G.C.E. (Advanced Level) Examination - April 2002
10 - Combined Mathematics - I
Provisional Scheme of Marking

(01) (i) α and β are roots of $f(x) = 0 \Rightarrow \alpha + \beta = -2$ and $\alpha\beta = 9$ (05)

Let $\alpha^2 - 1 = \gamma$ and $\beta^2 - 1 = \delta$

Then $\gamma + \delta = (\alpha^2 - 1) + (\beta^2 - 1)$
$= \alpha^2 + \beta^2 - 2$ (05)
$= (\alpha + \beta)^2 - 2\alpha\beta - 2$
$= 4 - 18 - 2$
$= -16$ (05)

and $\gamma\delta = (\alpha^2 - 1)(\beta^2 - 1)$
$= \alpha^2\beta^2 - (\alpha^2 + \beta^2) + 1$
$= (\alpha\beta)^2 - [(\alpha + \beta)^2 - 2\alpha\beta] + 1$ (05)
$= 81 - [4 - 18] + 1$
$= 96$ (05)

$\therefore$ The quadratic equation having γ and δ as roots is
$(x - \gamma)(x - \delta) = 0$

i.e. $x^2 - (\gamma + \delta) + \gamma\delta = 0$
$\Rightarrow x^2 + 16x + 96 = 0$ (05)

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(ii) $f(x) = k$ or $x^2 + 2x + 9 = k$
or $x^2 + 2x + (9 - k) = 0$ (05)
has exactly one real root if and only if
$4 - 4(9 - k) = 0$ i.e. $k = 8$ (05)
(10)

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(iii) $\frac{1}{f(x)} = \frac{1}{x^2 + 2x + 9} = \frac{1}{(x+1)^2 + 8}$ (05)

Take the greatest value when $x + 1 = 0$ i.e. When $x = -1$
(05) (05)

$\therefore$ the greatest value is $\frac{1}{8}$ (10)

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(iv) $f(x) = \lambda x$ or $x^2 + 2x + 9 = \lambda x$ (05)
or $x^2 + (2 - \lambda)x + 9 = 0$
has noreal solution if and only if
$(2 - \lambda)^2 - 36 < 0$ (10)
i.e. $(\lambda + 4)(8 - \lambda) > 0$ (05)
Required set of values $= (\lambda \in \mathbb{R} : -4 < \lambda < 8)$ (05)

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(02) (*a*) Number of ways the team can be selected
$= {}^{12}C_4 = 495$
(05) (05)

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(i) Number of ways the team can be selected if both Anura and Bhavan are in the team (05)
$= {}^{10}C_2 = 45$ (05)
(05)

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(ii) Number of ways the can be selected. if either Anura or Bhavan is in the team. (05)
$= {}^{10}C_3 + {}^{10}C_3 = 2 \times 120 = 240$ (05)
(10)

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(iii) Number of ways the team can be selected. If neither Anura nor Bhavan is in the team. (05)
$= {}^{10}C_4 = 210$ (05)
(05)

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(b) (i) $\left(\frac{7}{6x} - \frac{6x}{7}\right)^{13} = \sum_{r=0}^{13} {}^{13}C_r \left(\frac{7}{6x}\right)^{13-r} \left(-\frac{6x}{7}\right)^r$ (05)

$= \sum_{r=0}^{13} {}^{13}C_r (-1)^r \left(\frac{7}{6}\right)^{13-2r} x^{2r-13}$ (05)

Now, for each integer r, 2r - 13 is odd (05)
Hence the expansion does not contain even power of x or even powers of $\frac{1}{x}$ (05)

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(ii) In the expansion, the term $\frac{1}{x}$ is obtained when

$2r - 13 = -1$ or $r = 6$ (05)

Hence the coefficient of $\frac{1}{x} = {}^{13}C_6(-1)^6\left(\frac{7}{6}\right)^{13-12}$ (05)

$$= \frac{13!}{7!\ 6!}\frac{7}{6}$$

$$= \frac{13\times12\times11\times10\times9\times8\times7}{6\times5\times4\times3\times2\times1\times6}$$ (05)

$$= 13\times2\times11\times7 = 2002$$ (05)

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(03) (a) For each positive integer n, let $S(n)$: $n! \geq 2^{n-1}$

Note that $1! = 1$ and $2^{1-1} = 2^0 = 1$ Hence S(1) is true. (05)

