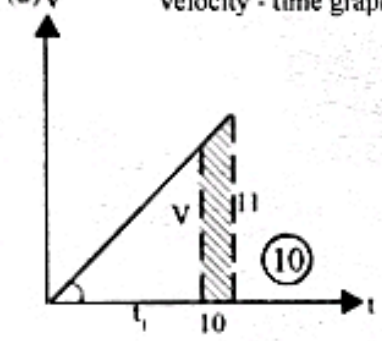
G.C.E. (Advanced Level) Examination - August 2000
10 - Combined Mathematics - II
Provisional Scheme of Marking

(01) (a)

(a) V — Velocity - time graph



10 10

Let V be the speed of the train as its front passes the signal post B.

(i) Then, the length of the train $= \frac{1}{2}(V+11)\times 10$

$= 100$ (10)

$\Rightarrow V = 9ms^{-1}$ (05)

If t_1 is the time, gradient of the graph

$= \frac{9}{t_1} = \frac{11}{t_1+10} \Rightarrow t_1 = 45\sec$ (05)

(05)

(ii) Acceleration $= \frac{9}{45} = \frac{1}{5} = 0.2ms^{-2}$ (05)

(05)

Distance $= \frac{1}{2}\times 11(45+10) = \frac{605}{2} = 302.5m$

(05) 40

Alternative Method :

Using $S = \frac{1}{2}(U+V)$, $100 = \frac{10}{2}(V+11) \Rightarrow$ (05)

(10)

Required velocity $V = 9$ ms-1

Using $V = u + at$, $9 = at_1$ and $11 = -9 + a \times 10$ (05)

(05) $a \Leftrightarrow 0.2ms^{-2}$ (05)

$t_1 = \frac{9}{0.2} = 45\sec$ (05)

Using $S = ut + \frac{1}{2}at^2$, required distance $= \frac{1}{2}(0.2)(55)^2$

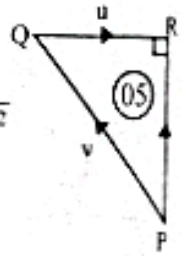
$= 302.5m$ (05)

40

(b) Velocity of man, **relative** to water = v;

Velocity of water, **relative** to bank = u →

Vel. M, B = ↑ = M, W + WB (05)



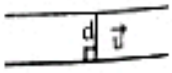
(05)

$= \overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR} = \sqrt{v^2-u^2}$ (05) (05)

Time $T = \frac{d}{\sqrt{v^2-u^2}}$ (05)

25

Time for Upstream and downstream swimming a distance d is



$\frac{d}{v-u} + \frac{d}{v+u} = \frac{2dv}{v^2-u^2}$ (05)

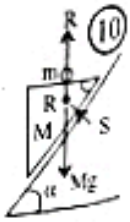
(05) (05)

$= \frac{2v}{\sqrt{v^2-u^2}} \cdot \frac{d}{\sqrt{v^2-u^2}} = \frac{2vT}{\sqrt{v^2-u^2}}$

(05)

The man can cross the river only if V > U. (05) 25

(02) (a)



Acceleration of m, M = f→

Acceleration of M, E = $\overrightarrow{\alpha F}$ (05)

Equations of Motion :

For m →0 = m (f - Fcosα)→(01) (10)

$\downarrow$ $mg - R = mF\sin\alpha \rightarrow (02)$ (10)

for M $\swarrow$ $(Mg + R)\sin\alpha = MF \rightarrow (03)$ (10)

(02) $\sin\alpha$ + (03) $\Rightarrow (M + m)\, g\sin\alpha = (M + m\sin^2\alpha)\, F \rightarrow (04)$ (05)

Acceleration of m, E = m, M + M, E

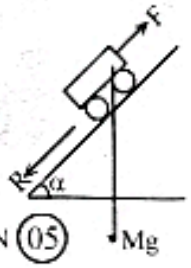
$= \overline{(f - F\cos\alpha)} + F\sin\alpha \downarrow$ (05)

$= (F\sin\alpha) \downarrow$ (05) using (01)

$= \dfrac{(M+m)\, g\sin^2\alpha}{M + m\sin^2\alpha}$. (05)

Vertically down wards. [65]

(b) Speed $v = \dfrac{24000}{3600} = \dfrac{20}{3}\, ms^{-1}$ (05)



