# G.C.E. (Advanced Level) Examination - August 2000
## 10 - Combined Mathematics - I
## Provisional Scheme of Marking

**(01)** Solution

**(a)** $\alpha + \beta = p$ and $\alpha\beta = q$ (05)

Let $x^2 - ax + b = 0$ the equation whose roots are $\alpha(\alpha + \beta)$ and $\beta(\alpha + \beta)$

Then

$a = \alpha(\alpha + \beta) + \beta(\alpha + \beta)$

$= (\alpha + \beta)^2$

$= p^2$ and

$b = \{\alpha(\alpha + \beta)\}\ \{\beta(\alpha + \beta)\}$

$= \alpha\beta\ (\alpha + \beta)^2$

$= qp^2$ (05)

$\therefore$ The equation is $x^2 - p^2x + qp^2 = 0$ (05) [15]

**(b)** Let

$2x^2 + \lambda xy + 3y^2 - 5y - 2 \equiv (ax + by + C)(px + qy + r)$ for all $x$ and $y$. (05)

When $x = 0$

$3y^2 - 5y - 2 \equiv (by + C)(qy + r)$ for all y. (05)

But $3y^2 - 5y - 2 = (3y + 1)(y - 2)$

$\therefore$ $b = 3$, $c = 1$, $q = 1$ and $r = -2$ (10)

$\therefore$ $2x^2 + \lambda xy + 3y^2 - 5y - 2 \equiv (ax + 3y + 1)(px + y - 2)$ for all $x$ and $y$.

When $y = 0$ we have

$2x^2 - 2 \equiv (ax + 1)(px - 2)$ for all $x$. (05)

comparing corresponding coefficients we have

$2 = ap$ and $-2a + b = 0$

$a^2 = 1 \Rightarrow a = \pm 1$ and hence $p = \pm 2$ (05) (05)

$\therefore$ $2x^2 + \lambda xy + 3y^2 - 5y - 2 = (x + 3y + 1)(2x + y - 2)$

or

$2x^2 + \lambda xy + 3y^2 - 5y - 2 = (-x + 3y + 1)(-2x + y - 2)$

Comparing corresponding coefficient of $xy$ in each case we have $\lambda = \pm 7$ (10)

[45]

Aliter

$2x^2 + \lambda xy + 3y^2 - 5y - 2 = (x + ay + b)(2x + py + q)$ (05)

$2x^2 + \lambda xy + 3y^2 - 5y - 2 = 2x^2 + (p + 2a)xy + (q + 2b)x$ (05) (05)

$+ (aq + pb)y + apy^2 + bq$

Comparing corresponding coefficients we have

$\lambda = p + 2a$, $ap = 3$, $q + 2b = 0$, $aq + bp = -5$ and $bq = -2$

$q^2 - 2bq = 4$ and hence $q = \pm 2$ and $b = \mp 1$ (05) (05)

$a^2q + abp = -5a$ and $a^2q + 3b = -5a$

When $q = 2$, $b = -1$ We have $2a^2 + 5a - 3 = 0$

$a = \frac{1}{2}$ or $-3$

When $a = \frac{1}{2}$ $P = 6$ and $\lambda = +7$

When $a = -3$ $p = -1$ and $\lambda = -7$ (05)

When $q = -2$, $b = 1$ we have

$-2a^2 + 5a + 3 = 0 \Rightarrow a = -\frac{1}{2}$ or $3$

When $a = -\frac{1}{2}$ $P = -6$ and $\lambda = -7$

When $a = 3$ $p = 1$ and $\lambda = 7$ (05)

The values of $\lambda$ are $-7$ and $7$. (05) (05)

[45]

**(c)** $\frac{2x^3 - x + 3}{x(x-1)^2} = 2 + \frac{4x^2 - 3x + 3}{x(x-1)^2}$ (05) (10)

$= 2 + \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$ (05)

When A, B, and C are constants

$$\frac{4x^2-3x+3}{x(x-1)^2}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$$ (05)

A = 3, (05) B = 1, (05) and C = 4 (05)

$$\frac{2x^3-x+3}{x(x-1)^2}=2+\frac{3}{x}+\frac{1}{x-1}+\frac{4}{(x-1)^2}$$ 40

(05)

$$\frac{2x^3-x+3}{x(x-1)^2}=A+\frac{B}{x}+\frac{C}{x-1}+\frac{D}{(x-1)^2}$$ (05)

