of gravity is at its midpoint. So the centre of gravity of the triangle, G must lie on the line (05) joining the mid points of these strips. that is on the median (05) through A (05)

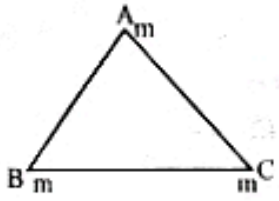
Similarly by considering the triangle as being made from a series of strips parallel to AC, G also lies on the median through B. (05)

Therefore the position of the centre of gravity of a uniform triangle is at the point of intersection of the medians. (05)

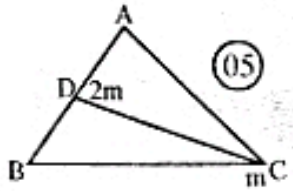
[30]

Let the mass of the particle at each vertex be m.

The resultant, 2mg of the weights at A and B Passes through D, the midpoint of AB, that is

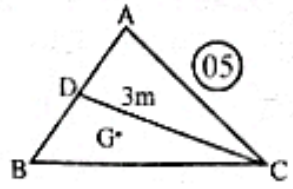


This is equivalent to



(05)

Which is equivalent to

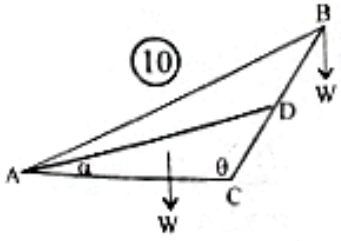


(05)

The resultant, 3mg, of the weights 2mg at D and mg at C passes through G, Where G is on DC such that DG : GC = 1 : 2 (05)

But CD is a median of the triangle so G is the centroid of the triangle and so coincides with G. (05)

[20]



(10)

Let W1 be the weight which if suspended from vertex B will not overfurn the lamina.

Taking moments about C. We get

$$W.a(-\cos\theta) \le W\left(b\cdot\frac{2}{3}(CD)\frac{\left(b-\frac{a}{2}\cos\theta\right)}{(CD)}\right)$$

$$= W\left(b\cdot\frac{2b}{3}-\frac{a}{3}\cos\theta\right)$$

$$= \frac{W}{3}(b+a\cos\theta)$$

$$\Rightarrow W_1 \le \frac{-W}{3}\left(\frac{b}{a}\sec\theta+1\right) \quad (05)$$

$$\Rightarrow W_1 \le \frac{-W}{3}\left[\frac{b}{a}\left(\frac{2ab}{a^2-b^2-c^2}\right)+1\right]$$

$$\Rightarrow W_1 \le \frac{W}{3}\left(\frac{a^2+3b^2-c^2}{c^2-a^2-b^2}\right) \quad (10)$$

$$\underset{(05)}{C}\ R.b + W_1.a(-\cos\theta) - W\left(b-\frac{2}{3}AD\cos\alpha\right)=0$$

(05) (05)

now $R \ge 0$ (05)

$$\cos\alpha - \left(\frac{b-\frac{a}{2}\cos\theta}{AD}\right) \quad (05)$$

[50]

(08) **(a)** If A.B independent then

$$p(A\cap B) = P(A)\times P(B) \quad (05)$$ [05]

$$P(A\cup B) = P(A)+P(B)-P(A\cap B) \quad (05)$$ [05]

$u = x + y : 0 + 0,\ 0 + 1,\ 1,\ 0,\ 1 + 1$

"$u = 0$": $A\cap B$

"$u = 1$": $\left(A\cap B'\right)\cup\left(A'\cap B\right)$

"$u = 2$": $A'\cap B'$

(i) $P(u=0) = P(A\cap B) = P(A)\times P(B)$

$$= \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$$

$$P(u=1) = P\left(A \cap B'\right) \cup \left(A' \cap B\right)$$

$$= P\left(A \cap B'\right) + P\left(A' \cap B\right) + P(\phi)$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{2} \quad (05)$$

$$P(u=2) = P\left(A' \cap B'\right)$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4} \quad (05)$$

10

(ii) V = XY

V = 0, (0, 0) (1, 0) (0, 1)

$Pr = \frac{3}{4}$ (05)

V = 1 : (1, 1)

$Pr = \frac{1}{4}$ (05)

10

(b) P(x/A) = 0.2 ; P(x/B) = 0.8

P(A) = 0.4 ; P(B) = 0.6

(05) (05)

P(x) = P(x/A) P(A) + P(x/B) P(B) (10)

= 0.2 x 0.4 + 0.8 x 0.6 (05)

= 0.08 + 0.48

= 0.56 (05)

30

$$P(B/x) = \frac{P(x/B)P(B)}{P(x)} \quad (15)$$

$$P(B/x) = \frac{0.8 \times 0.6}{0.56}$$

$$= \frac{6}{7} \quad (05)$$

20

Presence of illness x has increased the probability of symptom B. (10)