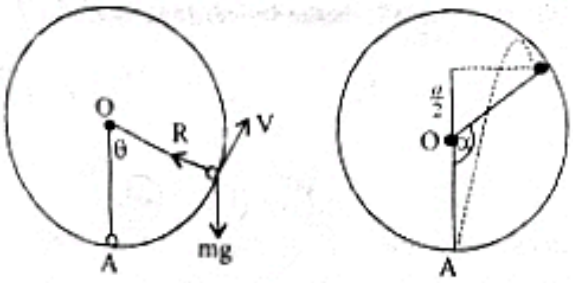
Tractive force $F = \dfrac{H}{V} = \dfrac{500,000}{20/3} = 75000N$ (05) (05)

$F - (R + Mg\sin\alpha) = M(0.2)$ (05)

$R = 75,000 - 250,000\left(0.2 + \dfrac{9.8}{196}\right) = 12,500N$ (05) (05) (05)

[35]

(03)



Energy conservation equation is

$\dfrac{1}{2}mv^2 - mga\cos\theta = \dfrac{1}{2}mu^2 - mga$ (10)

$\Rightarrow v^2 = ga(n - 2 + 2\cos\theta)$ (10)

$R - mg\cos\theta = m\dfrac{v^2}{a} \rightarrow R = mg[n - 2 + 3\cos\theta]$ (05) (05)

[30]

Particle leaves the surface when R = 0 (05)

$\cos\theta = \dfrac{(2-n)}{3} = \cos\alpha$ (05)

Then $v^2 = \dfrac{(n-2)ga}{3}$ (05) and α is an obtuse angle. (05)

[20]

Provided $2 < n < 5$

$\alpha = \pi - \dfrac{\pi}{3}, \cos\alpha = -\dfrac{1}{2} \Rightarrow \dfrac{n-2}{3} = \dfrac{1}{2}, n = \dfrac{7}{2}$ (05) (05) (05)

[15]

Then $v^2 = \dfrac{ga}{2}$ (05), and the time T taken to reach the vertical line OA is given by

T.V $\cos\dfrac{\pi}{3} = \dfrac{a\sqrt{3}}{2} \Rightarrow T = a\sqrt{3}\sqrt{\dfrac{2}{ga}} = \sqrt{\dfrac{6a}{g}}$ (05) (05) (05)

The vertical distance traveled, during time T is

$= -\sqrt{\dfrac{ga}{2}}\dfrac{\sqrt{3}}{2}T + \dfrac{1}{2}gT^2$ (05)

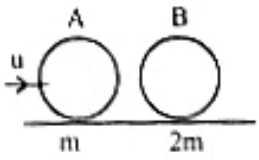
$= -\dfrac{3a}{2} + 3a$ (05)

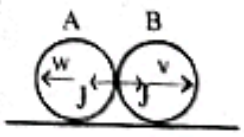
$= \dfrac{3a}{2}$

$\therefore$ Path of p Passes through A. (05)

[35]

(04)





Momentum conservation equation is $2mV - mw = mu$ (05)

$\Rightarrow 2v - w = u \rightarrow (01)$

Newton's law of restitution gives : $V + W = eu \rightarrow (02)$ (05)

From (01) & (02) Velocity v of B and W of A are.

$$v=\frac{(1+e)u}{3},\ W=\frac{(2e-1)u}{3}\rightarrow(03)\ (05)$$

The impulse

$$J-2mv=\frac{2mu(1+e)}{3}\ (05)\quad (iv)\ Also\ \ J=m(w+u)\ (05)$$

25

Loss of kinetic energy, $E=\frac{1}{2}m\left(u^2-W^2\right)-\frac{1}{2}(2m)V^2$ (05)

$$=\frac{1}{2}m[u-w](u+w)-\frac{1}{2}(2mv)v \quad (05)$$

$=\frac{J}{2}(u-w)-\frac{J}{2}$ v, using (04) & (05) and so (05)

$E=\frac{J}{2}u(1-e)=\frac{1}{3}mu^2(1-e^2)$ using (04) (05)

20

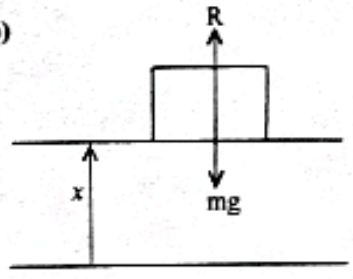
If the direction of motion of A gets reversed $W>0$ (05)

$\Rightarrow 2e-1>0$, or $e>\frac{1}{2}$, using (03) and (05)

$E=\frac{1}{3}mu^2(1-e^2)<\frac{1}{3}\left(1-\frac{1}{4}\right)mu^2=\frac{1}{4}mu^2$ (05) (05)

20

(b)



Equation of motion is

$R-mg=m\ddot{x}$ (05)

$=m\left(-\omega^2x\right)$, Where $\frac{2\pi}{\omega}$ (05)

