G.C.E. (Advanced Level) Examination - August 2001
10 - Combined Mathematics - I
Provisional Scheme of Marking

(01) (i) α and β are roots of $x^2 + px + 1 = 0$

$\Rightarrow (x - \alpha)(x - \beta) = x^2 + px + 1$ (05)

Then $(\alpha - x)(\beta - x) = x^2 + px + 1 = 0$

Hence $(\alpha - \gamma)(\beta - \gamma) = \gamma^2 + p\gamma + 1$ and (05)

$(\alpha - \delta)(\beta - \delta) = \delta^2 + p\delta + 1$ (05)

Taking the product of these two equations we get

$(\alpha - \gamma)(\beta - \gamma)(\alpha - \delta)(\beta - \delta) = (\gamma^2 + p\gamma + 1)$

$(\delta^2 + p\delta + 1)$ (05) **20**

Simplifying the R.H.S. of above equation we get

$(\alpha-\gamma)(\beta-\gamma)(\alpha-\delta)(\beta-\delta)$

$= \gamma^2\delta^2 + \gamma^2 p\delta + \gamma^2 + p\gamma\delta^2 + p^2\gamma\delta + p\gamma + \delta^2 + p\delta + 1$ (05)

$= (\gamma\delta)^2 + p\gamma\delta(\gamma + \delta) + p^2\gamma\delta + p(\gamma + \delta) + \gamma^2 + \delta^2 + 1$

$= (\gamma\delta)^2 + p\gamma\delta(\gamma+\delta) + p^2\gamma\delta + p(\gamma+\delta) + [(\gamma + \delta)^2 - 2\gamma\delta] + 1$ (10)

$= 1 + p\left(\frac{-1}{p}\right) + p^2(1) + p\left(\frac{-1}{p}\right) + \left(\frac{1}{p^2} - 2\right) + 1$ (05)

$= 1 - 1 + p^2 - 1 + \left(\frac{1}{p^2}\right) - 2 + 1$ (05)

$= \left(P - \frac{1}{p}\right)^2$

Where we have used the properties that $\gamma\delta = 1$

$\gamma + \delta = -1/p$, (05) **30**

(b)

let $\log_a b = y$ and $\log_b a = x$ (05)

Then $b = a^y$ and $a = b^x$ Therefore $b = \left(b^x\right)^y = b^{xy}$ (05)

and hence $xy = 1$ (05)

i.e. $y = \frac{1}{x}$ giving the required result $\log_a b = \frac{1}{\log_b a}$ **15**

Aliter let $\text{Log}_a\ b = y$, Then $b = a^y$, (05)

Now $1 = \log_b b = \log_b a^y = y\log_b a$ (05)

(05)

i.e., $\log_a b = \frac{1}{\log_b a}$ **15**

Applying the above result for each term We get

$\frac{1}{\log_2 2001} + \frac{1}{\log_3 2001} + \ldots\ldots + \frac{1}{\log_{100} 2001}$

$= \log_{2001} 2 + \log_{2001} 3 + \ldots\ldots + \log_{2001} 100$ (10)

$= \log_{2001}(2, 3, 4, \ldots\ldots 100)$ (10)

$= \log_{2001}(100!)$ (10)

$= \frac{1}{\log_{100} 2001}$ (05) **35**

(02) (*a*) When $n = 1$, $A_n = 1 - (1 - \beta)\alpha^{n-1}$ becomes

$A_1 = 1 - (1 - \beta)\alpha^{1-1} = 1 - (1 - \beta) = \beta$ (05)

and this is true.

Assume that the result holds when $n = k$,

i.e. $A_k = 1 - (1 - \beta)^{K-1}$ (05)

Then $A_{K+1} = (1 - \alpha)(1 - \beta)\alpha^{K-1} + [1 - (1 - \beta)\alpha^{K-1}]$ (05)

$= (1 - \beta)\alpha^{K-1}[(1 - \alpha) - 1] + 1$ (05)

$= (1 - \beta)\alpha^{K-1}(-\alpha) + 1$

$= 1 - (1 - \beta)\alpha^K$ (05)

$= 1 - (1 - \beta)\alpha^{(K+1)} - 1$ (05)

$\therefore$ The result holds when n = k + 1, Hence by Principle of Mathematical Induction,

$A_n = 1 - (1 - \beta)\,\alpha^{n-1}$ for every positive integer. (05)

