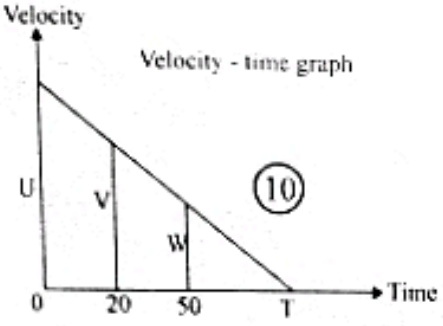
G.C.E. (Advanced Level) Examination - April 2002
Combined Mathematics - II
Provisional Scheme of Marking

(01) (i)



10

$\left(\frac{U+V}{2}\right)\times 20 = 750;\ U+V=75 \rightarrow (01)$ (05)

$\left(\frac{W+V}{2}\right)\times 30 = 750;\ W+V=50 \rightarrow (02)$ (05)

(01) - (02) $U - W = 25 \rightarrow (03)$ (05)

$$Retardation = \frac{U-W}{50}$$ (05)

$$= \frac{25}{50} = 0.5 ms^{-2}$$ (05) 25

Also, $\left(\frac{U+W}{2}\right)\times 50 = 1500,\ U+W=60 \rightarrow (04)$ (05)

$(03)\ \&\ (04) \Rightarrow W = 17.5 ms^{-2}$ (05) 10

$$\frac{W}{T-50} = 0.5$$ (05)

$$T - 50 = \frac{17.5}{0.5} = 35$$

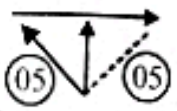
Value of t when train comes to rest is $50 + 35 = 85S$ (05) 10

(b) Vel C_1, relative to E = $\overrightarrow{V}$ (05) Vel. W, C_1 = ↑ (05)

Vel C_2, E = $\overrightarrow{2V}$ (05) Vel W, C_2 = 45° (05)

Adding vectorially, Vel W, E = WC + C, E (in both cases)

(Two velocity - traingles) (05)



Magnitude of Vel, W, E = $\sqrt{2}V$ (05)

Direction of wind is from SW (or towards NE) (05) 45

(02) (a) After time T, $x = u\cos\alpha$. (05) $x = (u\cos\alpha)T$ (05)

$\dot{y} = u\sin\alpha - gT$, (05) $y = (u\sin\alpha)T - \frac{1}{2}gT^2$ (05)

20

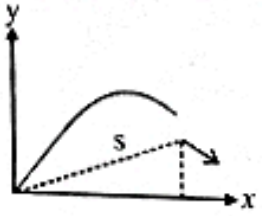
$\therefore$ direction of motion $\perp$ to the direction of projection

$$\frac{\dot{y}}{\dot{x}} = -\frac{\cos\alpha}{\sin\alpha}$$ (05)

$$\Rightarrow (u\sin\alpha - gT)\sin\alpha = -\cos\alpha(u\cos\alpha)$$ (05)

$$\Rightarrow gT\sin\alpha = u(\sin^2\alpha + \cos^2\alpha)$$ (05)

$$T = \frac{u}{g\sin\alpha}$$ (05) 20



$$S^2 = x^2 + y^2$$ (05)

$$= \begin{pmatrix} u^2T^2\cos^2\alpha + u^2T^2\sin^2\alpha \\ -gT^3 u\sin\alpha + \frac{1}{4}g^2T^4 \end{pmatrix}$$ (05)

$$= u^2T^2 - T^2u^2 + \frac{1}{4}g^2T^4$$ (05)

$$S = \frac{1}{2}gT^2$$ 15

(b) $54 kmh^{-1} = \frac{54 \times 1000}{3600} = 15 ms^{-1}$; (05)

Tractive Force = Resistance = 50 x 300 = 15000N; (05)

Power H = Fv = 15000 x 15 = 225kw (05)

15

$15000 - 50 \times 250 = 250,000a$ (10)

$\Rightarrow a = \frac{2500}{250,000} = 0.01 ms^{-2}$ (05)