10

$$P(B/x) = \frac{6}{7} = 0.86$$

Re*asons* $P(B/x) > P(B)$

(05) (05)

10

(09) Mean = $\frac{\text{Sum of observations}}{\text{No. of observations}}$ (05)

Median = Middle Value

(for odd no. of observations)

$= \frac{1}{2}$ (Sum of two middle values)

(for even no. of observations) (05)

When arranged in as cending order

Mode = Most frequently occuring value. (05)

15

$$S^2 = \frac{1}{N}\left[\sum_{i=1}^{N} x_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} x_i\right)^2\right]$$

$d_i = x_i - x, \qquad i = 1, 2, \ldots\ldots N$

$$\sum_{i=1}^{N} d_i^2 = \sum_{i=1}^{N} (x_i - \bar{x})^2$$

$$= \sum_{i=1}^{N} \left(\bar{x}_i^2 - 2x_i\bar{x} + \bar{x}^2\right)$$

$$= \sum_{i=1}^{N} \bar{x}_i^2 - 2\bar{x} \sum_{i=1}^{N} x_i + N\bar{x}^2$$

$$= \sum_{i=1}^{N} x_i^2 - \frac{2\sum_{i=1}^{N} x_i \sum_{i=1}^{N} x_i}{N} + N\frac{\left[\sum_{i=1}^{N} x_i\right]^2}{N^2}$$

$$= \sum_{i=1}^{N} x_i^2 - 2\frac{\left[\sum_{i=1}^{N} x_i\right]^2}{N} + \frac{\left[\sum_{i=1}^{N} x_i\right]^2}{N}$$

$$= \sum_{i=1}^{N} x_i^2 - \frac{\left(\sum_{i=1}^{N} x_i\right)^2}{N}$$

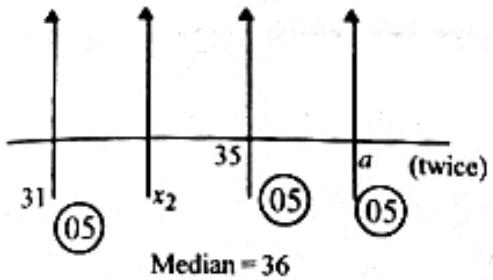
(10)

$$= \frac{\sum_{i=1}^{N} d_i^2}{N} = S^2$$

10

Let mode = a

Since the number of observations is odd then the median is the middle value = 36



(05) (05) (05)

Median = 36

Futher, mode $\neq$ median (=36) there fore there are two value to left including minimum value (=31) and remainning two values lye to the right.

$$\sum_{i=1}^{5} x_i = 5\bar{x} = 5 \times 35 = 175 \quad (05)$$

$\therefore \quad 31 + x_2 = 175 - 36 - 2a$

$\therefore \quad 2a + x_2 = 108 \quad (05)$

	a	x_2	
(i)	37	34	(05)
(ii)	38	32	

$\therefore \quad x_2 > 30$ only (i) , (ii) are possible

$\therefore$ Two possible sets are

(A) : 31 34 36 37 37 (05)

(B) : 31 32 36 38 38 (05) 40

Corresponding d_i . S are

(A) : -4 -1 1 2 2

(B) : -4 -3 1 3 3

$$S^2 = \frac{\sum_{i=1}^{5} d_i^2}{5}$$

For (A) : $\frac{26}{5} = 5.2$ (05)

For (B) : $\frac{44}{5} = 8.8$ (05)

$\therefore$ Correct set is (A) 10

Coefficient of skewness = $\frac{\text{mean - mode}}{\text{S.D.}}$ OR

= $\frac{3(\text{mean - median})}{\text{S.D.}}$

$= \frac{35.37}{\sqrt{5.2}}$ OR $\frac{3(35-36)}{\sqrt{5.2}}$

$= \frac{-2}{\sqrt{5.2}}$ OR $\frac{-3}{\sqrt{5.2}}$ (05)

$y_i = 55 - x_i \qquad i = 1, 2, 3, 4, 5 \rightarrow (01)$

$$\sum y_i = \sum 55 - \sum x_i$$

Dividing by 5. $\bar{y} = 55 - \bar{x} \rightarrow (02)$ (05)

(01) − (02) $y_i - \bar{y} = -(x_i - \bar{x}) = -d_i$ (05)

$$S_i^2 = \sum \frac{(-d_i)^2}{5} = \sum \frac{d_i^2}{5} = S^2 \quad (05)$$

Skewness = $\frac{\text{Mean - Mode}}{\text{S.D}}$

$= \frac{+2}{\sqrt{5.2}}$ or $\frac{+3}{\sqrt{5.2}}$

$\therefore$ Mode and Median are mirror images as compared with "ages" data set. (05)

25