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$$\sum_{r=1}^{n} A_r = \sum_{r=1}^{n}\left[1-(1-\beta)\alpha^{r-1}\right]$$

$$= \sum_{r=1}^{n} 1 - (1-\beta)\sum_{r=1}^{n}\alpha^{r-1} \quad (05)$$

$$= n - (1-\beta)\frac{(1-\alpha^n)}{(1-\alpha)} \quad (05)$$

(05)

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(b) $k\ {}^nC_k = \dfrac{kn!}{k!(n-k)!}$ (05) $= \dfrac{n(n-1)!}{(k-1)!(n-k)!}$ (05)

$$= \frac{n(n-1)!}{(k-1)!\left[(n-1)-(k-1)\right]!} \quad (05)$$

$$= n\ {}^{n-1}C_{k-1}$$

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$$\sum_{k=0}^{n} k\,{}^nC_k\, x^k (1-x)^{n-k} \quad (05)$$

$$= \sum_{k=1}^{n} k\,{}^nC_k\, x^k (1-x)^{n-k}$$

$$= \sum_{k=1}^{n} n\ {}^{n-1}C_{k-1}\ x^k (1-x)^{n-k} \quad (05)$$

$$= nx\sum_{k=1}^{n}\ {}^{n-1}C_{k-1}\ x^{k-1}(1-x)^{(n-1)-(k-1)} \quad (05)$$

$$= nx\sum_{j=0}^{n-1} {}^{n-1}C_j\ x^j (1-x)^{(n-1)-j}, \quad \textit{Where } j = k-1$$

(05) (05)

(05) $= nx\left[x + (1-x)^{n-1}\right]$ *by Binomial theotem*

$= nx$ (05)

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(03) **(a)** G, B, G, B, G, B.....G, B (or equivalent statement)

(05)

The above is how the arrangement will appear to be.

The number of ways in which the girls can be arranged = 7! (10)

For each such arrangement the ways in which the boys can be arranged = 7! (10)

Therefore the total number of arrangements = 7! x 7! (10)

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(b) $y = 2|x+1| - 3 \qquad x \in \mathbb{R}$

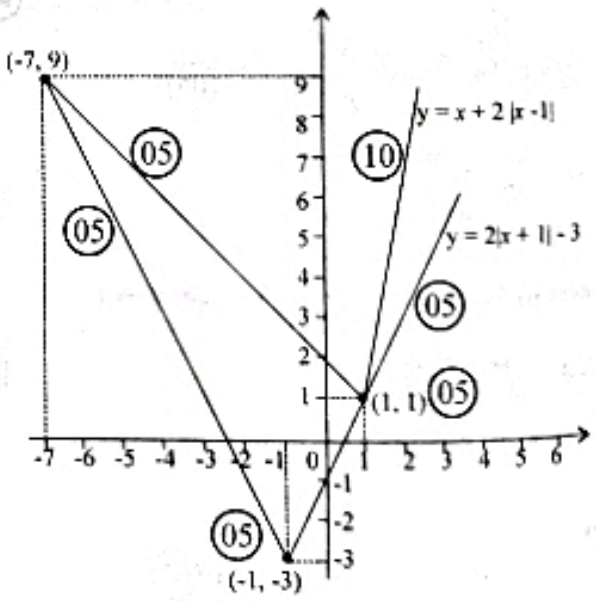
$$= \begin{cases} 2(x+1) - 3 & x+1 \geq 0 \\ -2(x+1) - 3 & x+1 < 0 \end{cases}$$

$$= \begin{cases} 2x - 1 & x \geq 1 \\ -2x - 5 & x < 1 \end{cases}$$

$y = x + 2|x-1| - 3 \qquad x \in \mathbb{R}$

$$= \begin{cases} x + 2(x-1) & x-1 \geq 0 \\ x - 2(x-1) & x-1 < 0 \end{cases}$$

$$= \begin{cases} 3x - 2 & x \geq 1 \\ -x + 2 & x < 1 \end{cases}$$



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The set of x values satisfying the given inequality is the union of two intervals $(-7, 1)$ and $(1, +\alpha)$

(10) (10) **20**

Points of intersection of two graphs:

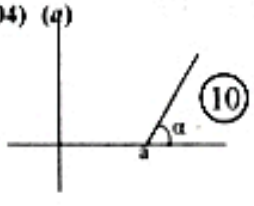
When $x < -1$, $-2x - 5 = -x + 2$ $\Rightarrow x = -7$

When $x \geq 1$, $3x - 2 = 2x - 1$ $\Rightarrow x = 1$

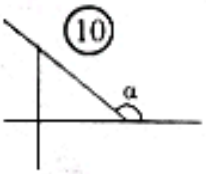
Solutions of the equations $x + 2|x - 1| = 2|x + 1| - 3$ are $x = 1$ and $x = -7$

(05) (05) **10**

(04) (a)

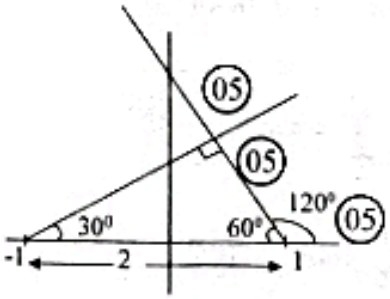


(10)



(10)

The locus is a half line passing throngh a and making an angle α with the positive direction of the x - axis **20**



(05) (05) (05)

$z = 1 \cdot 1 \cos 60° + i\,[1 \sin 60°]$

(05) (05)

$= \frac{1}{2} + \frac{\sqrt{3}}{2} i$ (05) **30**

(b) $\frac{5 - i}{2 - 3i} = \frac{(5 - i)(2 + 3i)}{(2 - 3i)(2 + 3i)} = \frac{13 + 13i}{13} = (1 + i)$ (10) (05) (05)

is of the form $\lambda(1+i)$ where $\lambda = 1$

(05) (05) **30**

$(1 + i)^2 = 1 + 2i - 1 = 2i$ (05)

$(1 + i)^6 = (2i)^3 = 8(-i) = -8i$ With is imergaraly

(05) (05) (05) **20**

(05) (a) $x = t - \sin t$, $y = 1 - \cos t$

$\frac{dx}{dt} = 1 - \cos t$, $\frac{dy}{dt} = \sin t$ (05)

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \sin t \cdot \frac{1}{1 - \cos t}$ (05)

$= \frac{2 \sin t/2 \cos t/2}{2 \sin^2 t/2}$

$= \sin t/2 \neq 0$

$= \cot t/2$ (05)

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dt}{dx} = \frac{d}{dt}\left(\cot \frac{t}{2}\right)\frac{1}{1 - \cos t}$

$= -cosec^2 t/2 \left(\frac{1}{2}\right) \cdot \frac{1}{2 \sin^2 t/2}$ (05)

$= -\frac{1}{4} cosec^4 t/2$ (05)

$\frac{d^3y}{dx^3} = \frac{d}{dt}\left(\frac{d^2y}{dx^2}\right)\frac{dt}{dx}$

$= \frac{-4}{4} cosec^3 \frac{t}{2}\left(-cosec \frac{t}{2} \cot \frac{t}{2} \cdot \frac{1}{2}\right)\frac{1}{1 - \cos t}$ (10)

$= \frac{1}{2} cosec^4 \frac{t}{2} \cot \frac{t}{2} \frac{1}{2 \sin^2 t/2}$

$= -\left[-\frac{1}{4} cosec^4 t/2\right] \cot \frac{t}{2} \cdot \frac{1}{\sin^2 t/2}$ (05)

$y \frac{d^3y}{dx^3} = -\left(\frac{d^2y}{dx^2}\right) y \cot \frac{t}{2} \cdot \frac{1}{\sin^2 \frac{t}{2}}$ (05)

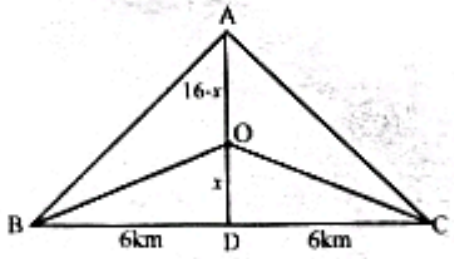
$= \frac{-d^2y}{dx^2}(1 - \cos t) \cot \frac{t}{2} \cdot \frac{2}{(1 - \cos t) \cdot \frac{1}{2}}$

$$= -2\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right) \quad (05)$$

$$y\,\frac{d^3y}{dx^3} + 2\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right) = 0$$

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(b)



(05)

Let OD = x, Then AO = 16 - x and $OC^2 = x^2 + 36$

Let ℓ denote the required length of pipe.