Where A B C and D are constants.

$$\frac{2x^3-x+3}{x(x-1)^2}=\frac{A(x^3-2x^2+x)+B(x^2-2x+1)+C(x^2-x)+Dx}{x(x-1)^2}$$ (05)

Comparing the corresponding coefficients We have

(05)

A = 2, 2A + B + C = 0, (05) A - 2B - C + D = -1 and (05) B = 3

C = -1 and (05) D = 4 (05)

$$\frac{2x^3-x+3}{x(x-1)^2}=2+\frac{3}{x}+\frac{1}{x-1}+\frac{4}{(x-1)^2}$$ 40

**(02) (a)** $U_n=\frac{1}{6}n(n+1)(n+2)\rightarrow(01)$

When n = 1

L. H. S. of $= U_1 = 1.1 = 1$

∴ R. H. S. of (01) $=\frac{1}{6}1.2.3=1$

∴ L. H. S. of (01) = R. H. S. of (01)

∴ The result is true for n = 1 (05)

Assume that the result is true for n = p

i.e Assume $U_p=\frac{1}{6}p(p+1)(p+2)$ (05)

When n = p + 1

$U_{p+1} = 1.(p+1)+2(p)+\ldots+p.2+(p+1).1$

$= 1.p+2(p-1)+\ldots+p.1+\{1+2+\ldots+(p+1)\}.1$

$= U_p+\{1+2+\ldots\ldots\ldots+(p+1)\}$

$=\frac{1}{6}p(p+1)(p+2)+\frac{1}{2}(p+1)(p+2)$

$=\frac{1}{6}(p+1)(p+2)(p+3)$

The result is true for n = p + 1 (10)

By the principle of Mathematical Induction.

$U_n=\frac{1}{6}n(n+1)(n+2)$ for any positive integer n. (05) 25

For any positive integer n

$$U_n=\frac{6}{n(n+1)(n+2)}$$

$$=\frac{3}{n(n+1)}-\frac{3}{(n+1)(n+2)}$$ (05)

$= -V_n - V_{n+1}$ When $V_n=\frac{3}{n(n+1)}$ (05) 10

$$\frac{1}{U_1}=V_1-V_2$$

$$\frac{1}{U_2}=V_2-V_3$$ (05)

.....

.....

.....

$$\frac{1}{U_{n-1}}=V_{n-1}-V_n$$

$$\frac{1}{U_n}=V_n-V_{n+1}$$ (05)

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$$\sum_{r=1}^{n}\frac{1}{U_r}=V_1-V_{n+1}$$ (05)

$$=\frac{3}{2}-\frac{3}{(n+1)(n+2)}$$ 15

$$\sum_{r=1}^{\infty} \frac{1}{U_r} = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{U_r} \quad (05)$$

$$= \lim_{n\to\infty} \left\{ \frac{3}{2} - \frac{2}{(n+1)(n+2)} \right\}$$

$$= \frac{3}{2} \quad (05) \qquad \therefore \lim_{n\to\infty} \frac{2}{(n+1)(n+2)} = 0 \qquad \boxed{10}$$

**(b)** $(1+kx)^{10} = {}^{10}C_0 + {}^{10}C_1 kx + {}^{10}C_2 k^2x^2 + \ldots\ldots +$

$${}^{10}C_r k^r x^r + \ldots\ldots\ldots + {}^{10}C_{10} k^{10} x^{10}$$

$$= a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_{10} x^{10}$$

$$\therefore \sum_{r=0}^{10} {}^{10}C_r\, k^r x^r = \sum_{r=0}^{10} a_r\, x^r : \textit{ for all } x \in \mathbb{R}$$

$$\therefore {}^{10}C_r k^r = a_r \quad \textit{for} \quad r = 0,1,2,\ldots.10 \quad (05)$$

In particular ${}^{10}C_2 k^2 = a_2 = \frac{20}{9}(given)$

$$But \ {}^{10}C_2 k^2 = \frac{10\cdot 9}{2} k^2 \quad (05)$$

$$\therefore k^2 = \left(\frac{2}{9}\right)^2$$

$$k = \frac{2}{9} \quad \therefore \text{ k is a possitive constant} \quad (05)$$

$$\therefore \left(1+\frac{2}{9}x\right)^{10} = a_0 + a_1 x + a_2 x^2 + \ldots. a_{10} x^{10}$$