Assume S(k) is true, i.e. $k! \geq 2^{k-1}$, (05)

Now $(k+1)! = k!(k+1) \geq 2^{k-1}(k+1) \geq 2^{k-1}\cdot 2 = 2^k$ (05)

as $k \geq 1 \Rightarrow k+1 \geq 2$

Hence $(k+1)! \geq 2^{(k+1)-1}$ and S(k + 1) true. (05)

∴ By principle of Mathematical Induction, S(n) is true for each positive integer. (05)

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Note that for each n, $\frac{1}{n!} \leq \frac{1}{2^{n-1}}$ (05)

$$\therefore \sum_{k=1}^{n}\frac{1}{k!} \leq \sum_{k=1}^{n}\frac{1}{2^{k-1}} = \frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}} = 2-2\left(\frac{1}{2}\right)^n = 2-\frac{1}{2^{n-1}}$$ (05) (05)

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Now $e = \lim_{n\to\infty}\sum_{k=0}^{n}\frac{1}{k!} = \lim_{n\to\infty}\left[1+\sum_{k=1}^{n}\frac{1}{k!}\right]$ (05)

$$= 1 + \lim_{n\to+\infty}\sum_{k=1}^{n}\frac{1}{k!}$$ (05)

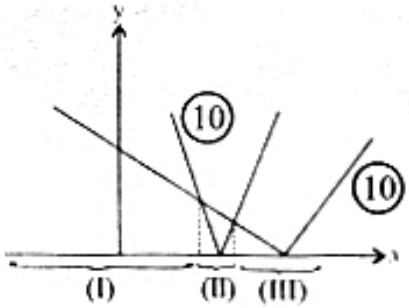
$$\leq 1 + \lim_{n\to+\infty}\left(2-\frac{1}{2^{n-1}}\right)$$

$$= 1 + 2 - \lim_{n\to+\infty}\left(\frac{1}{2^{n-1}}\right)$$

$$\leq 3$$ (05)

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(b)



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Observe that for arbitrary a and inequality of the given type holds either on I ∪ III or on II.

i.e. either on union of two disjoint infinite intervals or on a finite interval.

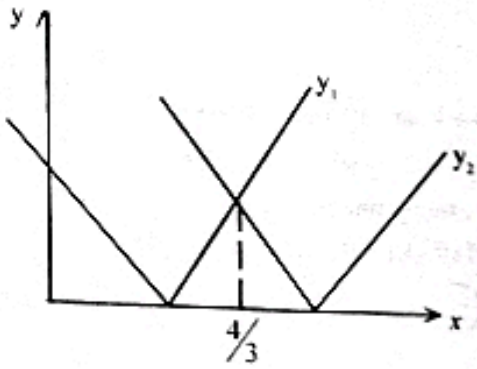
However, it is given that the inequality holds on

$\left(\frac{4}{3}, +\infty\right)$

This is possible only if each of the two segments is parallel. as indicated, where (05)

$y_1 = |bx - 2|$ and $y_2 = |3x - a|$

Then $b = 3$ (05)



Now,

y_1 and y_2 intersect at $x = \frac{4}{3}$ (05)

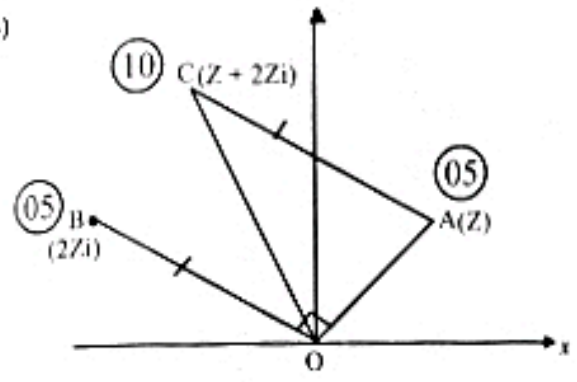
i.e. $bx - 2 = -(3x - a)$ at $x = \frac{4}{3}$

$a = (b + 3)x - 2$ at $x = \frac{4}{3}$

$a = (3+3)\times\frac{4}{3} - 2 = 6$ (05)

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(04)



The effect of multiplying z by i is to turn OZ through a right angle to a new position OZ,

Where Z^1 = Zi O(2Zi) is obtained by extending O(2i) to twice its length without any rotation.