Then $\ell = (16 - x) + 2\sqrt{36 + x^2}$ (10)

Differentiating with respect to x we get

$$\frac{d\ell}{dx} = -1 + \frac{2}{2}\left|36 + x^2\right|^{\frac{-1}{2}}.2x \quad (05)$$

$$= -1 + \frac{2x}{\sqrt{36 + x^2}} = \frac{-\sqrt{36 + x^2} + 2x}{\sqrt{36 + x^2}}$$

$$= \frac{4x^2 - (36 + x^2)}{\sqrt{36 + x^2}\left[2x + \sqrt{36 + x^2}\right]}$$

$$= \frac{3(x^2 - 12)}{\sqrt{36 + x^2}\left(2x + \sqrt{36 + x^2}\right)} \quad (05)$$

$$= \frac{3(x - 2\sqrt{3})(x + 2\sqrt{3})}{\sqrt{36 + x^2}\left(2x + \sqrt{36 + x^2}\right)}$$

$\frac{d\ell}{dx} = 0$ When $(x - 2\sqrt{3})(x + 2\sqrt{3}) = 0$

i.e. When $x = 2\sqrt{3}$ (*Because* $x \neq -2\sqrt{3}$) (05)

Note that

$\frac{d\ell}{dx} < 0$ When $0 < x < 2\sqrt{3}$, $\frac{d\ell}{dx} = 0$ When $x = 2\sqrt{3}$ (05)

$\frac{d\ell}{dx} > 0$ When $2\sqrt{3} < x < 16$

Therefore ℓ decteases in the interval $0 < x < 2\sqrt{3}$ (05)

and increases in the interval $2\sqrt{3} < x < 16$ (05)

Hence ℓ is minimum when $x = 2\sqrt{3}$ and the Well should be located $(16 - 2\sqrt{3})\,km$ from along the altitude. (05)

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(06) (*a*)

Let $I = \int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4 - x^2}}\,dx$ and $x = 2\sin\theta$. (05)

Then $dx = 2\cos\theta\,d\theta$

$x = 1 \Rightarrow \sin\theta = \frac{1}{2}$, $\theta = \frac{\pi}{6}$ *and*

$x = \sqrt{2} \Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$, $\theta = \frac{\pi}{4}$ (05)

$$I = \int_{\pi/6}^{\pi/4} \frac{2\cos\theta\ d\theta}{4\sin^2\theta\sqrt{4 - 4\sin^2\theta}}$$

$$= \frac{1}{2}\int_{\pi/6}^{\pi/4} \frac{\cos\theta\ d\theta}{\sin^2\theta\ 2\cos\theta} \quad (05)$$

$$= \frac{1}{4}\int_{\pi/6}^{\pi/4} \operatorname{cosec}^2\theta\ d\theta \quad (05)$$

$$= -\frac{1}{4}(\cot\theta)_{\pi/6}^{\pi/4} \quad (05)$$

$$= -\frac{1}{4}\left[\cot\frac{\pi}{4} - \cot\frac{\pi}{6}\right]$$

$$= -\frac{1}{4}\left[1 - \sqrt{3}\right]$$

$$= \frac{1}{4}(\sqrt{3} - 1) \quad (05)$$

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(b) $\int_2^4 x\,\ell nx\,dx$

$= \int_2^4 \ell n\,x \cdot \frac{d}{dx}\left(x^2/2\right)dx$ (05)

$= \left[(\ell nx)\frac{x^2}{2}\right]_2^4 - \int_2^4 \frac{x^2}{2}\cdot\frac{1}{x}-dx$ (05)

$= \left[(\ell\, nx)\frac{x^2}{2}\right]_2^4 - \frac{1}{2}\left[\frac{x^2}{2}\right]_2^4$ (05)

$= \left[\ell n4\left(\frac{16}{2}\right)\right] - \left[\ell n2\left(\frac{4}{2}\right)\right] - \left[\frac{16}{2}-\frac{4}{2}\right]$ (05)

$= 8\ell n4 - 2\ell n2 - \frac{1}{2}\left(\frac{12}{2}\right)$

$= 2\ell n4^4 - 2\ell n2 - 3$

$= 2\ell n\frac{4^4}{2} - 3$

$= 2\ell n128 - 3$ (05)