$=T$ (05)

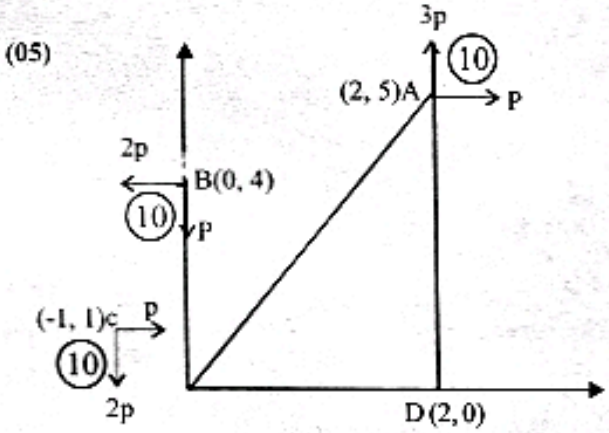
Hence $R=m\left[g-\frac{4\pi^2}{T^2}x\right]$ (05) R takes

its least value when $x=a$, so the block will not be thrown off the platform, if $m\left[g-\frac{4\pi^2}{T^2}a\right]\geq 0\Rightarrow a\leq 0.25m$, (05) (05)

provided that $\pi^2\equiv 9.8$ and $g=9.8ms^{-2}$.

35

(05)



Resolving along OX

→ x = p + p - 2p = O (05)

along OY ↑

y = 3p - p - 2p = 0 (05)

Hence the system is equivalent to a couple, of moment

G = (2 x3p) - (5 x p) + (4 x 2p) + (1 x 2p) - (1 x p) (10)

= 10P Nm, (05) in the anticlockwise sense (05)

60

Additional forces are $\lambda\overrightarrow{OA}$ (05), $\lambda\overrightarrow{AD}$ (05), $\lambda\overrightarrow{DO}$ (05) acting along the respective sides of the triangle OAD.

The resultant of the first two forces $=\lambda\overrightarrow{OD}$, acting at A.

The three forces therefore form a couple of moment

$= 2 \times 2\lambda$, in the clockwise sense. (05)

Hence $10P-10\lambda=0\Rightarrow\lambda=P$ (05)

The forces are $P\overrightarrow{OA}=p(2\underline{i}+5\underline{i})$, (05)

$P\overrightarrow{AD}=-5P\underline{j}$ and $P\overrightarrow{DO}=-2P\underline{i}$ (05) (05)

40

Second Part (Alt Method)

Additional forces are $(2\underset{\sim}{i} + 5\underset{\sim}{j})$ Q acting along $\overrightarrow{OA}$ (05)

$-5Q\underset{\sim}{j}$ acting $\overrightarrow{AD}$ (05)

$-2Q\underset{\sim}{j}$ acting along $\overrightarrow{DO}$ (05)

Moment of former couple is G = 10P Nm in the anticlockwise sence.

There fore, in order to bring the system to equilibrium, moment of additional forces about O_D

$G^1 = 2(-5Q) = -10P$ Nm $\Rightarrow Q = P$ (05) (05)

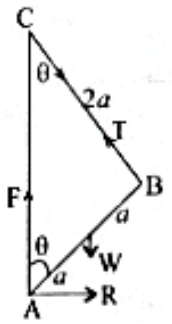
There fore forces required are

$(2\underset{\sim}{i}+5\underset{\sim}{j})p$ *along* $\overrightarrow{OA}$ (05)

$-5P\underset{\sim}{j}$ along $\overrightarrow{AD}$ *and* (05)

$-2P\underset{\sim}{j}$ along $\overrightarrow{DO}$ (05) 40

(06) (a)



Resolving horizontally

$\rightarrow T\sin\theta = R \rightarrow (01)$ (05)

Resolving vertically

$\uparrow\ F + T\cos\theta = W \rightarrow (02)$

Taking the moment about (05)

$\overset{A}{\curvearrowleft}\quad T\cdot 2a\sin 2\theta = W\,a\sin\theta$ (10)

$$\Rightarrow T = \frac{W}{4}\sec\theta \rightarrow (03)$$ (05)

Using (01) and (03) $R = \frac{W}{4}\tan\theta$ (05)

Using (02) and (03) $F = \frac{3}{4}W$ (05)

For equilibrium $\frac{F}{R} \le \mu$ (05)

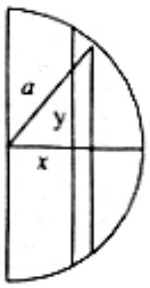
Therefore, $3\cot\theta \le \mu$ or $\cot\theta \le \frac{\mu}{3}$ (05)