C is an shown. AC is equal and parallel to OB [20]

$\hat{AOB} = \frac{\pi}{2}$ (05) *and* $\tan \hat{AOC} = \frac{AC}{OA} = \frac{2}{1} = 2$ (05) [10]

(i) If C lies on the imaginary axis

$\arg(z) = \frac{\pi}{2} - \hat{AOC}$ (05)

i.e. $\frac{y}{x} = \tan\left(\frac{\pi}{2} - \hat{AOC}\right) = \frac{1}{2}$ (05) (05)

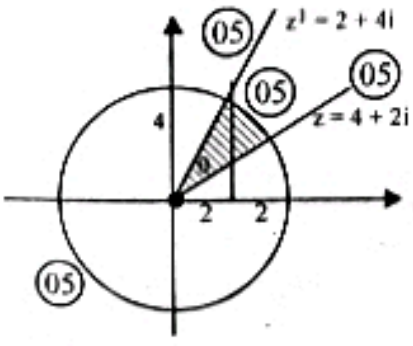
$\therefore \quad x - 2y = 0$ (05) [20]

(ii) If $y = 2x$, $\hat{XOA} = \hat{AOC}$ (05)

i.e. $arg(z + 2iz) = 2arg\, z = \arg z^2$ (05) (05)

$\therefore$ z^1 lies on OC (05) [20]

(iii)



[20]

$Area = \left\{\frac{1}{4}(4)^2\theta\right\} = 8\theta$, *where* $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ (05) (05)

θ can be obtained by, either

$$\tan\theta = \frac{2 - \frac{1}{2}}{1 + 2\cdot\frac{1}{2}} = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow \theta = \tan^{-1}\left(\frac{3}{4}\right)$$

$$\therefore \quad \frac{z^1}{z} = \frac{2 + 4i}{4 + 2i} = \frac{1 + 2i}{2 + i}$$

$$= \frac{(1 + 2i)(2 - i)}{(2 + i)(2 - i)}$$

$$= \frac{4 + 3i}{5}$$

$\arg\left(\frac{z^1}{z}\right) = \theta$, *where* $\tan\theta = \frac{3}{4}$ [10]

(05) (*a*) $y = e^{4x}\sin 3x$

$$\Rightarrow \frac{dy}{dx} = e^{4x}(\cos 3x)3 + (\sin 3x)e^{4x}.4$$ (05)

$$= 3e^{4x}\cos 3x + 4y \quad \rightarrow (01)$$ (05)

$$\frac{d^2y}{dx^2} = 3e^{4x}(-\sin 3x) + 3 + 3\cos 3x\, e^{4x}\cdot 4 + 4\frac{dy}{dx}$$ (05)

$$= -9y + 4\left(\frac{dy}{dx} - 4y\right) + 4\frac{dy}{dx}$$ (05)

$$\Rightarrow \frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 25y = 0 \quad \rightarrow (02)$$ (05) [25]

Note that $y(0) = 0$ (05)

From (i) $\left(\frac{dy}{dx}\right)_{x=0} = 3 + 4y(0) = 3$ (05) [10]

From (ii) $\left(\frac{d^2y}{dx^2}\right)_{x=0} = 8\left(\frac{dy}{dx}\right)_{x=0} - 25y(0) = 8\times 3 - 0 = 24$ (05)

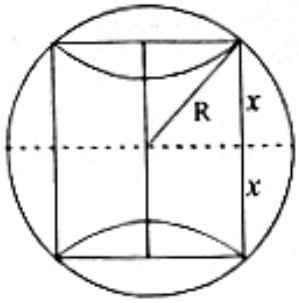
[05]

Differentiating (ii), we set $\frac{d^3y}{dx^3} = 8\frac{d^2y}{dx^2} - 25\frac{dy}{dx}$ (05)

Then $\left(\frac{d^3y}{dx^3}\right)_{x=0} = 8\left(\frac{d^2y}{dx^2}\right)_{x=0} - 25\left(\frac{dy}{dx}\right)_{x=0}$

$= 8\times 24 - 25\times 3$

$= 117$ (05) [10]



Let R be the radius of sphere and let $2x$ be the height of the cylinder. Then the volume of the cylinder. (10)

$V(x) = \pi(R^2 - x^2)\ 2x$

$= 2\pi(R^2 - x^2)\,x$ (10)