$= a\ell nb + c,$

with $a = 2,\ b = 128,\ c = -3$ (05) **30**

(c) Let

$$\frac{7x-x^2}{(2-x)(x^2+1)} = \left[\frac{A}{2-x} + \frac{Bx+C}{x^2+1}\right]$$

Then, $7x - x^2 = A(x^2+1) + (Bx+C)(2-x)$

$x = 2 \Rightarrow 14 - 4 = 5A \Rightarrow A = 2$ (10)

Coefficient of $x^2 \Rightarrow -1 = A - B$ and hence $B = 2+1 = 3$

Coefficient of $x \Rightarrow 7 = 2B - C$ and hence $\Rightarrow C = 6 - 7 = -1$

$$\int_0^1 \frac{(7x-x^2)}{(2-x)(x^2+1)}dx$$

$= \int_0^1 \frac{2}{2-x}dx + \int_0^1 \frac{3x-1}{x^2+1}\,dx$ (05)

$= -2\left[\ell n|2-x|\right]_0^1 + \frac{3}{2}\int_0^1 \frac{2x}{x^2+1}\,dx - \int_0^1 \frac{dx}{x^2+1}$ (05)

$= -2\left[\ell n|2-x|\right]_0^1 + \frac{3}{2}\left[\ell n\left|x^2+1\right|\right]_0^1 - \left[\tan^{-1}x\right]_0^1$ (05) (05)

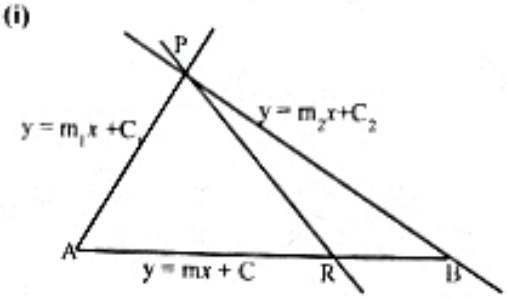
$= -2\left[\ell n1 - \ell n2\right] + \frac{3}{2}\left[\ell n2 - \ell n1\right] - \left(\tan^{-1}1 - 0\right)$ (05)

$= 2\ell n2 + \frac{3}{2}\ell n2 - \frac{\pi}{4}$ (05)

$= \frac{7}{2}\ell n2 - \frac{\pi}{4}$

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(07) (i)



Let $A \equiv (x_1, y_1)$ and $B = (x_2, y_2)$ Since A and B are on $y = mx + C$

$y_1 = mx_1 + C_1$, $y_2 = mx_2 + C \Rightarrow y_1 - y_2 = m(x_1 - x_2)$

A is on (05)

$y = m_1x + C_1 \Rightarrow y_1 = m_1x_1 + C_1 \Rightarrow y_1 - m_1x_1 - C_1 = 0$

and B is on,

$y = m_2x + C_2 \Rightarrow y_2 = m_2x_2 + C_2 \Rightarrow y_2 - m_2x_2 - C_2 = 0$

We also have, (05)

$$R = \left[\frac{kx_2 + x_1}{k+1},\ \frac{ky_2 + y_1}{k+1}\right]$$ (05)

Equation of PR is the of the form $u_1 + \lambda u_2 = 0$

i.e. $(y - m_1x - C_1) + \lambda(y - m_2x - C_2) = 0$ (05)

This goes through R.

$$\Rightarrow \left\{\begin{matrix}\frac{ky_2 + y_1}{k+1} - m_1\left(\frac{kx_2 + x_1}{k+1}\right) - C_1 \\ +\lambda\left(\frac{ky_2 + y_1}{k+1} - m_2\left(\frac{kx_2 + x_1}{k+1}\right) - C_2\right)\end{matrix}\right\} = 0$$ (05)

$(ky_2 + y_1) - m_1(kx_2 + x_1) - C_1(k+1) + \lambda[(ky_2 + y_1) - m_2(kx_2 + x_1) - C_2(k+1)] = 0$

$(y_1 - m_1x_1 - C_1) + k(y_2 - m_1x_2 - C_1) + \lambda[(y_1 - m_2x_1 - C_2) + k(y_2 - m_2x_2 - C_2)] = 0$ (05)