15

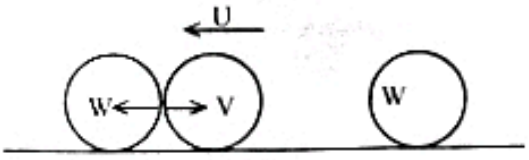
For the coach, loss in KE, $\frac{1}{2} mv^2$ = $R_1 x$ (= work done by resis tance)

$\frac{1}{2} \times 50,000 \times 15^2 = 50 \times 50x$ (10)

$\Rightarrow x = 2250m$ (05)

15

(03) 1st collision



Momentum conservation :

← $\lambda mw - mv = mu$

$\lambda w - v = u \rightarrow (01)$ (05)

Newton's law of restitution : $v + w = u \rightarrow (02)$ (05)

Solving (02) x λ - (01) $v(1 + \lambda) = u(\lambda - 1)$

Since $\lambda > 1$, Velocity of B after 1st collision is

$$v = \frac{(\lambda - 1)u}{1 + \lambda} \longrightarrow$$ (05)

Similarly, Velocity of B after 2nd collision is

$$v_1 = \left(\frac{\lambda - 1}{\lambda + 1}\right)^2 u \longleftarrow$$ (05)

20

Also, from (01) & (02) Velocity of A after 1st collision

$w = \frac{2u}{\lambda + 1}$ (05)

B will not strike A again if

$v_1 \le w \Rightarrow \left(\frac{\lambda - 1}{\lambda + 1}\right)^2 \le \frac{2}{\lambda + 1}$ (05)

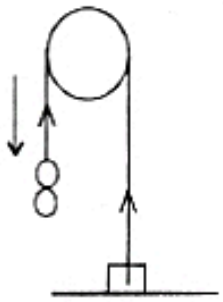
$\Rightarrow (\lambda - 1)^2 \le 2(\lambda + 1) \Rightarrow \lambda^2 - 4\lambda - 1 \le 0;$ (05)

Since $\lambda > 1$, the condition for no further impact becomes

$\lambda \le \frac{4 + \sqrt{16 + 4}}{2} = 2 + \sqrt{5}$ (05)

20

(b)



If J is the impulse in the string common speed V is given by equations:

J = 3m V, (05)

-J = 2mV - mu (05)

Where u is the freely falling mass just before collision

$\therefore u^2 = 2gh; \quad u = \sqrt{2gh}$ (05)

solving, $v = \frac{u}{5}$; $J = \frac{3mu}{5}$ (05) (05)

25

For the ensuing motion of the system, equations are

$3mg - T = 3mf$, (05) $T - 2mg = 2mf$ (05)

$\rightarrow$ *acceleration* $f = \frac{(3-2)mg}{3m+2m} = \frac{g}{5} \downarrow$ (05)

Maximum height reached by larger mass $= \frac{v^2}{2f}$ (05)

$= \frac{u^2/25}{2g/5} = \frac{h}{5}$ (05)

Time taken from the instant of jerk $= \frac{v}{f}$ (05)

$= \frac{u/5}{g/5} = \frac{u}{g}$ (05) [35]

(04) (*a*) Energy conservation Equation:

$\frac{1}{2}m(a\dot{\theta})^2 + mga\cos\theta = mga\cos\alpha$ (05)

$\Rightarrow a\dot{\theta}^2 = 2g(\cos\alpha - \cos\theta)$ (05)



↙ Equation of motion

$mg\cos\theta - R = ma\dot{\theta}^2$ (05)

$\Rightarrow R = mg(3\cos\theta - 2\cos\alpha)$; (05)

$R = 0$ *When* $3\cos\theta = 2\cos\alpha$ (05) [40]

(*b*)

$T^1 - T = m\ddot{x} \rightarrow (01)$ (10)

$T = mg\frac{(\ell + x)}{\ell}$ (05) $T^1 = mg\frac{(\ell - x)}{\ell}$ (05)

$\Rightarrow T - T^1 = \frac{2mgx}{\ell} \rightarrow (02)$

$(01) + (02) \Rightarrow \ddot{x} + \frac{2g}{\ell}x = 0; \ddot{x} + \omega^2 x = 0, \omega^2 = \frac{2g}{\ell}$ (05) (05) [30]

(05)

$\dot{x}^2 = \omega^2(\quad - x^2)$ Where a is the amplitude.