Now $V'(x) = 2\pi\,[R^2 - x^2 + x(-2x)]$

$= 2\pi[R^2 - 3x^2]$ (05)

$= 6\pi\left[\frac{R^2}{3} - x^2\right] = 6\pi\left[\frac{R}{\sqrt{3}} - x\right]\left[\frac{R}{\sqrt{3}} + x\right]$ (05)

Note that $V'(x) > 0$, when $x < \frac{R}{\sqrt{3}}$ and $V'(x) < 0$,

When $x > \frac{R}{\sqrt{3}}$ (05)

Hence V(x) has a maximum at $x = \frac{R}{\sqrt{3}}$ (05) and the maximum

valume $= V\left(\frac{R}{\sqrt{3}}\right) = 2\pi\left(R^2 - \frac{R^2}{3}\right)\frac{R}{\sqrt{3}}$

$= \frac{4\pi R^3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}\left(\frac{4\pi R^3}{3}\right)$ (05)

$= \frac{1}{\sqrt{3}}S$, *Where* $S = \frac{4}{3}\pi R^3$ (05)

is the volume of the sphere.

Hence the volume of the cylinder cannot exceed $\frac{1}{\sqrt{3}}$ times that of the sphere. [50]

(06) (*a*) Method I : (05)

Let $x^2 - 1 = u$ Then $2xdx = du$, (05)

When $x = 1$, $u = 0$ and when $x = 2$, $u = 3$ (05)

$\int_1^2 \frac{x^3}{\sqrt{x^2-1}}dx = \int_0^2 \frac{x^2(xdx)}{\sqrt{x^2-1}} = \frac{1}{2}\int_0^3 \frac{(1+u)du}{\sqrt{u}}$ (05)

$= \frac{1}{2}\int_0^3 \left[\frac{1}{\sqrt{u}} + \sqrt{u}\right]du$

$= \frac{1}{2}\left[\frac{-u\frac{1}{2}}{\frac{1}{2}} + \frac{u\frac{3}{2}}{\frac{3}{2}}\right]_0^3$ (05)

$= \left[u^{\frac{1}{2}} + \frac{1}{3}u^{\frac{3}{2}}\right]_0^3$

$= \sqrt{3} + \frac{1}{3}\cdot 3\sqrt{3}$

$= 2\sqrt{3}$ (05) [30]

Method II :

Let $x = \sec\theta$ Then $dx = \sec\theta\tan\theta\,d\theta$, (05)

When $x = 1$, $\theta = 0$ and when $x = 2$, $\theta = \frac{\pi}{3}$ (05)

$\int_1^2 \frac{x^3}{\sqrt{x^2-1}}dx = \int_0^{\pi/3} \frac{\sec^3\theta\sec\theta\tan\theta}{\tan\theta}d\theta = \int_0^{\pi/3} \sec^4\theta\cdot d\theta$ (05)

$= \int_0^{\pi/3} (1+\tan^2\theta)\,d(\tan\theta) = \left[\tan\theta + \frac{\tan^3\theta}{3}\right]_0^{\pi/3}$ (05) (05)

$= \sqrt{3} + \frac{3\sqrt{3}}{3}$

$= 2\sqrt{3}$ (05) [30]

(b) $\int_0^1 x\tan^{-1}x\,dx = \int_0^1 \tan^{-1}x\,\frac{d}{dx}\left(\frac{x^2}{2}\right)dx$ (05)

$= \left[\left(\tan^{-1}x\right)\frac{x^2}{2}\right]_0^1 - \frac{1}{2}\int_0^1 x^2\frac{1}{1+x^2}dx$ (05)

$= \frac{\pi}{8} - \frac{1}{2}\int_0^1 \frac{x^2}{1+x^2}dx$

$$= \frac{\pi}{8} - \frac{1}{2}\int_0^1 \frac{x^2+1-1}{1+x^2}\,dx \quad (05)$$

$$= \frac{\pi}{8} - \frac{1}{2}\int_0^1 dx + \frac{1}{2}\int_0^1 \frac{1}{1+x^2}\,dx$$

$$= \frac{\pi}{8} - \frac{1}{2}[x]_0^1 + \frac{1}{2}\left[\tan^{-1}x\right]_0^1 \quad (05)\ (05)$$

$$= \frac{\pi}{8} - \frac{1}{2} + \frac{1}{2}\cdot\frac{\pi}{4}$$

$$= \frac{\pi}{4} - \frac{1}{2} \quad (05)$$

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(c) $$\frac{5x-4}{(2+x)(1-x+x^2)} = \frac{A}{2+x} + \frac{Bx+C}{1-x+x^2} \quad (05)$$