Since $\cot\theta$ decreases as θ increases (When acute)

$\theta \ge \cot^{-1}\left(\frac{\mu}{3}\right)$ (05)

50

(b)



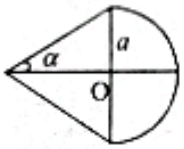
Let the distance of the centre of gravity from the plane base be $\bar{x}$.

$$\bar{x} = \frac{\delta\int_0^a x\pi y^2\,dx}{\delta\int_0^a \pi y^2\,dx}$$ (05) (05)

Where $y^2 = a^2 - x^2$ and δ is density.

$$\bar{x} = \frac{\int_0^a x\left(a^2-x^2\right)dx}{\int_0^a\left(a^2-x^2\right)dx} = \frac{\left[\frac{a^2x^2}{2} - \frac{x^4}{4}\right]_0^a}{\left[a^2x - \frac{x^3}{3}\right]_0^a} = \frac{\frac{a^4}{4}}{\frac{2a^3}{3}}$$ (05) (05)

There fore, $\bar{x} = \frac{3a}{8}$ 25



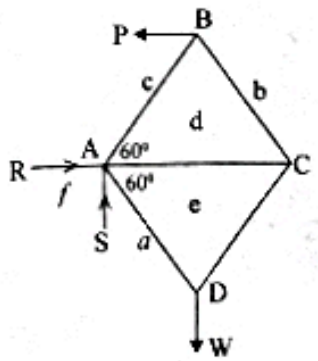
Since the distance of the centre of gravity of the right circular cone of semi vertical angle α is $\left(\frac{a}{4}\right)$ cot α, from o,

$\overset{o}{\curvearrowleft}\quad 0 = \frac{3}{8}a\frac{2}{3}\pi a^3\rho - \frac{a}{4}\cot\alpha\frac{\pi}{3}a^2(a\cot\alpha)$ (15)

Simplifying $\cot^2\alpha = 3$ or $\tan\alpha = \frac{1}{\sqrt{3}}$ (05)

That is $\alpha = \frac{\pi}{6}$ or $\alpha = 30^0$ (05) 25

(07)



For AB

Resolving horizontally $\rightarrow R = P$ (05)

Taking the moment about $\overleftarrow{A}\curvearrowright P$ $a\sqrt{3} = Wa$

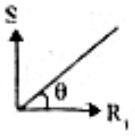
Where $2a$ is the lenghth of each rod.

$$\Rightarrow P = \frac{W}{\sqrt{3}} \quad (05)$$

For AD

Resolving vertically $\uparrow S = W$ (05)

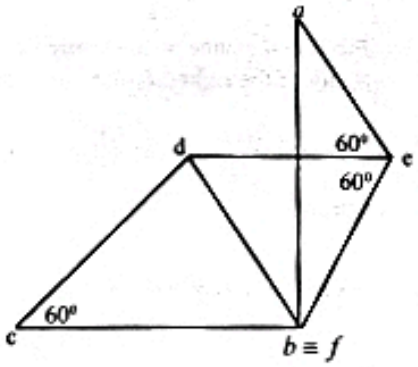
Since $R = P$, $R = \frac{W}{\sqrt{3}}$ (05)



Inclination of reaction θ is given by

$$\tan\theta = \frac{S}{R} = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3} \quad (05)$$ (05)

Therefore, reaction is along the rod AB. [30]

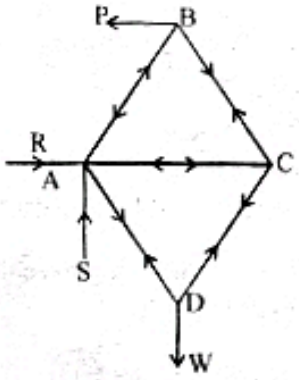


(20)

$$ae = eb = \left(\frac{W}{2}\right)\left(\frac{2}{\sqrt{3}}\right) = \frac{W}{\sqrt{3}}$$

$$cd = db = cb = \frac{W}{\sqrt{3}}$$

$$de = db = \frac{W}{\sqrt{3}}$$ [20]



Rod	Stress	Magnitude
AB	Thrust	$W/\sqrt{3}$ (10)
AC	Thrust	$W/\sqrt{3}$ (10)
AD	Tension	$W/\sqrt{3}$ (10)
BC	Tension	$W/\sqrt{3}$ (10)
CD	Tension	$W/\sqrt{3}$ (10)

[50]

(08) (a) (i) $P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{5}{12}} = \frac{3}{5}$ (10) [10]