When $x = 1$ we have

$$\left(\frac{11}{9}\right)^{10} = a_0 + a_1 + a_2 + \ldots\ldots + a_{10} \quad \rightarrow (01) \quad (05)$$

When $x = -1$ we have

$$\left(\frac{7}{9}\right)^{10} = a_0 - a_1 + a_2 + \ldots\ldots - a_9 + a_{10} \quad \rightarrow (02) \quad (05)$$

$(01)-(02) \Rightarrow$

$$\left(\frac{11}{9}\right)^{10} - \left(\frac{7}{9}\right)^{10} = 2(a_1 + a_3 + \ldots\ldots + a_9) \quad (05)$$

$$i.e. \quad a_1 + a_3 + a_5 + a_7 + a_9 = \frac{11^{10} - 7^{10}}{2\cdot 9^{10}} \quad \rightarrow (03) \qquad \boxed{30}$$

$(02)+(03) \Rightarrow$

$$i.e. \quad a_0 + a_2 + a_4 + a_6 + a_8 + a_{10}$$

$$= \left(\frac{7}{9}\right)^{10} + \frac{11^{10} - 7^{10}}{2\times 9^{10}} \rightarrow (03) \quad (05)$$

$$= \frac{11^{10} + 7^{10}}{2\times 9^{10}} \quad (05) \qquad \boxed{10}$$

**(03)** $$\frac{(-1+i)^3}{(1+i)^4} = \frac{(-1+i)^3(1-i)^4}{(1+i)^4(1-i)^4} \quad (05)$$

$$= -\frac{1}{2^4}(1-i)^7 \quad (05)$$

$$= -\frac{1}{2^4}(1-7i-2i+35i+35-21i-7+i) \quad (15)$$

$$= -\frac{1}{2} - \frac{1}{2}i \quad (05)$$

$$= \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) (05)$$

$\therefore$ The modulus is $\frac{1}{\sqrt{2}}$ and the argument is $\frac{5\pi}{4}$ (05) (05) $\boxed{45}$

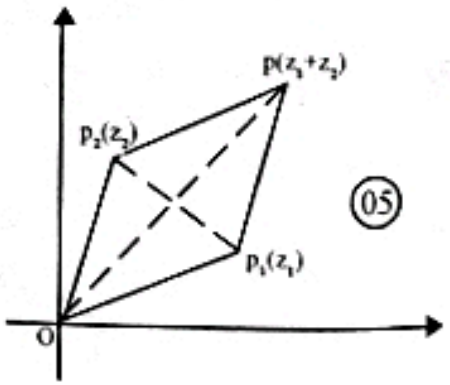
Aliter : $$\frac{(-1+i)^3}{(1+i)^4} \quad (10) \quad \frac{(i)^3(1+i)^4}{(1+i)^4} = \frac{-i(1-i)^4}{2} \quad (15)$$

$$= -\frac{1}{2} - \frac{i}{2}i \quad (05)$$

$$= \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) (05)$$

$\therefore$ The modulus is $\frac{1}{\sqrt{2}}$ and the argument is $\frac{5\pi}{4}$ (10)

$\boxed{45}$




(05)

Complete the parallelogram $OP_1PP_2$ with $OP_1$ and $OP_2$ as the adjacent sides. Then P represents the complex number $z_1 + z_2$. (05)

For

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

Then $p_1 = (x_1, y_1)$ and $p_2 = (x_2, y_2)$

Let $p(x, y)$

Then $\frac{x+0}{2} = \frac{x_1 + x_2}{2}$ and $\frac{y+0}{2} = \frac{y_1 + y_2}{2}$ (05)

i.e. $x = x_1 + x_2$ and $y = y_1 + y_2$

$\therefore$ $p = (x_1 + x_2, y_1 + y_2)$

$\therefore$ p represents the complex number

$(x_1 + x_2) + i(y_1 + y_2)$

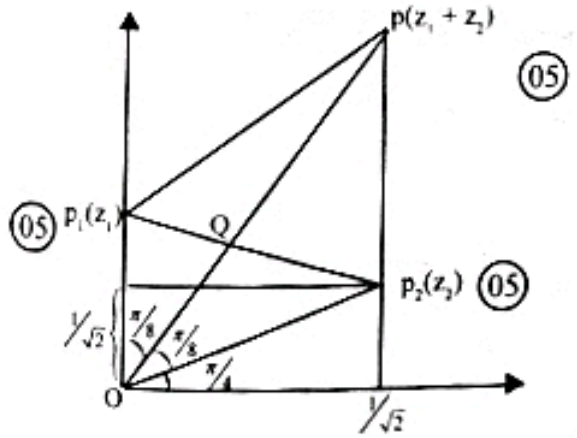
i.e. p represents the complex number (05)