$k(y_2 - m_1x_2 - C_1) + \lambda(y_1 - m_2x_1 - C_2) = 0$

$k[y_2 - m_1x_2 - (y_1 - m_1x_1)] + \lambda[y_1 - m_2x_1 - (y_2 - m_2x_2)] = 0$

$k[(y_2 - y_1) + m_1(x_1 - x_2)] + \lambda[(y_1 - y_2) + m_2(x_2 - x_1)] = 0$ (05)

$$k\left[-\frac{(y_1 - y_2)}{x_1 - x_2} + m_1\right] + \lambda\left[\frac{y_1 - y_2}{x_1 - x_2} - m_2\right] = 0$$

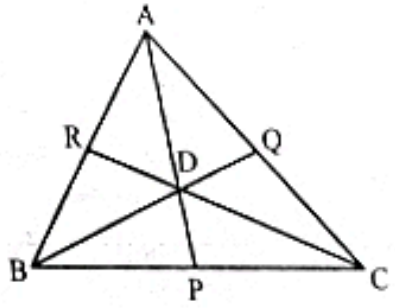
$\Rightarrow k[-m + m_1] + \lambda[m - m_2] = 0$

$\lambda = k\frac{(-m_1 + m)}{m - m_2}$ (05)

Therefore equation of PR

$$u_1 + k\left(\frac{m - m_1}{m - m_2}\right)u_2 = 0$$

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Let $U_{AB} \equiv y + \frac{3}{2}x - 3 = 0$,

$U_{BC} \equiv y + 2x - 2 = 0$ *and*

$U_{CA} \equiv y + x - 3 = 0$,

Let $m_{AB} = \frac{-3}{2}$,

$m_{BC} = -2$, and $m_{AC} = -1$

Note that $A \equiv (x_1\ y_1)$ is on U_{AB} and U_{CA},

Thus $3x_1 + 2y_1 - 6 = 0$ and $x_1 + y_1 - 3 = 0$ (05)

Hence $x_1 = 0$ and $y_1 = 3$ Therefore A = (0, 3) (05) [10]

(ii)

Equation of CR is $U_{CA} + k\left[\frac{m_{AB} - m_{CA}}{m_{AB} - m_{BC}}\right]u_{BC} = 0$

with $k = \frac{1}{2}$, *as* $AR = \frac{1}{2}RB$

Then $(x + y - 3) + \frac{1}{2}\left[\frac{-\frac{3}{2}+1}{-\frac{3}{2}+2}\right](2x + y - 2) = 0$ (05)

(05)

$x + y - 3 + \frac{1}{2}\left(\frac{-1}{1}\right)(2x + y - 2) = 0$

$\Rightarrow 2x + 2y - 6 - 2x - y + 2 = 0$

Equation of CR is $y - 4 = 0$ (05)

Equation of BQ is $U_{AB} + k\frac{(m_{AC} - m_{AB})}{m_{CA} - m_{BC}}U_{BC} = 0$

With $k = \frac{2}{3}$ as $AQ = \frac{2}{3}QC$

Then $\frac{3}{2}x + y - 3 + \frac{2}{3}\left[\frac{-1+\frac{3}{2}}{-1+2}\right](2x + y - 2) = 0$

i.e. $\frac{3}{2}x + y - 3 + \frac{1}{3}(2x + y - 2) = 0$ (05)

or (05)

$9x + 6y - 18 + 2(2x + y - 2) = 0$

Therefore the equation of BQ is $13x + 8y - 22 = 0$ (05) [30]

(iii)

By solving $U_{AB} \equiv y + \left(\frac{3}{2}\right)x - 3 = 0$ and

$U_{BC} \equiv y + 2x - 2 = 0$

We get $x = -2$ and $y = 6$, (the coordinates of point B)

Then B ≡ (-2, 6) (05)

Equation of any straight line passing through point D can be written as $U_{BQ} + \lambda U_{CR} = 0$, Where

$U_{BQ} \equiv 13x + 8y - 22 = 0$ and $U_{CR} \equiv y - 4 = 0$

Hence the equation of AD can be obtained by substituting the coordinates of A.