When $\quad = 0 \Rightarrow a = \ell$ (05)

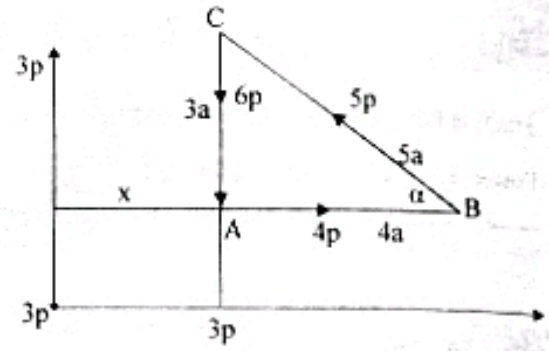
Center is $x =$ (mid-point of AB); Period

(05) $= \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{2g}{\ell}}} = \pi\sqrt{\frac{2\ell}{g}}$ (05)

Maximu speed $= \omega\ell\sqrt{2g\ell}$ (05)

Time to achieve this speed is $= \frac{\pi}{2\omega} = \frac{\pi}{2}\sqrt{\frac{\ell}{2g}}$ (05) [30]

(05)



→ $x = 4p - 5p\cos\alpha = 0$, Since $\cos\alpha = \frac{4a}{5a}$ (05) (05) (05)

↑ $y = -6p + 5p\sin\alpha = -3p$, Since $\sin\alpha = \frac{3}{5}$

Magnitude of the resultant is 3P; Direction is the same as that of $\overrightarrow{CA}$ (05) (05) [30]

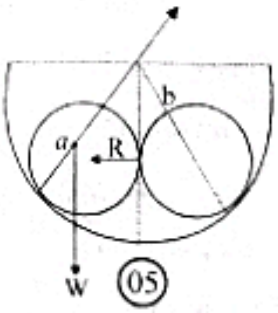
If the line of action of the resultant meets BA produced at X, where AX = *x* , then,

↶ X $(5p\sin\alpha) \times (x + 4a) - 6p \times x = 0$ (10)

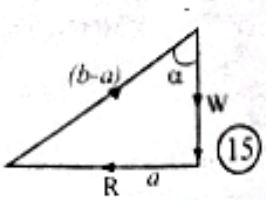
$\Rightarrow x + 4a - 2x = 0, \quad x = 4a$ (05)

For the new system, ↶ A $3p \times x + M = 0$ (05)

$\Rightarrow$ $M = -12pa$ clockwise (05) (05) [30]



(05)



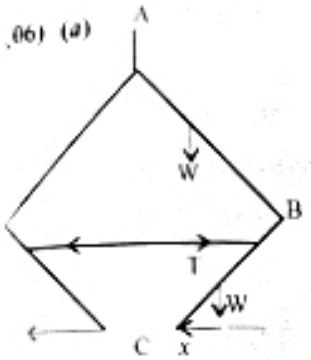
(15) [20]

$\sin\alpha = \frac{a}{b-a}$, (05) $\tan\alpha = \frac{a}{\sqrt{b(b-2a)}}$ (05)

$\frac{R}{\sin\alpha} = \frac{W}{\cos\alpha}$ (05)

$R = \frac{W\sin\alpha}{\cos\alpha} = W\tan\alpha = \frac{Wa}{\sqrt{b(b-2a)}}$ (05) [20]

(06) (a)