$\Rightarrow A = -2,\ B = 2,\ C = -1$ (10)

$$\therefore \int_1^2 \frac{5x-4}{(1-x+x^2)(2+x)}\,dx$$

$$= \int_1^2 \frac{2x-1}{1-x+x^2}\,dx - 2\int_1^2 \frac{dx}{x+2} \quad (05)$$

$$= \left[\ell n\left|1-x+x^2\right|\right]_1^2 - 2\left[\ell n|x+2|\right]_1^2 \quad (05)\ (05)$$

$$= (\ell n3 - \ell n1) - 2(\ell n4 - \ell n3)$$

$$= 3\ell n3 - 2\ell n4 \quad (05)$$

$$= \ell n\frac{27}{16}$$

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(07) Let $P(x_0, y_0)$ be the point of intersection of two non parallel straight lines $U_1 = 0$ and $U_2 = 0$

Then $a_1x_0 + b_1y_0 + c_1 = 0$ and

$a_2x_0 + b_2y_0 + c_2 = 0$ (05)

Note that

$$0 = u_1 + \lambda u_2 = (a_1x + b_1y + c_1) + \lambda(a_2x + b_2y + c_2) \quad (05)$$
$$= (a_1 + \lambda a_2)x + (b_1 + \lambda b_2)y + (c_1 + \lambda c_2)$$

denotes a straight line for every value of λ. (05)

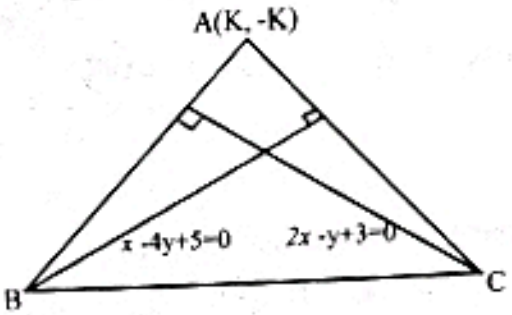
also, $(a_1 + \lambda a_2)x_0 + (b_1 + \lambda b_2)y_0 + (c_1 + \lambda c_2) = 0$

$(a_1x_0 + b_1y_0 + c_1) + \lambda(a_2x_0 + b_2y_0 + c_2) = 0 + \lambda 0 = 0$ (05)

implies that the straight line $u_1 + \lambda u_2 = 0$ (05)

Passes through $P(x_0y_0)$ for every value of λ.

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AB is perpendicular to

$2x - y + 3 = 0 \Rightarrow$ AB has the form $x + 2y + a = 0$ (05)

AB passes through $A(k, -k) \Rightarrow a = -(k - 2k) = k$

Equation of AB is $x + 2y + k = 0$ (05)

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AC is perpendicular to $x - 4y + 5 = 0 \Rightarrow$ AC has the form $4x + y + b = 0$ (05)

AC Passes through $A(k, -k) \Rightarrow b = -(4k - k) = -3k$

Equation of AC is $4x + y - 3k = 0$ (05)

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By Solving $2x - y + 3 = 0$ and $4x + y - 3k = 0$ (05)

We get $C = \left[\frac{(k-1)}{2}, k+2\right]$ (05) (05)

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(05)

and by Solving $x - 4y + 5 = 0$ and $x + 2y + k = 0$

We get $B = \left[-\left(\frac{2k+5}{3}\right), \frac{5-k}{6}\right]$ (05) (05)

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Let $G = (\bar{x}, \bar{y})$ be the centroid.

