Combined Mathematics II – 2000 Advanced Level Examination

❖ Answer six questions only.

(01) (a) A train, 100 metres long, starts from rest at a station A and moves with constant acceleration. Later, it takes 10 seconds to pass a signal post B. The train is moving with velocity 11 ms^{-2} when the rear of the train passes B. Draw a velocity-time graph for the motion of the train. Using this graph or otherwise,

(i) find the velocity with which and the time at which the front of the train passed B.

(ii) find the acceleration of the train and show that the total distance travelled is 302.5 metres, when the rear of the train is at B.

(b) Water in the straight river of breadth d flows with uniform velocity u. A man capable of swimming at a speed v relative to the water swims so that he moves right across the river perpendicular to the bank. Find the time T taken by the man to cross the river. Show that the time he takes to swim a distance d parallel to the bank, upstream and back to the starting place is $\frac{2vT}{\sqrt{v^2-u^2}}$ .Why should v be greater than u?

(02) (a) A smooth wedge of mass M and angle α is placed on a fixed smooth plane of inclination α to the horizontal, in such a way that the upper face of the wedge is horizontal; on this horizontal face is placed a particle of mass m, and the system is released from rest. Write down the equations of motion, to determine the accelerations of the particle and the wedge. Prove that the acceleration of the particle is of magnitude $\frac{(M+m)gsin^2\alpha}{M+msin^2\alpha}$. What is its direction?

(b) An engine working at constant power 500 kW is pulling a train up an incline of 1 in 196 (measured along the track). The total mass of the train with the engine is 2.5×10^5 kg. When, its speed is 24 kmh^{-1} the acceleration is 0.2 ms^{-2}. Find the constant resistance against the motion of the train, in newtones.

[Take acceleration due to gravity, $g = 9.8\ ms^{-2}$]

(03) A particle P, free to move on the smooth inner surface of a fixed hollow sphere of centre O and internal radius a is placed at the lowest point A of this surface. It is then projected horizontally with initial speed $\sqrt{nga}$, where $n > 0$.

While the particle is still in contact with the surface, find the reaction on the particle from the surface, when the angle turned by OP is θ.

Show that, if $2 < n < 5$ the particle P will leave the surface with speed $\sqrt{\frac{(n-2)ga}{3}}$.

If P leaves the surface when it is at a height $\frac{a}{2}$ above the level of O, show that

(i) $n = \frac{7}{2}$,

(ii) the path of P in the subsequent free motion under gravity passes through A.

(04) (a) A small smooth sphere A of mass m moving on a smooth horizontal table with velocity u collides directly with another small smooth sphere B, of equal size and mass $2m$, which is at rest on the table. The coefficient of restitution is e.

(i) Show that the velocity acquired by B is $(1+e)\frac{u}{3}$, and find the impulse J between the spheres.

(ii) Express the loss of kinetic energy due to the impact in the form $E = \frac{J}{2}(1+e)u$.

(iii) If the direction of motion of A gers reversed due to the impact, show that $e > \frac{1}{2}$ and $E < \frac{1}{2}mu^2$

(b) A block of mass m on a horizontal platform is at relative rest, while the platform perform vertical simple harmonic oscillations of amplitude a and period T. Show that, when the displacement of the platform from its mean position is x, measured in the upward vertical direction, the reaction of the platform on the block is $m\left(g - \frac{4\pi^2 x}{T^2}\right)$.

If $T = 1s$, deduce the greatest amplitude, in metres, so that the block is not thrown off the platform.

[Assume that $\pi^2 \Omega 9.8$ and that the acceleration due to gravity g, takes the same value in ms^{-2}]

(05) A system of coplanar forces consists of three forces. (measured in newtons) acting respectively at points specified as follows:

Point	Position Vector	Force
A	$2i + 5j$	$P(i + 3j)$
B	$4j$	$-P(2i + j)$
C	$-i + j$	$P(i - 2j)$

Here i, j denote unit vectors along rectangular Cartesian axes Ox, Oy respectively, the unit of length being the metre.

Mark these forces, in component form in a representative diagram, indicating the coordinates of the respectively points of application.

Hence or otherwise, show that the system is equivalent to a couple of moment $10\ PNm$, and indicate the sense of the couple.

D is the point with position vector $2i$. The given system of forces can be brought to equilibrium by introducing three additional forces along the sides of the triangle OAD, taken in order and proportional to their lengths. Find these forces in vector form.

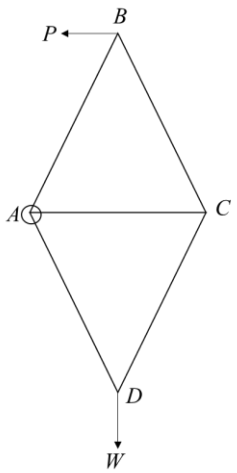
(06) (a) A uniform rob AB of weight W and length $2a$ is kept in equilibrium with the end A in contact with a rough vertical wall; it is supported by a light inextensible string of equal length $2a$ connecting the other end B to a point C of the wall, vertically above A. The rod is inclined at an angle θ to the upward vertical and it lies in a vertical and it lies in a vertical place perpendicular to the wall.

Find the tension in the string and show that $\theta \geq \cot^{-1}\left(\frac{\mu}{3}\right)$ where μ is the coefficient of friction.

(b) Show, by integration that the distance of the centre of gravity of a uniform solid hemisphere whose plane base is of radius a, from the base is $\frac{3a}{8}$.

A uniform solid body is formed by welding together, at coincident bases of radii a, a hemisphere and a right circular cone of semi-vertical angle α. If the body can rest in equilibrium with any point of the curved surface of the hemisphere in contact with a horizontal table, find the value of α.

(07) The adjoining figure represents a framework consisting of five smoothly jointed light rods of equal length. The framework is smoothly hinged at a fixed point A, and carries a load W at D; it is kept in equilibrium in a vertical plane with AC horizontal by a force P applied at B in a direction parallel to CA.



Find the magnitude of P and the horizontal and vertical components of the reaction at the hinge A. Deduce the direction of this reaction.

Draw a stress diagram for the framework, using Bow's Notation. Hence determine the stresses in all the five rods in terms of W, distinguishing between tensions and thrusts.

(08) (a) A and B are two random events such that $P(A \cap B) = \frac{1}{4}$ and $P(A) = (\mathrm{A}|\mathrm{B}') = \frac{5}{12}$, where B' is the complementary event of B.

Find

(i) $P(\mathrm{B}|\mathrm{A})$

(ii) $P(B)$

(iii) $P(\mathrm{A}|\mathrm{B})$ and

(iv) $P(A \cup B)$.

Are the events A and B mutually exclusive? Are they independent? Justify your answer in each case.

(b) "The birth of a boy' and "The birth of a girl" are assumed to be equally likely and independent events. In a family of two children, it is given that one child is a boy. Find the probability that the other child is

(i) also a boy,

(ii) a girl.

(09) The following table gives a grouped frequency distribution of the lifetimes of a random sample of 200 electric bulbs taken from the output of a particular factory.

Life time (in weeks)	Number of bulbs
95-99	10
90-94	14
85-89	16
80-84	21
75-79	35
70-74	41
65-69	38
60-64	15
55-59	7
50-54	3

(a) Estimate, to one decimal place,

(i) the median,

(ii) the lower quartile Q_1

(iii)the upper quartile Q_2 of these lifetimes.

(b) Evaluate, to one decimal place,

(i) the mean,

(ii) the standard deviation,

(iii) the coefficient of skewness of this distribution.

What is the shape of this distribution?