$(x_1 + iy_1) + (x_2 + iy_2)$

i.e. p represemts the complex number $z_1 + z_2$ [20]

$z_1 = \frac{1+i}{1-i} = i$ and $z_2 = \frac{\sqrt{2}}{1-i} = \frac{1}{\sqrt{2}}(1+i)$ (05)

(05)




25

$\tan\left(\frac{\pi}{8}\right) = \frac{QP_2}{OQ} = \frac{1 - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \sqrt{2} - 1$ (10) [10]

**(04)** *(a)* $\lim_{x \to 0} \frac{1 - \cos^2(2\sin x)}{2\sin^2 x}$ (05)

$= \lim_{x \to 0} \frac{2\sin^2(2\sin x)}{(2\sin x)^2}$ (05)

$= \lim_{x \to 0} 2\left[\frac{\sin(2\sin x)}{2\sin x}\right]^2$ (05)

$= \lim_{x \to 0} 2\left[\frac{\sin t}{t}\right]^2$ $\quad t = 2\sin x \Rightarrow t \to 0$ as $x \to 0$

$= 2$ (05) [20]

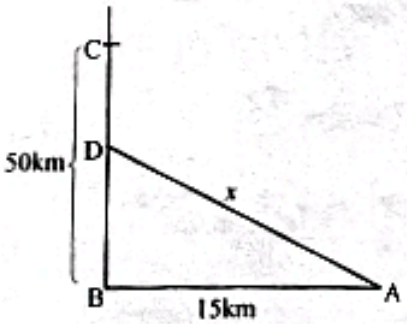
**(b)** $y = e^{k\sin^{-1}x}$

(05) $\frac{dy}{dx} = e^{k\sin^{-1}x} \cdot k\frac{1}{\sqrt{1-x^2}}$ (05)

$\sqrt{1-x^2}\,\frac{dy}{dx} = ky$ (05) [15]

When $x = \frac{1}{2}$ $\quad y = e^{k\frac{\pi}{6}}$ (05)

$\frac{dy}{dx} = \frac{2}{\sqrt{3}} k\, e^{k\frac{\pi}{6}}$ (05) [10]




(05)

$DC = 50 - \sqrt{x^2 - 225}$ *for* $x \geq 15$

$T(x) = \frac{x}{40} + 1 - \frac{\sqrt{x^2 - 225}}{50}$ (10)

[15]

$$\frac{dT(x)}{dx} = \frac{1}{40} - \frac{x}{50\sqrt{x^2-225}}$$ (10)

$$\frac{1}{200\sqrt{x^2-225}}\left\{5\sqrt{x^2-225} - 4x\right\} = 0$$ (05) When $x = 25$

| $15 < x < 25$ | $x = 25$ | $25 < x < 50$ |
|---|---|---|
| $\frac{dT(x)}{dx}$ (05) | 0 | (+) (05) |
| (-) (05) | | |

T(x) decreases as x increases from 15 to 25 (05)

T(x) increases as x increases from 25 to 50 (05)

∴ T(x) is least at x = 25 and hence the shortest time is (05)

T(25) which is 1.225 hours or 73.5 minutes. [40]

**(05) (a)**

Let $I = \int_1^8 \frac{1}{x^{\frac{4}{3}} + x^{\frac{2}{3}}}\,dx$

Take $y = x^{\frac{1}{3}}$ (05) then $dy = \frac{1}{3}\cdot\frac{1}{x^{\frac{2}{3}}}\,dx$ (05)

If $x = 1$ then $y = 1$ and if $x = 8$ then $y = 2$

$$I = 3\int_1^2 \frac{1}{1+y^2}\,dy = \left[3\tan^{-1} y\right]_1^2 = 3\left\{\tan^{-1} 2 - \frac{\pi}{4}\right\}$$ (05) (05) (05) [25]

**(b)** $I = \int_0^{\pi} e^{-2x}\cos x\,dx$

$= \int_0^{\pi} e^{-2x}\frac{d}{dx}(\sin x)\,dx$ (05)