(ii) $P(A/B') = \frac{P(A\cap B')}{P(B')}$ and thus

$$P(A\cap B') = \frac{5}{12}P(B') \rightarrow (01) \quad (05)$$

$A = (A\cap B)\cup(A\cap B')$ AND $(A\cap B)\cup(A\cap B') = \phi$

$\therefore P(A) = P(A\cap B) + P(A\cap B')$

$\therefore P(A\cap B') = P(A) - P(A\cap B) = \frac{5}{12} - \frac{1}{4} = \frac{1}{6} \rightarrow (02)$ (05)

From (01) & (02) we have $P(B') = \frac{2}{5}$ and thus $P(B) = \frac{3}{5}$

(05) (05)

[20]

(iii) $P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{4}}{\frac{3}{5}}=\frac{5}{12}$ (10)

10

(iii) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$P(A\cup B)=\frac{5}{12}+\frac{3}{5}-\frac{1}{4}=\frac{23}{30}$ (10)

10

A and B are not mutually exclusive (05)

05

For $P(A\cap B)=\frac{1}{4}\neq 0\Rightarrow (A\cap B)\neq\phi$ (05)

05

A and B are independent. (05)

05

For $P(A)P(B)=\frac{5}{12}\times\frac{3}{5}=\frac{1}{4}=P(A\cap B)$ (05)

05

(b) Define B = the birth of a boy, G = The birth is a girl

A = one of the children is a boy and

C = The other one is a boy

$P(B)=P(G)=\frac{1}{2}$ (05)

$P(A)=P(BB)+P(BG)+P(GB)$

$=\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}=\frac{3}{4}$ (05)

$P(A\cap C)=P(BB)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$ (05)

$P(C/A)=\frac{P(A\cap C)}{P(A)}=\frac{\frac{1}{4}}{\frac{3}{4}}=\frac{1}{3}$ (05)

20

$P(C'/A)=1-P(C/A)=\frac{2}{3}$ (10)

10

(09)

Life time	No. of bulbs (f)	cumulative frequenxy (c.f)
50 - 54	3	3
55 - 59	7	10
60 - 64	15	25
65 - 69	38	63
70 - 74	41	104
75 - 79	35	139
80 - 84	21	160
85 - 89	16	176
90 - 94	14	190
95 - 99	10	200
Total	200	

(05)

(i) Median : Med = Size of $\frac{200th}{2}$ item = 100th item

$\therefore$ Median lies in the class 70 - 74 (05)

But the real limit of this class is 69'5 - 74'5

$$\text{Med} = L+\left[\frac{\frac{N}{2}-c.f}{f}\right]\times i$$

$$=69.5+\frac{\frac{200}{2}-63}{41}\times 5 \text{ (05)}$$

$$=74.01\approx 74.0 \text{ (05)}$$

20

(ii) The lowest quartile = Q_1 = size of $\frac{200th}{4}$ item

= 50th item

$\therefore$ Q_1 lies in the class 65 - 69. But the real limit of this class is 64.5 - 69.5 (05)

$$Q_1=64.5+\frac{\frac{200}{4}-25}{38}\times 5=67.79\approx 67.8 \text{ (05)}$$

(05)

iii) The upper quartile = Q_3 = size of $\frac{3(200)}{4}th$ item

$= 150^{th}$ item

$\therefore$ Q_3 lies in the class 80 - 84, But the real limit of this class is79.5 - 84.5 (05)

$$Q_3 = 79.5 + \frac{150-139}{21} \times 5 = 82.12 \approx 82.1$$ (05) (05)

15

(b)

Life time	Mid-point (x)	$\frac{x-72}{5} = d$	f	fd	fd^2
50 - 54	52	-4	3	-12	48
55 - 59	57	-3	7	-21	63
60 - 64	62	-2	15	-30	60
65 - 69	67	-1	38	-38	38
70 - 74	72	0	41	0	0
75 - 79	77	1	35	35	35
80 - 84	82	2	21	42	84
85 - 89	87	3	16	48	144
90 - 94	92	4	14	56	224
95 - 99	97	5	10	50	250
Total			200	130	946

(05) (05)

(05)

Mean = $\bar{x} = 72 + \frac{130}{200} \times 5$ (05)

$= 75.25 \approx 75.3$ (05)

25

Standard deviation = $\sqrt{\frac{946}{200} - \left(\frac{130}{200}\right)^2} \times 5$ (05)

$= 10.377 \approx 10.4$ (05)

10

Coefficient of Skewness = $\frac{3(mean - median)}{\sigma}$

$= \frac{3(75.25-74)}{10.377} \approx 0.4$ (05)

(05)

10

The shape of the distribution is positive skewed. (05) 05