Then $0 + 24 - 22 + \lambda(3 - 4) = 0$ $\Rightarrow$ $\lambda = 2$,

Therefore the equation of AD is $13x + 10y - 30 = 0$ (05)

By solving this equation and $U_{BC} \equiv y + 2x - 2 = 0$

We get $x = \frac{-10}{7}$ and $y = \frac{34}{7}$ (the coordinates of P)

i.e. $P = \left[\frac{-10}{7}, \frac{34}{7}\right]$

Now, $A = (0,3)$ $B = (-2,6)$ and $P = \left(\frac{-10}{7}, \frac{34}{7}\right)$ imply (05)

That $AP^2 = \frac{269}{49}$ *and* $PB^2 = \frac{80}{49}$

Hence $AP : PB = \sqrt{269} : \sqrt{80}$ (05) [20]

(08) Let the tangents drawn from external point (x_0, y_0) touch the circle at (x_1, y_1) and (x_2, y_2)

Tangent to the circle at (x_1, y_1) is

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + C = 0$ (05)

and the tangent to the circle at $(x_2 y_2)$ is

$xx_2 + yy_2 + g(x + x_2) + f(y + y_2) + C = 0$ (05)

These two straight lines go through $(x_0 y_0)$

Therefore

$x_0x_1 + y_0y_1 + g(x_0 + x_1) + f(y_0 + y_1) + C = 0$ $\rightarrow$(01) (05)

$x_0x_2 + y_0y_2 + g(x_0 + x_2) + f(y_0 + y_2) + C = 0$ $\rightarrow$(02) (05)

(01) and (02)

$\Rightarrow$ $xx_0 + yy_0 + g(x_0 + x) + f(y_0 + y) + C = 0$

is the equation of a straight line passes through point (x_1, y_1) and (x_2, y_2). Therefore the equation of chord of contact from an external point (x_0, y_0) is

$xx_0 + yy_0 + g(x_0 + x) + f(y_0 + y) + C = 0$ (10) **30**

(05)

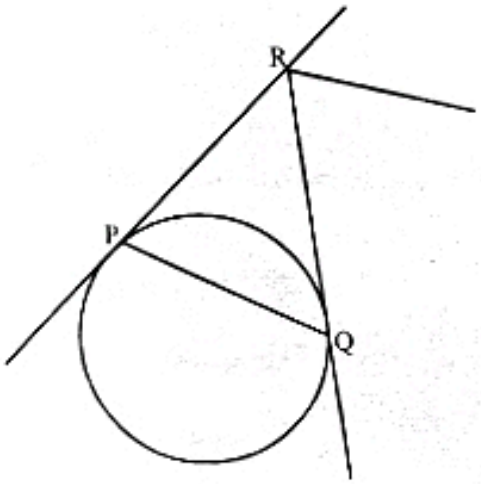
$S \equiv x^2 + y^2 + 2x + 6y + 1 = 0$ and $\ell \equiv 4x + 3y - 5 = 0$ imply that centre of $S = 0$ is $(-1, -3)$ and the radius

$r = \sqrt{1+9-1} = 3$ (05)

Perpemdoculer dostance d of from (-1, -3) to ℓ is given by

$$d = \frac{|4(-1)+3(-3)-5|}{\sqrt{16+9}} = \frac{|-18|}{5} = \frac{18}{5} = 3\frac{3}{5}$$ (05)

Therefore $d > r$ and hence the straight line does not out the circle. (05) **20**



Let R be the intersection point of two tangents.

Since R is on $4x + 3y - 5 = 0$,

$R = \left[t, \frac{5-4t}{3}\right]$ (05)

Let $ax + by + C = 0$ be the variable straight line. (05)

But this is the same as the chord of contact corresponding to (x_0, y_0). From the first result, this equation can be written as

$$xt + y\left(\frac{5-4t}{3}\right) + (x+t) + 3\left(y + \frac{5-4t}{3}\right) + 1 = 0$$ (05)

$3xt + y(5 - 4t) + 3(x + t) + 9y + 3(5 - 4t) + 3 = 0$

$x(3t + 3) + y(14 - 4t) + 18 - 9t = 0$ (05)

Then

$$\frac{3t+3}{a} = \frac{14-4t}{b} = \frac{18-9t}{c}$$ (05)

Therefore

$$a = \frac{3b(t+1)}{2(7-2t)} \text{ (05) } \text{and } c = \frac{9b(2-t)}{2(7-2t)}$$

Hence theequation of variable straight line is

$$\frac{3b(t+1)}{2(7-2t)}x + by + \frac{9b(2-t)}{2(7-2t)} = 0$$

i.e. $3b(t + 1)x + 2by(7 - 2t) + 9b(2 - t) = 0$ (05)

$\therefore$ $3(t + 1)x + 2y(7 - 2t) + 9(2 - t) = 0$ or

$3x + 4y + 18 + t(3x - 4y - 9) = 0$

(05)

This shows that the variable straight line passes through the intersection point of two lines.