$= \left[e^{-2x}\sin x\right]_0^{\pi} + 2\int_0^{\pi} e^{-2x}\sin x\,dx$ (05) (05) [15]

$= 2J$

$J = \int_0^{\pi} e^{-2x}\sin x\,dx$ (05)

$= \int_0^{\pi} e^{-2x}\frac{d}{dx}(-\cos x)\,dx$

$= \left[-e^{-2x}\cos x\right]_0^{\pi} - 2\int_0^{\pi} e^{-2x}\cos x\,dx$ (05) (05)

$= e^{-2\pi} + 1 - 2I$ [15]

$$\frac{x^2-5x}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$$ (05)

Where A, B, and C are constants

∴ A = -1, and C = -3 (05) (05)

When $x = 0$ We have $0 = 1 + B - 3$ Which gives $B = 2$ (05)

$$\int\frac{x^2-5x}{(x-1)(x+1)^2}dx = \int\frac{1}{(x-1)}dx + 2\int\frac{1}{(x+1)}dx - 3\int\frac{1}{(x+1)^2}dx$$ (05)

$= -\ln|x-1| + 2\ln|x+1| + \frac{3}{(x+1)} + D$ (05) (05)

Where D is an arbitrary constant. [35]

**(06)**

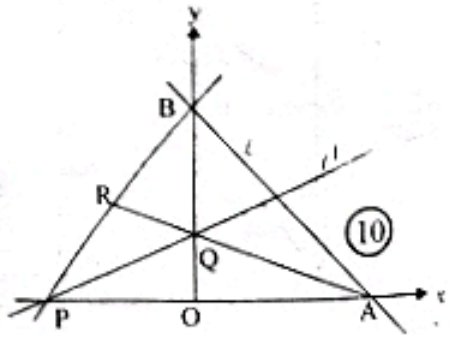
Let $y = mx + C$ be the equation.

Since $(a, 0)$ and $(0, b)$ lie on the line we have $0 = am + c$

and $b = c$ (10)

$\therefore m = -\frac{b}{a}$

Hence the equation $\frac{x}{a} + \frac{y}{b} = 1$ (05) [15]



(10)

A = (h, 0) and B(0, k)

Let $\frac{x}{h^1} + \frac{y}{k^1} = 1$ (05) be the equation

of the line perpendicular to the line given by $\frac{x}{h} + \frac{y}{k} = 1$

$\therefore \left(\frac{-k^1}{h^1}\right)\left(-\frac{k}{h}\right) = -1$ Which implies that (10)

$\frac{k^1}{h} = \frac{-h^1}{k} = t\,(say)$ (05)

i.e. $k^1 = ht$, $h^1 = kt$ and hence We have $p = (-kt, 0)$ and $Q = (O, ht)$ (05) (05)

The equation of AQ is $\frac{y}{x-h} = \frac{ht}{-h} = -t$ (05)

i.e. $y + tx = ht$ (05)

The equation of BP is $\frac{y-k}{x} = \frac{k}{kt} = \frac{1}{t}$ (05)

i.e. $yt - x = kt$ (05)

Let $R = (x_0, y_0)$ Then $y_0 = t(h - x_0)$ and $x_0 = t(y_0 - k)$ (05) (05)

$\frac{y_0}{h - x_0} = \frac{x_0}{y_0 - k}$, *provided* $x_0 \neq h$ *and* $y_0 \neq k$ (05) (05)

i.e. $x_0^2 + y_0^2 - hx_0 - ky_0 = 0$ (05)

Hence R lies on the circle $x^2 + y^2 - hx - ky = 0$ with the point (h, k) deleted. 85

(07) Let $C_1$ and $C_2$ be the centres and $r_1$ and $r_2$ be the radius of the two circles.

Then $C_1 = (-g_1, -f_1) : r_1 = \sqrt{g_1^2 + f_1^2 - C_1}$ (05) *and*

$C_2 = (-g_2, -f_2) : r_2 = \sqrt{g_2^2 + f_2^2 - C_2}$ (05)

If the two circles intersect orthogonally then

$(C_1C_2)^2 = r_1^2 + r_2^2$ (05)

i.e.

$(g_1 - g_2)^2 + (f_1 - f_2)^2 = (g_1^2 + f_1^2 - c_1) + (g_2^2 + f_2^2 - c_2)$ (05)

i.e. $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ 20

Let (p, 0) be the centre and r be the radius of the circle.

