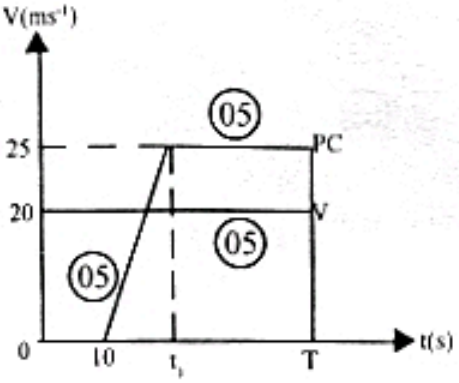
G.C.E. (Advanced Level) Examination - August 2001
Combined Mathematics - II
Provisional Scheme of Marking

(01) (a) $72kmh^{-1} = \frac{72\times1000}{3600}ms^{-1} = 20ms^{-1}$ (05)

$90kmh^{-1} = \frac{90\times10}{36}ms^{-1} = 25ms^{-1}$ (05)



25

area of triangle = Distance moved by P.C. in attainning its maximum velocity;

$\frac{1}{2}(25)(t_1 - 10) = 200$ (05)

$25t_1 = 400 + 250, \quad t_1 = 26s$ (05)

Gradient = acceleration; $f = \frac{25}{t_1 - 10} = \frac{25}{16}ms^{-2}$ (05)

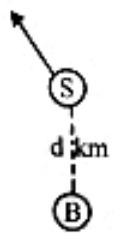
15

If T is the time of overtaking, equating areas under the two curves, $20T = 200 + 25(T - 26)$ (05)

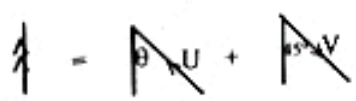
$5T = 650 - 200 = 450, \quad \Rightarrow T = 90S$ (05)

10

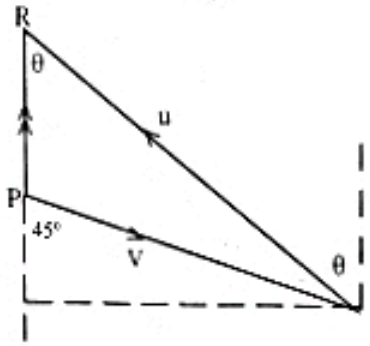
(b)



Veloity : (B, S) = B, E + E, S



Velocity triangle



From the velocity triangle, velocity of B makes an angle θ^0 west of North,

Where $u\sin\theta = v\sin 45^0$ $\qquad \theta = \sin^{-1}\left(\frac{v}{\sqrt{2}u}\right)$

Also, Vel of (B, S) [in the nothern direction] is of magnitude.

$$(PR) = u\cos\theta - v\cos 45^0 = \frac{1}{\sqrt{2}}\left[\sqrt{2u^2 - V^2} - V\right]kmh^{-1}$$

Time to intercept the ship

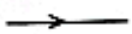
$$= \frac{d}{PR} = \frac{\sqrt{2}d}{\sqrt{2u^2 - V^2} - V}$$

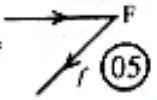
$$= \frac{\sqrt{2}d\left[\sqrt{2u^2 - V^2} + V\right]}{\left(2u^2 - V^2\right) - V^2}$$

$$= \frac{\sqrt{2}d\left[\sqrt{2u^2 - V^2} + V\right]}{2\left(u^2 - V^2\right)}hours$$

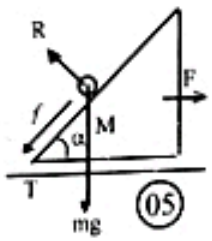
(02) (a)

Acceleration of m, M = f 

Acceleration of M, T = 

m, T = 

(05)



(05)

Equations of motion: For the system

→ $MF + m(F - f\cos\alpha) = 0$ →(01) (10)

from ↙ $m(f - F\cos\alpha) = mg\sin\alpha$ →(02) (10)

(01) + (02) $\cos\alpha$ ⇨

$MF + mF(1 - \cos^2\alpha) = mg\sin\alpha\cos\alpha$ (05)

$$F = \frac{mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}$$

35

For M ⇨

$R\sin\alpha = MF$, (10) $\quad R = \dfrac{Mmg\cos\alpha}{M + m\sin^2\alpha}$ (05)

15

(b) Tractive force F when the speed is maximum (zero acceleration) is given by

$F - 400 = 0$; $F = 400N$ (05)

Maximum speed $V = 144\ kmh^{-1} = \dfrac{144\times10}{36} = 40ms^{-1}$ (05)