B, for BC:

$$\frac{T}{\sqrt{2}}+\frac{W}{\sqrt{2}}-\frac{2x}{\sqrt{2}}=0 \quad (10)$$

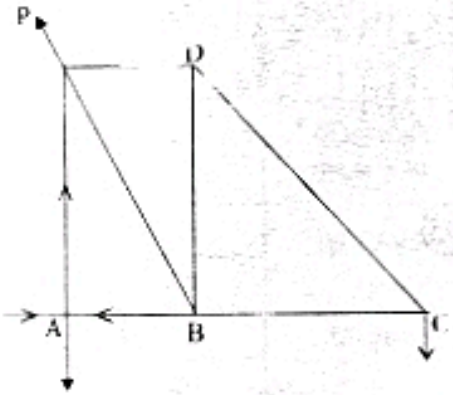
A, for AB, BC:

$$\frac{3T}{\sqrt{2}}-\frac{2W}{\sqrt{2}}-\frac{4x}{\sqrt{2}}=0 \quad (10)$$

$$\Rightarrow T=4W, \quad x=\frac{5W}{2} \quad (05)$$

(05)

30



A, $P\times\frac{a}{2}=400\left(\frac{a}{\sqrt{3}}+a\right)$ (05)

$\Rightarrow P=800\times\left(\frac{1}{\sqrt{3}}+1\right)$ N (05)

$\rightarrow x=\frac{P}{2}$ (05) $\downarrow y=\frac{P\sqrt{3}}{2}-400=400\sqrt{3}$ N (05)

Stress in AB = X - $400\left(\frac{1}{\sqrt{3}}+1\right)$ N (05)

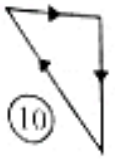
= Thrust (05)

Stress in AE = $y=400\sqrt{3}$ N (05)

= Tension (05)

40

For joint C



(05) (05)

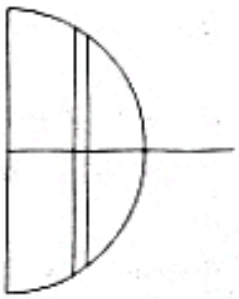
Stress in BC = 400N; Thrust

Stress in CD = 400N; Tension

(05) (05)

30

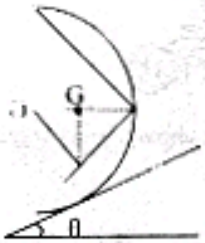
(07) $\bar{x}=\dfrac{\int_0^a \rho x\cdot\pi\left(a^2-x^2\right)dx}{\int_0^a \rho\pi\left(a^2-x^2\right)dx}$ (05) (05)



$=\dfrac{\rho\pi\left[a^2\cdot\frac{x^2}{2}-\frac{x^4}{4}\right]_0^a}{\rho\pi\left[a^2x-\frac{x^3}{3}\right]_0^a}$ (05) (05)

$=\dfrac{a\left[\frac{1}{4}\right]}{\left[\frac{2}{3}\right]}=\frac{3}{8}a$ (05)

25



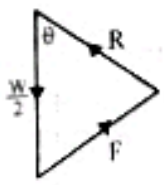
$\bar{x}=\dfrac{\frac{3a}{8}\cdot\frac{2}{3}\pi a^3\rho-\frac{a}{4}\cdot\frac{1}{3}\pi a^3\rho}{\frac{1}{3}\pi a^3\rho}$ (10) (10) (05)

$=\dfrac{\frac{a}{4}\left(1-\frac{1}{3}\right)}{\frac{1}{3}}=\frac{a}{2}$ (05) (05)

35

If OG is horiaontal, $\sin\theta=\frac{OG}{a}=\frac{1}{2}$; $\theta=30^0$ (05) (05)

10



Frictional force, $F=\frac{W}{2}\sin\theta=\frac{W}{4}$ (05) (05)

Normal reaction, $R=\frac{W}{2}\cos\theta=\frac{\sqrt{3}W}{4}$ (05) (05)

For equilibrium, $\frac{F}{R}\le\mu\Rightarrow\frac{1}{\sqrt{3}}\le\mu$ (05) (05)

30

(08) The two event A and B are independent.