Then the equation of the circle can be written as

$(x - p)^2 + y^2 = r^2$

i.e $x^2 + y^2 - 2px + p^2 - r^2 = 0$ (05)

Since this circle intersects orthogonally the circle

$x^2 + y^2 - 8x - 6y + 21 = 0$ We have

$2(-p)(-4) = 21 + p^2 - r^2$

i.e $r^2 = p^2 - 8p + 21 \rightarrow (01)$ (05)

Since this circle touches the circle

$x^2 + y^2 + 4x + 6y + 9 = (x + 2)^2 + (y + 3)^2 - 4\ 0$

We have $|\pm r + 2| = \sqrt{[p - (-2)]^2 + [0 - (-3)]^2}$

$= \sqrt{(p + 2)^2 + 9} \rightarrow (02)$ (05)

$r^2 \pm 4r = p^2 + 4p + 9 \rightarrow (03)$ (05)

(03) - (01) $\Rightarrow$ $r = \pm 3(p - 1) \rightarrow (04)$ (05)

From (01) We have $p = 2$ or $\frac{-3}{4}$ (05)

If $p = 2$ then from (04) with plus sign we have $r = 3$ and

if $p = \frac{-3}{4}$ (05)

Then from (04) with minus sign we have $r = \frac{21}{4}$ (05)

$\therefore$ There are two such circles.

When $p = 2$ and $r = 3$ from (01) with plus sing

We have

$3 + 2 = \sqrt{(2 + 2)^2 + 9} = 5$ (05)

i.e. the sum of the two radius = the distance between the two centres. (05)

$\therefore$ The two circles touch each other ternally. (05)

$\therefore$ The equation of the circle is

$(x - 2)^2 + y^2 = 9$

i.e. $y^2 + x^2 - 4x - 5 = 0$ (05)

When $p = \frac{-3}{4}$ and $r = \frac{21}{4}$ from (01) with minus sign we have

$\left|\frac{21}{-4} + 2\right| = \sqrt{\left(-\frac{3}{4} - (-2)\right)^2 + [0 - (-3)]^2} = \frac{13}{4}$ (05)

i.e. the difference of the two radius = the distance between the two centres. (05)

$\therefore$ The two circles touch each oter internally. (05)

$\therefore$ The equation of the circle is

$\left(x + \frac{3}{4}\right)^2 + y^2 = \left(\frac{21}{4}\right)^2$ 80

i.e. $2y^2 + 2x^2 + 3x - 54 = 0$ (05)

**(08)** **(a)**

$$\frac{1+\cos\theta+\sin\theta}{1-\cos\theta+\sin\theta} = \frac{(1+\cos\theta+\sin\theta)^2}{(1+\sin\theta)^2-\cos^2\theta} \quad (05)$$

$$= \frac{(1+\sin\theta)^2+2\cos\theta(1+\sin\theta)+\cos^2\theta}{2\sin\theta(1+\sin\theta)} \quad (05)$$

$$= \frac{(1+\sin\theta)+2\cos\theta+1-\sin\theta}{2\sin\theta} \quad (05)$$

$$= \frac{1+\cos\theta}{\sin\theta} \quad (05)$$

20

**(b)** $8(\cos^6 x + \sin^6 x)$

$= 8[\sin^6 x + (1 - \sin^2 x)^3]$ (05)

$= 8[1 - 3\sin^2 x + 3\sin^4 x]$ (05)

$= 8 - 24\sin^2 x(1 - \sin^2 x)$ (05)

$= 8 - 24\sin^2 x\cos^2 x$ (05)

$= 8 - 6\sin^2 2x$ (05)

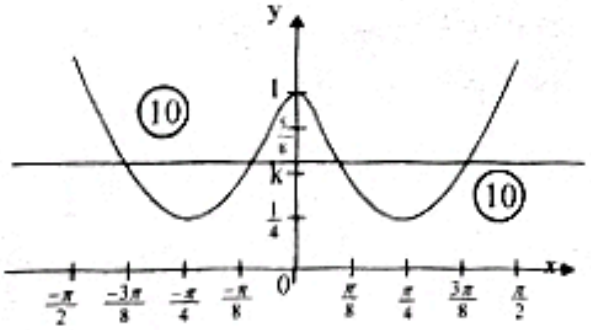
$= 8 - 3(1 - \cos 4x)$ (05)

$= 5 + 3\cos 4x$

30

The graph of $y = (\cos^6 x + \sin^6 x)$ is the same as the graph of

$y = \frac{5}{8} + \frac{3}{8} = \cos 4x$ (10)