Power of the engine, $H = Fv = 16000Js^{-1}$ (walts) = 16kw (05)

15

At the same power, When the speed is $24kmh^{-1}$

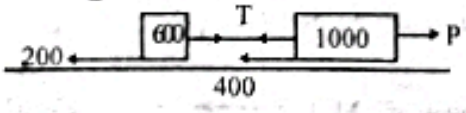
$$= \frac{20}{3}ms^{-1}$$ (05)

The tractive force P is given by $P\times\dfrac{20}{3} = 16000$ (05)

$P = 2400N$ (05)

The acceleration of the system is given by

$1600f = 2400 - 400 - 200$; $f = \dfrac{18}{16} = \dfrac{9}{8}ms^{-2}$ (05) (05)



Tension in the rope T satisfies $T - 200 = 600\times\dfrac{9}{8}$ (05)

$T = 200 + 675 = 875\ N$ (05)

35

(03) (a) For the motion of particle;

→ $x = ut$ (05)

↑ $y = vt - \dfrac{1}{2}gt^2$ (05)

Eliminating t: $y = v\cdot\dfrac{x}{u} - \dfrac{g}{2}\left(\dfrac{x}{u}\right)^2$ (05)

15

When $x = a$, $y = \dfrac{a}{2}$, $\quad u^2 = 2uv - ga$ (05)

$x = 4a$, $y = 0$, $\quad 0 = 8uv - 16ga$ (05)

$uv = 2ga$ (05) $\quad u^2 = 3ga$ (05)

$u = \sqrt{3ga}$ (05) $\quad v = 2\sqrt{\dfrac{ga}{3}}$ (05)

The angle of Projection $= \tan^{-1}\left(\dfrac{uv}{u^2}\right) = \tan^{-1}\left(\dfrac{2}{3}\right)$ (05)

35

(b) Momentum conservation equation :

$2mV + 3mW = 14u - 9u$ (10)

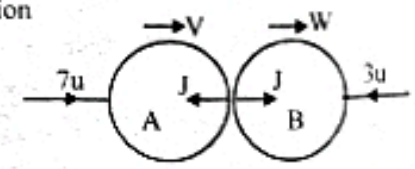
$2V + 3W = 5u$

Newton's law of re stituation $\omega - v = e(7u + 3u)$ (10)

$2\omega - 2v = 20eu$, $\omega = u(1 + 4e)$ (05)

Impulse of the collision

$J = 3m(W + 3u)$



$J = 3m(4e + 1 + 3)u = 12mu(e + 1)$ (05)

30

Velocity of A is $v = w - 10eu$ $\quad = (1 - 6e)u$ (05)

$= o$, if $e = \dfrac{1}{6}$

Then $W = \dfrac{5u}{3}$ and the kinetic energy after impact is

$T = \dfrac{1}{2}\cdot3m\left(\dfrac{5u}{3}\right)^2 = \dfrac{mu^2}{2}\cdot\dfrac{25}{3}$, (05)

KE before impact is $T_0 = \frac{1}{2}\cdot 2m(7u)^2 + \frac{1}{2}\cdot 3m(3u)^2$

$= \frac{mu^2}{2}(98+27)$ (05)

Fraction of kinetic energy retained $\frac{T}{T_0} = \frac{25}{3}\cdot\frac{1}{125}$

$= \frac{1}{15}$ (05)

20

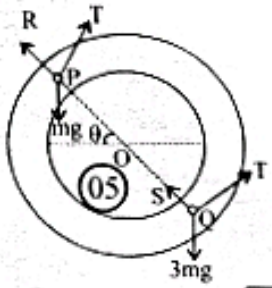
(04) (*a*) Energy conservation equation is

$\frac{1}{2}m(a\dot{\theta})^2 + \frac{1}{2}(3m)(a\dot{\theta})^2 + mga\sin\theta - 3mga\sin\theta = 0;$

(05) (05) (05) (05)

20

$a\dot{\theta}^2 = g\sin\theta$



Force R exerted by the tube on the particle is given by.