(05)

Then $\bar{x} = \frac{1}{3}\left[k - \frac{(2k+5)}{3} + \frac{(k-1)}{2}\right] = \frac{5k-13}{18}$

$$\text{and} \quad \bar{y} = \frac{1}{3}\left[-k + \left(\frac{5-k}{6}\right) + (k+2)\right] = \frac{17-k}{18} \quad (05)$$

$$\bar{x} + 5\bar{y} = \frac{5k-13}{18} + 5\frac{(17-k)}{18}$$

$$= \frac{1}{18}(5k - 13 + 85 - 5k) = 4 \quad (10)$$

$\therefore$ G lies on the line $x + 5y - 4 = 0$ (05) [25]

(08) Condition that they should touch is

$C_1C_2 = |r_1 \pm r_2|$ (05)

depending on whether they touch externally or internally i.e.

$$\sqrt{(g_1 - g_2)^2 + (f_1 - f_2)^2}$$

$$= \left|\sqrt{g_1^2 + f_1^2 - c_1} \pm \sqrt{g_2^2 + f_2^2 - c_2}\right| \quad (10)$$ [15]

If they touch then the point of contact $(x_0\ y_0)$ lies on the line of centres $C_1\ C_2$ (05)

Given by,

$$y - (-f_1) = \frac{(-f_1) - (-f_2)}{(-g_1) - (-g_2)}[x - (-g_1)] \quad (05)$$

i.e. $(g_2 - g_1)y + f_1g_2 = (f_2 - f_1)x + f_2g_1$

i.e. $(f_1 - f_2)x - (g_1 - g_2)y + f_1g_2 - f_2g_1 = 0$ [10]

Also,

$x_0^2 + y_0^2 + 2g_1x_0 + 2f_1y_0 + c_1 = 0$ and

$x_0^2 + y_0^2 + 2g_2x_0 + 2f_2y_0 + c_2 = 0$ (05)

$\Rightarrow \quad 2(g_1 - g_2)x_0 + 2(f_1 - f_2)y_0 + c_1 - c_2 = 0$ (05)

i.e. the point of contact lies on the line.

$2(g_1 - g_2)x + 2(f_1 - f_2)y_0 + c_1 - c_2 = 0$ [10]

(05)

$$C_1C_2 = \sqrt{\{(+1) - 5\}^2 + \{(-2) - (0)\}^2} = \sqrt{20} = 2\sqrt{5} \quad (05)$$

$$r_1 = \sqrt{(-1)^2 + 2^2 - 0} = \sqrt{5} \quad \text{and} \quad (05)$$

$$r_2 = \sqrt{(-5)^2 - 20} = \sqrt{5} \quad (05)$$

$\therefore \quad r_1 + r_2 = 2\sqrt{5}$ and $C_1C_2 = r_1 + r_2$ (05)

Hence circles touch externally. (05)

Since the radial are equal, the point of contact is the mid point of C_1C_2

$$\left[\frac{(1)+(-5)}{2}_{(05)}, \frac{(-2)+(0)}{2}_{(05)}\right] = (3, -1)$$ [40]

Let P(a variable point) be denoted by (x, y)

Length of the tangent from (x, y) to the first circle is $\sqrt{x^2 + y^2 - 2x + 4y}$ and to the second circle is $\sqrt{x^2 + y^2 - 10x + 20}$ (05)

$$\sqrt{x^2 + y^2 - 2x + 4y} = k\sqrt{x^2 + y^2 - 10x + 20} \quad (05)$$

$$\Leftrightarrow \quad (x^2 + y^2 - 2x + 4y) = k^2(x^2 + y^2 - 10x + 20)$$

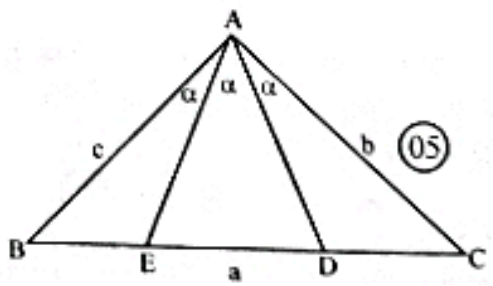
$$\Leftrightarrow (k^2 - 1)(x^2 + y^2) - (2k^2 - 10)x + 4k^2y - 20 = 0 \quad (05)$$

$\therefore$ if $k^2 - 1 \neq 0$, the locus of P is a circle through A. (05)

Its equation is

$$x^2 + y^2 - \frac{(2k^2 - 10)}{k^2 - 1}_{(05)}x + \frac{4k^2}{(k^2 - 1)}y - \frac{20}{(k^2 - 1)} = 0$$ [25]

(09)