30

(i) If $k < \frac{1}{4}$ or $k > 1$ the graphs of $y = (\cos^6 x + \sin^6 x)$ and $y = k$ do not intersect each other and hence $(\cos^6 x + \sin^6 x) = k$ has no real solution (05)

(ii) If $k = \frac{1}{4}$ then the straight line $y = k$ is a tangent to the curve $y = (\cos^6 x + \sin^6 x)$ at $x = -\frac{\pi}{4}$ and $\frac{\pi}{4}$

$\therefore$ $(\cos^6 x + \sin^6 x) = k$ has two real solutions. (05)

(iii) If $k = 1$ then the straight line $y = k$ is a tangent to the curve $y = (\cos^6 x + \sin^6 x)$ at $x = \frac{-\pi}{2}, 0$ and $\frac{\pi}{2}$

$\therefore$ $(\cos^6 x + \sin^6 x) = k$ has three real solutions. (05)

(iv) If $\frac{1}{4} < k < 1$ the graphs of $y = (\cos^6 x + \sin^6 x)$ and $y = k$ intersect each other at the four point and hence $(\cos^6 x + \sin^6 x) = k$ has four real solutions. (05)

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**(09)** $4\sin^2 x + 12\sin x\cos x - \cos^2 x + 5$

$= 2(1 - \cos 2x) + 6\sin 2x - \left(\frac{\cos 2x + 1}{2}\right) + 5$

$= 12\sin 2x - 5\cos 2x + 13$ (05)

$= 0 \Rightarrow \frac{5}{13}\cos 2x - \frac{12}{13}\sin 2x$

$= 1$

Let

$\cos\alpha = \frac{5}{13}$ *and* $\sin\alpha = \frac{12}{13}$ $\cos\alpha > 0$ *and* $\sin > 0$

$\therefore 0 < \alpha < \frac{\pi}{2}$ (05)

$\cos\alpha\cos 2x - \sin\alpha\sin 2x = 1$ i.e. $\cos(2x + \alpha) = 1$ (05)

$\therefore$ $2x + \alpha = 2n\pi + 0$ Where $n \in \mathbb{Z}$

i.e. $x = n\pi - \frac{\alpha}{2}$ (05)

When n = 1, $x = \pi - \frac{\alpha}{2}$ and $0 < \pi - \frac{\alpha}{2} < 2\pi$ (05)

n = 2, $x = 2\pi - \frac{\alpha}{2}$ and $0 < 2\pi - \frac{\alpha}{2} < 2\pi$ (05)

These are the only two solutions that lie betweem 0 and $2\pi$.

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Sine Rule : $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ (05) 05

Cosine Rule : $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ (05) 05

$\frac{b+c}{2k-1} = \frac{c+a}{2k} = \frac{a+b}{2k+1}$

$\frac{b+c}{c+a} = \frac{2k-1}{2k}$ (05) *and* $\frac{a+c}{b+a} = \frac{2k}{2k+1}$ (05)

$\frac{b+c}{a-b} = \frac{2k-1}{1}$ (05) *and* $\frac{a+c}{b-c} = \frac{2k}{1}$ (05)

$\frac{b+c}{2k-1} = a-b = \frac{a+b}{2k+1}$ (05) *and* $\frac{a+c}{2k} = b-c = \frac{b+c}{2k-1}$ (05)

$ka = b(k+1)$ (05) *and* $b(k-1) = kc$ (05)

Hence we have $\frac{a}{k+1} = \frac{b}{k} = \frac{c}{k-1}$

Applying sine Rule we have $\frac{\sin A}{k+1} = \frac{\sin B}{k} = \frac{\sin C}{2k+1}$

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$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{k^2 + (k-1)^2 - (k+1)^2}{2k(k-1)}$ (05)

$= \frac{k-4}{2(k-1)}$ (05)

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$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{(k+1)^2 + (k-1)^2 - k^2}{2k(k-1)}$

$= \frac{k^2+2}{2(k^2-1)}$ (05)

$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{(k+1)^2 + k^2 - (k-1)^2}{2k(k+1)}$

$= \frac{k+4}{2(k+1)}$ (05)

$\frac{\cos A}{(k-4)(k+1)} = \frac{\cos B}{k^2+1} = \frac{\cos C}{(k+4)(k-1)}$ 10