$mg\sin\theta - R = ma\dot{\theta}^2$ (10)

$R = 0$ (05)

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(*b*) $T_0 = mg = \lambda\cdot\frac{\ell}{\ell}$, $\lambda = mg$ (05)

$\therefore$ Tension, $T = mg\left(\frac{x-\ell}{\ell}\right)$ Equatio of motion is (05)

$mg - T = m\ddot{x}$, $x > \ell$ (10)

$mg - mg\left(\frac{x-\ell}{\ell}\right) = m\ddot{x}$, $\ddot{x} + \frac{g}{\ell}(x-2\ell) = 0$ (05) 25

Writing $x - 2\ell = x$ and $\frac{g}{\ell} = \omega^2$, We get $\ddot{x} + \omega^2 x = 0$

The particle performs (05)

S.H.M. with center $x = 0$ or $x = 2\ell$ and period $\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g}}$ (05) (05)

Also, $\dot{x}^2 = \omega^2(a^2 - x^2)$, where the constant a is given by the condition $\dot{x} = \sqrt{9\ell}$, when $x = 0$, $g\ell = \frac{g}{\ell}a^2$ (05)

Amplitude $a = \ell$ (05)

Maximum value of $x = 2\ell + \ell = 3\ell$ (05)

Minimum value of $x = 2\ell - \ell = \ell$ (05)



30

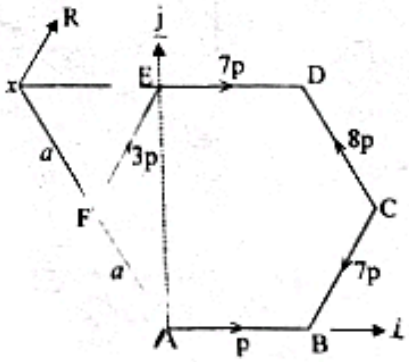
(05) (i) Forces are Pi-along AB; (05)

(05) $+7p\left(\frac{1}{2}i - \frac{\sqrt{3}}{2}j\right)$ *along* CB,

(05) $8p\left(-\frac{1}{2}i + \frac{\sqrt{3}}{2}j\right)$ *along* CD,

(05) $7pi$ *along* ED,

(05) $3p\left(\frac{1}{2}i + \frac{\sqrt{3}}{2}j\right)$ *along* FE,



25

(ii) Vector sum of forces,

$R = pi\left(1 - \frac{7}{2} - 4 + 7 + \frac{3}{2}\right) + \sqrt{3}pj\left(-\frac{7}{2} + 4 + \frac{3}{2}\right)$ (05)

$= 2p(i + \sqrt{3}j) \neq 0$ (05)

$\Rightarrow$ Single resultant force (05)

$\rightarrow$ Also, $BC = \frac{a}{2}(i + \sqrt{3}j) \parallel R$ and magnitude of (05)

$R = 2p\sqrt{1^2 + 3} = 4p$ (05) 25

(iii) Taking moments about the point of intersection of DE & AF (both produced)

$\curvearrowleft X$ $\left[P(2a)\frac{\sqrt{3}}{2} - 7p(3a)\frac{\sqrt{3}}{2} + 8p(2a)\frac{\sqrt{3}}{2} + 3p(a)\frac{\sqrt{3}}{2} \right]$ (05) (05) (05) (05)

$= a\frac{\sqrt{3}}{2}[2-21+16+3] = 0$ (05)

Resultant R Passes through x [25]

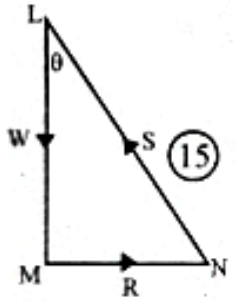
(iv) Introduce forces R and -R through the vertex A.
Then the system is equivalent to a force R at A together with a couple formed by forces R at X and - R at A.

Moment of the couple is $= -4p \cdot 2a \times \frac{\sqrt{3}}{2} = 4\sqrt{3}\ Pa$ Nm (05) (05)

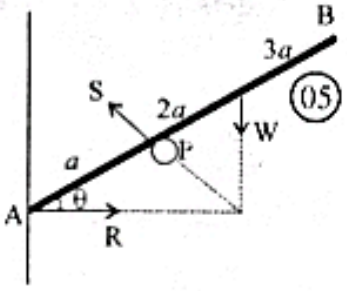
anticlockwise (05) (05)

[25]

(06) (a) Triangle of forces : LMN



(15)



(05)

$S = W\sec\theta$; (05)

$\curvearrowleft A$ $S \cdot a\sec\theta = W.3a\cos\theta$ (10)

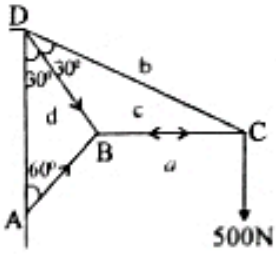
$\sec^2\theta = 3\cos\theta$ (05)

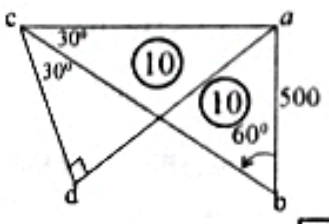
$\Rightarrow \quad 3\cos^3\theta = 1$

[40]