$3x + 14y + 18 = 0$ and $3x - 4y - 9 = 0$ **40**

The point of intersection of these two lines is $\left(1, \frac{-3}{2}\right)$.

Hence the coordinates of fixed point is $\left(1, \frac{-3}{2}\right)$ (10) **10**

(09) (*a*) $\sin^3 2x\cos 6x + \cos^3 2x \sin 6x$

$$= \left[\frac{3\sin 2x - \sin 6x}{4}\right]\text{(05)}\cos 6x + \left[\frac{\cos 6x + 3\cos 2x}{4}\right]\text{(05)}\sin 6x$$

$$= \frac{1}{4}\left(\begin{matrix}3\sin 2x\cos 6x - \sin 6x\ \cos 6x \\ +\cos 6x\ \sin 6x + 3\cos 2x\sin 6x\end{matrix}\right)$$ (05)

$$= \frac{1}{4}\left[3(\sin 2x\cos 6x + \cos 2x\sin 6x)\right]$$ (05)

$$= \frac{3}{4}\sin(2x + 6x)$$ (05)

$$= \frac{3}{4}\sin 8x$$ (05) **30**

Now $\sin^3 2x \cos 6x + \cos^3 2x \sin 6x = a$

(05) $\Leftrightarrow \quad \frac{3}{4}\sin 8x = a \quad i.e. \quad \sin 8x = \frac{4a}{3}$

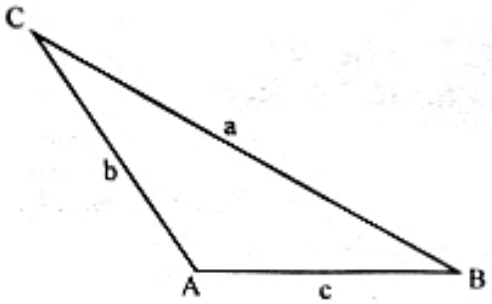
To solve the equation, $-1 \le \sin 8x \le 1$ (05)

i.e. $-1 \le \frac{4a}{3} \le 1$ (05)

Thus the equation is solvable for values a satistying

$-\frac{3}{4} \le a \le \frac{3}{4}$ (05) [20]

(*b*)



Let the largest and the smallest angles of triangle be A & C, respectively

Then the lengths of longest and the shortest sides of triangle are a & c respectively. (05)

It is given that A = 2C and $a = \frac{3}{2}c$.

Applying the sine rule to triangle ABC,

$\frac{\sin A}{a} = \frac{\sin C}{c}$ (05) *i.e.* $\frac{\sin 2C}{\frac{3}{2}C} = \frac{\sin C}{C}$

We get $2\sin C \cos C = \frac{3}{2}\sin C$ (05)

Now $\sin C \neq 0 \Rightarrow 2\cos C = \frac{3}{2} \Rightarrow \cos C = \frac{3}{4}$ (05)

$\Rightarrow C = \cos^{-1}\left(\frac{3}{4}\right)$ [20]

Note that $A = 2C \Rightarrow B = \pi - 3C$ (05)

Applying the sine rule again we get $\frac{\sin B}{b} = \frac{\sin C}{C}$

i.e. $\frac{\sin(\pi - 3C)}{10} = \frac{\sin C}{C}$ (05)

By simplifying this we get $\frac{\sin 3C}{10} = \frac{\sin C}{C}$ or

$\frac{3\sin C - 4\sin^3 C}{10} = \frac{\sin C}{C}$

$\frac{\sin C(3 - 4\sin^2 C)}{10} = \frac{\sin C}{C} \Leftrightarrow 3 - 4(1 - \cos^2 C) = \frac{10}{C}$ (05)

Hence $-1 + 4\cos^2 C = \frac{10}{C} \Leftrightarrow -1 + 4.\frac{9}{16} = \frac{10}{C} \Leftrightarrow \frac{5}{4} = \frac{10}{C}$ (05)

This C = 8cm and $a = \frac{3}{2}C = 12cm$ (05) (05) [30]