Therefore $P(A \cap B) = P(A) \cdot P(B)$ →(01) (05)

Now, $P(B) - P(B)$ →(02)

(02) - (01) Gives $P(A' \cap B) = P(A') P(B)$ →(03) (05)

Hence A' and B are independent.

$P(A') = P(A')$ →(04)

(04) - (03) Gives $P(A' \cap B') = P(A') P(B')$ (10)

Hence A' and B' are independent. [20]

(a) Let A and B be the event that engine A fails and engine B fails respectively

Then $P(A) = P(B) = P$

For a successful flight the following event,

E should happen

$E = (A \cap B') \cup (A' \cap B) \cup (A' \cap B')$ (10)

Because of mutually exclusiveness of the above three events in brackets we have

$P(E) = P(A \cap B') + P(A' \cap B) + P(A' \cap B')$

OR $P(E) = 1 - P(A \cap B')$

Hence $P(E) = P(1 - P) + (1 - P) P + (1 - P)^2 = 1 - P^2$ (20)

For a safe flight with prob > 0.999999 we should have

$P(E) > 0.999999$ (05)

i.e. $1 - P^2 > 0.999999$

or $P^2 < 0.000001 = 10^{-6}$

Therefore $P < 0.001$ (05) [40]

(b)

Mother

Father	A	B	C
A	0	0	0
B	0	$\frac{1}{4}$	$\frac{1}{2}$
C	0	$\frac{1}{2}$	1

Let x denote the event that a randoomly chosen child's eye colour is brown.

Let y_i (1 = 1, 2,...9) denote the event of pairs of characteristics present in the parents as given below.

y_i : (A, A) (A, B) (A, C) (B, A) (B, B) (B, C) (C, A) (C, B) (C, C)

Now, $P(x) = P(x \mid y_1) P(y_1) + \ldots\ldots + P(x \mid y_9) P(y_9)$ (05)

Bec. of independence, $P(y_i) = P(A) \cdot P(B)$ etc. for all 9 (05)

$$P(x) = \left(\begin{array}{l} \left[0 \times \frac{1}{4} \times \frac{1}{4}\right] + \left[0 \times \frac{1}{4} \times \frac{1}{2}\right] + \left(0 \times \frac{1}{4} \times \frac{1}{4}\right) \\ + \left(0 \times \frac{1}{2} \times \frac{1}{4}\right) + \left(\frac{1}{4} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{4}\right) \\ + \left(0 \times \frac{1}{4} \times \frac{1}{4}\right) + \left(\frac{1}{2} \times \frac{1}{4} \times \frac{1}{2}\right) + \left(1 \times \frac{1}{4} \times \frac{1}{4}\right) \end{array} \right) \quad (10)$$

$$= 4 \times \frac{1}{16} \quad (05)$$

$$= \frac{1}{4}$$

[25]

Let z denote the event that both the parents have black eyes. Then

$$p(z) = p(z|y_1)\,p(y_1) + \ldots\ldots + p(z|y_9)\,p(y_9)$$

$$P(y) = \left[1 \times \frac{1}{4} \times \frac{1}{4}\right] + \left[\frac{3}{4} \times \frac{1}{4} \times \frac{1}{2}\right] + 0 + \left[\frac{3}{4} \times \frac{1}{2} \times \frac{1}{4}\right] + \left[\frac{3}{4} \times \frac{3}{4} \times \frac{1}{2} \times \frac{1}{2}\right] + 0 + 0 + 0 + 0 \quad (10)$$

$$= \frac{25}{64}$$

Required probability is $P(x \mid z)$

Now $$P(x/z) = \frac{P(x \cap z)}{P(z)}$$

$$= \frac{\frac{3}{4} \times \frac{3}{4} \times \frac{1}{2} \times \frac{1}{2}}{\frac{25}{64}}$$

$$= \frac{9}{25} \quad (05)$$

[15]

(09)