Eliminating S, between the two equations :

(b)





Stress Diagram [20]

bc = 1000N; (05) Tension in CD (05)

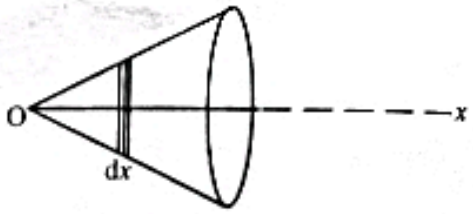
ca = $500\sqrt{3}$N ; (05) Thrust in BC (05)

cd = $250\sqrt{3}$N ; (05) Thrust in BD (05)

da = 750 N; (05) Thrust in AB (05)

[40]

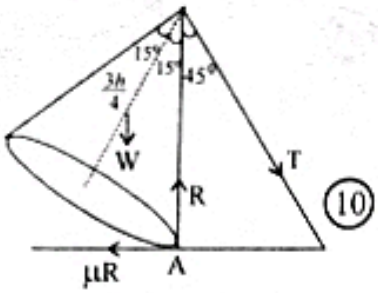
(07)



By symmetry center of mass lies on the axis of the cone. (05)

Also, $\bar{x} = \frac{\int_0^h x \cdot \rho\pi(x\tan\alpha)^2\,dx}{\int_0^h \rho\pi(x\tan\alpha)^2\,dx} = \frac{\left(\frac{h^4}{4}\right)}{\left(\frac{h^3}{3}\right)} = \frac{3h}{4}$ (05) (05) (05) (05) (05)

[30]



(10)

Resolving horizontally (→) $\frac{T}{\sqrt{2}} = \mu R$, Where μ is the coefficient of friction. (10)

Resolving vertically $(\uparrow) R = W + \frac{T}{\sqrt{2}}$ (10)

Taking the moment about $\curvearrowleft A$ $T \cdot \frac{h}{\sqrt{2}}\sec(15^0)$ (10)

$= W \cdot \frac{3}{4}h\sin(15^0)$ (10)

[50]

and since $\sin(15^0)\cos(15^0) = \frac{1}{2}\sin(30^0) = \frac{1}{4}$, (05)

$$T = \frac{3\sqrt{2}W}{16}$$ (05)

There fore $\mu R = \frac{3W}{16}$ (05) *and* $R = W + \frac{3}{16}W$

$$= \frac{19}{16}W$$ (05)

$$\mu = \frac{3}{19}$$

20

(08) (a) Definitions

P(A∪B) is the probability of either A or B or both A and B occurring. (05)

P(A∩B) is the probability of both A and B occurring. (05)

P(A/B) is the probability that A occurs given that B has occurred. (05)

15

Given : P(A) = 0.6, P(B) = 0.2 and P(A|B) = 0.1;

$$\frac{P(A \cap B)}{P(B)} = 0.1$$ (05)

(i) Probability of both A and B occurring

P(A∩B) = 0.1 x 0.2 = 0.02

(∴ P(B) = 0.2) (05)

10

(ii) Probability of exactly one of A or B occurring

= P(A∩B′) + P(A′∩B) (05)

Now P(A∩B′) + P(A∩B) = P(A),

P(A∩B′) = 0.6 - 0.02

= 0.58 (05)

Also, Since P(A′∩B) + P(A∩B) = P(B)

P(A′∩B) = 0.2 - 0.02

= 0.18 (05)

Adding the two values, the probability of exactly one event occurring = 0.76 (05)

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(iii) Probability of neither A nor B occuring

= P(A′∩B′)

= 1 - P(A∪B) (05)

and P(A∪B) = P(A) + P(B) - P(A∩B)

= 0.6 + 0.2 - 0.02

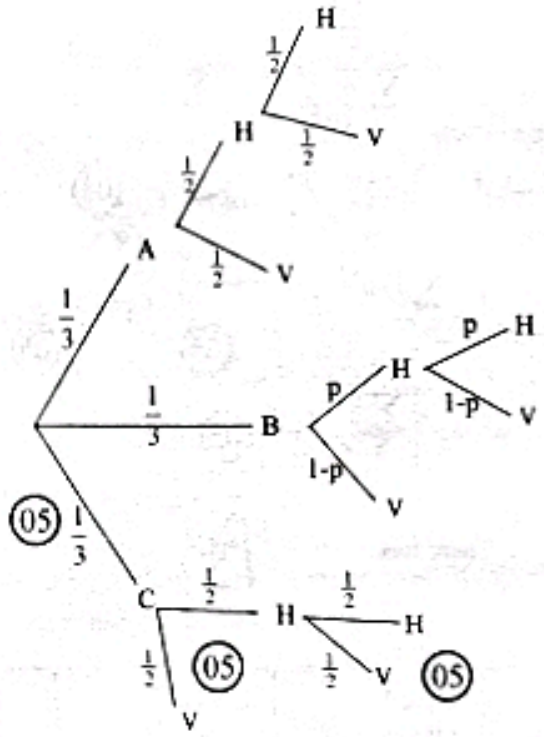
= 0.78 (05)

using the formula, the probability of neither A nor B occurring = 1 - 0.78

= 0.22 (05)