(05)

$$BD = DC = \frac{a}{2}$$

Sine rule for ABD Δ

$$\frac{BD}{\sin 2\alpha} = \frac{AB}{\sin \hat{ADB}} \quad (05)$$

$$\Rightarrow \sin A\hat{D}B = \frac{AB}{BD}\sin 2\alpha$$

Sine rule for ADC $\Delta \Rightarrow \frac{AC}{\sin A\hat{D}C} = \frac{DC}{\sin\alpha}$

$$\Rightarrow \sin A\hat{D}C = \frac{AC}{DC}\sin\alpha \quad (05)$$

$$\sin A\hat{D}B = \sin A\hat{D}C \Rightarrow \frac{AC}{DC}\sin\alpha \overset{(05)}{=} \frac{AB}{BD}\sin 2\alpha$$

$$\Rightarrow \cos\alpha = \frac{AC}{2AB} = \frac{b}{2C} \quad (05)$$

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$DE : EB = 1 : k \Rightarrow DE = \frac{1}{1+k}BD$ and

$$\Rightarrow EB = \frac{k}{1+k}BD$$

$$\Rightarrow DE = \frac{a}{2(1+k)} \text{ and } EB = \frac{ak}{2(1+k)} \quad (05)$$

Also,

$$EC = DE + DC = \frac{a(2+k)}{2(1+k)} \quad (05)$$

Sine rule for AEC Δ

$$\Rightarrow \frac{EC}{\sin 2\alpha} = \frac{AC}{\sin A\hat{E}C} \overset{(05)}{\Rightarrow} \sin A\hat{E}C = \frac{AC}{EC}\sin 2\alpha$$

Sine rule for AEB Δ

$$\Rightarrow \frac{BE}{\sin\alpha} = \frac{AB}{\sin A\hat{E}B} \overset{(05)}{\Rightarrow} \sin A\hat{E}B = \frac{AB}{BE}\sin\alpha$$

$$\sin A\hat{E}C = \sin A\hat{E}B \Rightarrow \frac{AC}{EC}\sin 2\alpha = \frac{AB}{BE}\sin\alpha \quad (05)$$

$$\Rightarrow \cos\alpha = \frac{1}{2}\frac{AB}{BE}\cdot\frac{EC}{AC} = \frac{1}{2}\frac{2C(1+k)}{ak}\cdot\frac{a(2+k)}{2b(1+k)}$$

$$= \frac{(2+k)C}{2kb} \quad (05)$$

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If $k = 1$, $\cos\alpha = \frac{3c}{2b} = \frac{b}{2c} \Rightarrow b^2 = 3c^2 \Rightarrow \left(\frac{b}{2c}\right)^2 = \frac{3}{4}$ (05)

$$\Rightarrow \cos\alpha = \frac{b}{2c} = \frac{\sqrt{3}}{2} \Rightarrow \alpha = \frac{\pi}{6} \quad \therefore A = 3\alpha = 90^0 \quad (05)$$

If $k = 2$, $\cos\alpha = \frac{4c}{4b} = \frac{b}{2c} \Rightarrow b^2 = 2c^2 \Rightarrow \left(\frac{b}{2c}\right)^2 = \frac{1}{2}$ (05)

$$\Rightarrow \cos\alpha = \frac{b}{2c} = \frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{\pi}{4} \quad (05)$$

$$\therefore \quad A = 3\alpha = 135^0$$

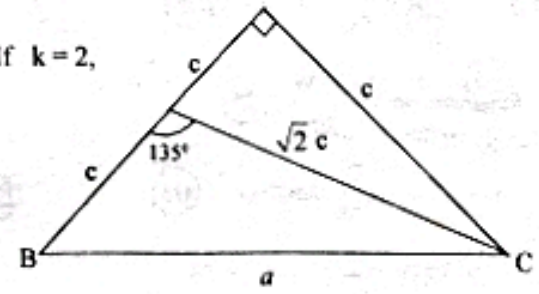
20

If $k = 1$,

$a^2 = b^2 + c^2 = 3c^2 + c^2 = 4c^2 \Rightarrow c = \frac{a}{2}$, $b = \frac{\sqrt{3}}{2}a$ (05)

(05)

If $k = 2$,



$$a^2 = (AC)^2 + (AB)^2 + 2(AB)(AC)\cos 45^0 \quad (05)$$

$$= 2C^2 + C^2 + 2\sqrt{2}C^2 \cdot \frac{1}{\sqrt{2}} = 5C^2$$

$$C^2 = \frac{a^2}{5}$$

$$\therefore C = \frac{a}{\sqrt{5}}, \quad b = \sqrt{\frac{2}{5}}a \quad (05)$$

(05)

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