Class	class limit- scl.	MK (x^1)	freq. (f)	CODED $\frac{(x^1-40)}{C}=d$	fd	fd^2
1	28-32	30	7	-2	-14	28
2	33-37	35	6	-1	-6	6
3	38-42	40	4	0	0	0
4	43-47	45	2	1	2	2
5	48-52	50	1	2	2	4
			20	(05)	-16	40
					(05)	(05)

OTHERWISE	
fx^1	$f(x^1)^2$
210	6300
210	7350
160	6400
90	4050
50	2500
720	26600

Class width C = 5 middle of class mark = 40

Transformation $d=\frac{x^1-40}{5}$

$$Mean=\frac{fx^1}{f}=40+5d$$

$$d=\frac{-16}{20}=\frac{-4}{5}=-0.8$$

$$Mean=40+5\left(\frac{-4}{5}\right)$$

$$=36gm \quad (10)$$

$$mean=\frac{720}{20}$$

$$=36gm$$

$$Median=L+C\left(\frac{N}{2}-F_{m-1}\right)\Big/f(m)$$

$$Fm=f_1+f_2+.....+f_m$$

L = upper C / bdy. in med. cl which is cl. no. 2 (05)

$$Median=32.5+5\frac{\left(\frac{20}{2}-7\right)}{6}$$

$$=35gm$$

Modal class is class r.

Mode = Modal class $mk+\frac{C}{2}\left\{\frac{(f_2-f_1)-(f_2-f_3)}{(f_2-f_1)+(f_2-f_3)}\right\}$

Where f_2 is the freq. in modal cl. and others previous and following classes.

$$Mode=30+\frac{5}{2}\left\{\frac{(7-0)-(7-6)}{(7-0)+(7-6)}\right\} \quad (05)$$

$$=30+\frac{5.6}{2.8} \quad (05)$$

$$=31.9gm$$

$$=32gm(approx) \quad (05)$$

Since Mean > Median > Mode shape of above distribution is positively skewed. (05)

$$S^2=C^2\frac{\left(\sum f_i d_i^2-Nd^2\right)}{N}$$

$$=25\frac{\left(40-20\left(\frac{-16}{20}\right)^2\right)}{20}$$

$$=34gm^2 \quad (10)$$

$$S^2=\frac{\left(\sum f_i x_i^2-Nx^2\right)}{N}$$

$$=\frac{26600-20.36^2}{20}$$

$$=34gm^2$$

Coefficient of skewness $=\frac{3(Mean-Median)}{stdDev}$ (05)

$$=\frac{3\cdot(36-35)}{sqrt(34)}$$

$$=0.5(approx)$$

6

It is observed clearly that frequencies in the second situation are in exact reverse (order)

(i) The shape is negatively skewed since it is the mirror image of the earlier (distn.) (05)

(ii) Variance is the same since abs. values of devns. of cl. mks. from mean on either side are the same. (05)

(iii) Mean in 1st dist is cl mk of 2nd cl + one unit, i.e. 35 + 1 = 36; for 2nd dist bec. of 'mirror image' situation new mean is cl mk (05) of 4th cl - one unit i.e. 45 - 1 = 44gm (05)

Similarly median will be 45gm the cl mk of 4th cl opposed to 2nd cl of 1st distn and new mode would be cl mk of last cl - 1.9 units or 50 - 2 = 48gm approx. (05)

Distn	Mean	Median	Mode
1st	36 gm	35gm	32 gm
Estd Wt.	7.2 kg	7.0 kg	6.4 kg
2nd	44gm	45 gm	48gm
Estdwt.	8.8 kg	9.0 kg	9.6 kg

Retailar always prefers to buy least estimated.

Weight while the opposite is true for the wholesaler.

Acc. to the 1st distn leastest is obtd from Mode while the Mode gives the highest estimate for the 2nd sample.

Therefore Mode is not liked by the retailer in case plantains he buys belong to the second distribution. The same situation arises with the Mean. There fore the compromise is to use the Median. (10)

10