15

(b)



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From the tree diagram

$$P(H,H) = \frac{1}{3}\left(P^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right) = \frac{17}{54}$$ (05) (given) $\therefore P^2 + \frac{1}{4} + \frac{1}{4} = \frac{17}{18}$

$P^2 = \frac{17-9}{18} = \frac{8}{18} = \frac{4}{9}$ (05) $P = \frac{2}{3}$ $(\because 0 \le P \le 1)$ (05)

$$P(B/HH) = \frac{P(B \cap H,H)}{P(H,H)}$$ (05) $= \frac{1}{3} \times P^2 \div \frac{17}{54}$

$= \frac{4}{27} \times \frac{54}{17} = \frac{8}{17}$ (05)

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(09) Mean $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i}$ (05), Where f_i is the frequency of the ith class in which the mid - class value is x_i. (05)

coding $y = \dfrac{x-a}{c} \rightarrow x = a + cy$

$\bar{x} = \dfrac{\sum f_i(a+cy_i)}{\sum f_i}$ (05) $= a + c\dfrac{\sum f_i y_i}{\sum f_i}$, $\bar{x} = a + c\bar{y}$ (05)

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Given, $\sigma^2 = \dfrac{\sum f(x-\bar{x})^2}{\sum f} = \dfrac{\sum f\left(x^2 - 2\bar{x}x - \bar{x}^2\right)}{\sum f}$ (05)

$= \dfrac{\sum fx^2}{\sum f} - \bar{x}^2 \left(\because \dfrac{\sum f(x)}{\sum f} = \bar{x}\right)$ (05)

$\sigma^2 = \dfrac{\sum f(a+cy)^2}{\sum f} - (a+c\bar{y})^2$ (05)

$= \dfrac{\sum f\left(a^2 + 2acy + c^2y^2\right)}{\sum f} - \left[a^2 + 2ac\bar{y} + c^2\bar{y}^2\right]$ (05) (05)

$\sigma^2 = c^2\left[\dfrac{\sum fy^2}{\sum f} - \bar{y}^2\right]$ (05)

There fore, $\sigma = c\sqrt{\dfrac{\sum fy^2}{\sum f} - \bar{y}^2}$

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Class	Class Mid-value (x)	Frequency (f)	Deviation from 45 d = x - 45	$y = \frac{x-45}{10}$	fy	fy^2
0 < x <10	5	4.2	-40	-4	-16.8	67.2
10 ≤ x <20	15	3.9	-30	-3	-11.7	35.1
20 ≤ x <30	25	3.4	-20	-2	-6.8	13.6
30 ≤ x <40	35	3.2	-10	-1	-3.2	3.2
40≤ x < 50	45	2.8	0	0	0	0
50≤ x < 60	55	2.8	10	1	2.8	2.8
60≤ x < 70	65	2.5	20	2	5.0	10.0
70≤ x < 80	75	1.6	30	3	4.8	14.4
80≤ x < 90	85	0.6	40	4	2.4	9.6
	Total	25.0		(05)	-23.5	155.9

(10) (10)

25

$\sum fy = -23.5$ and $\sum fy^2 = 155.9$

Since $x = 10y + 45$

The mean of the population is $\mu = 45 + 10\dfrac{\sum fy}{\sum f}$

$= 45 + 10\left(\dfrac{-23.5}{25}\right)$ (05)

$= 45 - \dfrac{235}{25} = 45 - 9.4$

$= 35.6$ *years* (05)

10

Veriance $\sigma^2 = c^2\left[\dfrac{\sum fy^2}{\sum f} - \bar{y}^2\right]$.

$\bar{y} = \dfrac{\sum fy}{\sum f} = \dfrac{-23.5}{25} = -0.94$

$= 100\left[\dfrac{155.9}{25} - (0.94)^2\right]$ (05)

$= 623.60 - 88.36$

$= 535.24$ (05)

Therefore, the standard deviation of the population is

$\sigma = 23.1$ years (05)

15